OLUME 107 PART A NUMBER 35 OCTOBER 1960

The Proceedings

OF

THE INSTITUTION OF ELECTRICAL ENGINEERS

FOUNDED 1871: INCORPORATED BY ROYAL CHARTER 1921

PART A

POWER ENGINEERING

SAVOY PLACE · LONDON W.C.2

Price Ten Shillings and Sixpence

THE INSTITUTION OF ELECTRICAL ENGINEERS

FOUNDED 1871 INCORPORATED BY ROYAL CHARTER 1921

PATRON: HER MAJESTY THE QUEEN

COUNCIL 1960–61

President

SIR HAMISH D. MACLAREN, K.B.E., C.B., D.F.C.*, LL.D., B.Sc.

Past-Presidents

W. H. ECCLES, D.Sc., F.R.S.
THE RT. HON. THE EARL OF MOUNT EDGCUMBE, T.D.
J. M. DONALDSON, M.C.
PROF. E. W. MARCHANT, D.Sc.
H. T. YOUNG.
SIR GEORGE LEE, O.B.E., M.C.
J. R. BEARD, C.B.E., M.Sc.
SIR NOEL ASHBRIDGE, B.Sc.(Eng.).
SIR HARRY RAILING, D.Eng.
P. DUNSHEATH, C.B.E., M.A., D.Sc. (Eng.), LL.D.
SIR VINCENT Z. DE FERRANTI, M.C.
T. G. N. HALDANE, M.A.
PROF. E. B. MOULLIN, M.A., Sc.D., LL.D.
SIR ARCHIBALD J. GILL, B.Sc.(Eng.).
SIR JOHN HACKING.
COL. B. H. LEESON, C.B.E., T.D.
SIR HAROLD BISHOP, C.B.E., B.Sc.(Eng.), F.C.G.I.
SIR JOSIAH ECCLES, C.B.E., D.Sc.
THE RT. HON. THE LORD NELSON OF STAFFORD.
SIR W. GORDON RADLEY, K.C.B., C.B.E., Ph.D.(Eng.).
S. E. GOODALL, M.Sc. (Eng.), F.Q.M.C.
SIR WILLIS JACKSON, D.Sc., F.R.S.

Vice-Presidents

B. DONKIN, B.A. O. W. HUMPHREYS, C.B.E., B.Sc. G. S. C. LUCAS, O.B.E., F.C.G.I. C. T. MELLING, C.B.E., M.Sc.Tech. A. H. MUMFORD, O.B.E., B.Sc.(Eng.).

Honorary Treasurer

C. E. STRONG, O.B.E., B.A., B.A.I.

Ordinary Members of Council

J. C. ARKLESS, B.Sc.
PROF. H. E. M. BARLOW, Ph.D., B.Sc. (Eng.).
D. A. BARRON, M.Sc.
C. O. BOYSE, B.Sc.(Eng.).
F. H. S. BROWN, C.B.E., B.Sc.
PROF. M. W. HUMPHREY DAVIES, M.Sc.
SIR JOHN DEAN, B.Sc.
L. DRUCQUER.
J. M. FERGUSON, B.Sc.(Eng.).
D. C. FLACK, B.Sc.(Eng.), Ph.D.
R. J. HALSEY, C.M.G., B.Sc.(Eng.), F.C.G.I.
R. A. HORE, M.A., B.Sc.
J. S. MCCULLOCH.
PROF. J. M. MEEK, D.Eng.
THE HON. H. G. NELSON, M.A.
H. V. PUGH.
J. R. RYLANDS, M.Sc., J.P.
R. L. SMITH-ROSE, C.B.E., D.Sc., Ph.D.
G. A. V. SOWTER, Ph.D., B.Sc.(Eng.).
H. G. TAYLOR, D.Sc.(Eng.).
D. H. TOMPSETT, B.Sc.(Eng.).

Chairmen and Past-Chairmen of Sections

Electronics and Communications:
T. B. D. TERRONI, B.Sc.
†M. J. L. PULLING, C.B.E., M.A.

Measurement and Control:
C. G. GARTON.
†PROF. A. TUSTIN, M.Sc.

Supply:
J. E. L. ROBINSON, M.Sc.
†J. R. MORTLOCK, Ph.D., B.Sc.(Eng.).

Utilization:
J. M. FERGUSON, B.Sc.(Eng.).
†T. E. HOUGHTON, M.Eng.

Chairmen and Past-Chairmen of Local Centres

East Midland Centre:
LT.-COL. W. E. GILL, T.D.
†D. H. PARRY, B.Sc.

North-Western Centre:
F. LINLEY.
†F. J. HUTCHINSON, M.Eng.

Mersey and North Wales Centre:
D. A. PICKEN.
†T. A. P. COLLEDGE, B.Sc.(Eng.).

Northern Ireland Centre:
J. MCA. IRONS.
†T. S. WYLIE.

Southern Centre:
R. GOFORD.
†W. D. MALLINSON, B.Sc.(Eng.).

North-Eastern Centre:
D. H. THOMAS, M.Sc.Tech., B.Sc.(Eng.).
†H. WATSON-JONES, M.Eng.

Scottish Centre:
R. B. ANDERSON.
†J. A. AKED, M.B.E.

Western Centre:
A. C. THIRTLE.
†H. JACKSON, B.Sc.(Eng.).

North Midland Centre:
F. W. FLETCHER.
†PROF. G. W. CARTER, M.A.

South Midland Centre:
BRIGADIER F. JONES, C.B.E., M.Sc.
†G. F. PEIRSON.

† Past Chairman.

MEASUREMENT AND CONTROL, SUPPLY AND UTILIZATION SECTION COMMITTEES 1960–1961

Measurement and Control Section

Chairman
C. G. GARTON.

Vice-Chairmen
W. S. ELLIOTT, M.A.; A. J. MADDOCK, D.Sc.

Past-Chairmen
PROFESSOR A. TUSTIN, M.Sc.; J. K. WEBB, M.Sc.(Eng.), B.Sc.Tech.

Ordinary Members of Committee

S. S. CARLISLE, M.Sc.
W. J. JEFFERSON.
C. A. LAWS.
A. C. LYNCH, M.A., B.Sc.
R. E. MARTIN.
A. NEMET, Dr.Sc.Techn.
S. N. POCOCK.
W. RENWICK, M.A., B.Sc.
G. F. TAGG, Ph.D., B.Sc.(Eng.)
R. D. TROTTER, B.Sc.(Eng.).
J. H. WESTCOTT, B.Sc.(Eng.), Ph.D.
F. C. WIDDIS, B.Sc.(Eng.), Ph.D.

And

The President (*ex officio*).
The Chairman of the Papers Committee.
G. A. V. SOWTER, Ph.D., B.Sc.(Eng.) (representing the Council).
C. C. BAXENDALE (representing the North-Eastern Measurement and Electronics Group).
A. CHORLTON, B.Sc.Tech. (representing the North-Western Measurement and Control Group).
D. L. A. BARBER, B.Sc.(Eng.) (nominated by the National Physical Laboratory).
H. M. GALE, B.Sc.(Eng.) (representing the South Midland Electronics and Measurement Group).
W. H. P. LESLIE, B.Sc. (representing the Scottish Electronics and Measurement Group).

Supply Section

Chairman
J. E. L. ROBINSON, M.Sc.

Vice-Chairmen
J. S. FORREST, D.Sc., M.A.; D. B. IRVING, B.Sc.; C. H. FLURSCHIEM, B.A.

Past-Chairmen
J. R. MORTLOCK, Ph.D., B.Sc.(Eng.); D. P. SAYERS, B.Sc.

Ordinary Members of Committee

E. S. BOOTH, M.Eng.
G. S. BUCKINGHAM, B.Sc.(Eng.).
W. J. CARFRAE, B.Sc.
PROF. M. W. HUMPHREY DAVIES, M.Sc.
J. L. EGGINTON, B.Sc.
C. J. O. GARRARD, M.Sc.
W. D. HORSLEY.
D. MCDONALD, B.Sc.
O. W. MINSHULL, B.Sc.
N. C. SIMPSON.
F. C. WALMSLEY.
T. W. WILCOX.

And

The President (*ex officio*).
The Chairman of the Papers Committee.
H. V. PUGH (representing the Council).
J. B. KILSHAW the North-Western Supply Group.
E. V. HARDAKER, M.C., B.Sc. (representing the South Midland Supply and Utilization Group).
A. H. MCQUEEN (representing the Western Supply Group).

Utilization Section

Chairman
J. M. FERGUSON, B.Sc.(Eng.).

Vice-Chairmen
H. G. TAYLOR, D.Sc.(Eng.); W. G. THOMPSON, Ph.D., B.Sc.

Past-Chairmen
T. E. HOUGHTON, M.Eng.; R. A. MARRYAT, B.Sc.(Eng.).

Ordinary Members of Committee

J. R. ANDERSON, T.D., B.Sc.(Eng.).
S. L. M. BARLOW.
J. W. BINNS.
J. E. BOUL, A.C.G.I.
J. A. BROUGHALL, B.Sc.(Eng.).
C. M. COCK.
O. T. EVANS.
A. V. LAWRY.
P. M. MARTIN, B.Sc.
A. V. MILTON.
PROF. G. H. RAWCLIFFE, M.A., D.Sc.
R. F. RICHARDSON, M.B.E.
J. TOZER.
C. D. WILKINSON.

And

The President (*ex officio*).
The Chairman of the Papers Committee.
B. L. METCALF, B.Sc.(Eng.) (representing the Council).
E. HAWKESLEY (representing the Western Utilization Group).
C. AYERS, B.Sc.(Eng.) (representing the North-Western Utilization Group).
W. PATERSON, B.Sc. (representing the North Midland Utilization Group).

The following nominees of Government Departments:

Admiralty: P. SMITH, B.Sc.
Air Ministry: H. F. INNOCENT.
Ministry of Labour and National Service (Factory Dept.): S. J. EMERSON, M.Eng.
Ministry of Works: M. WOOLFSON
War Office: C. L. CHAMPION, B.Sc.

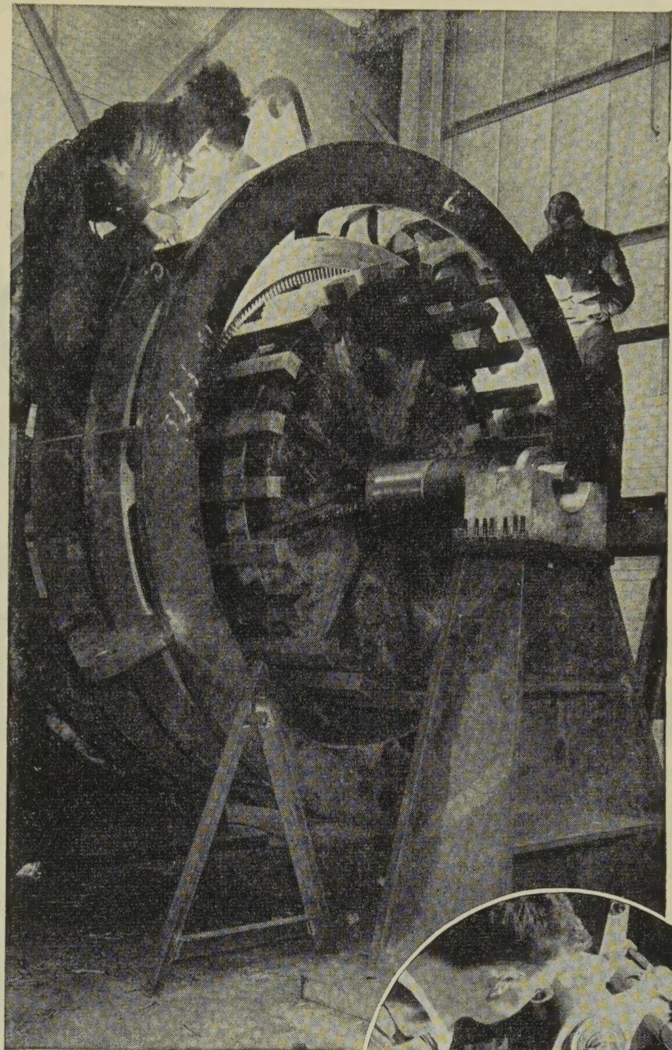
Secretary

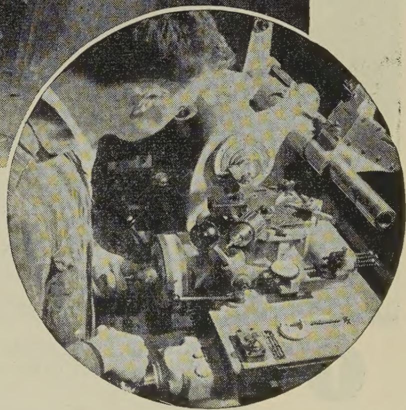
W. K. BRASHER, C.B.E., M.A., M.I.E.E.

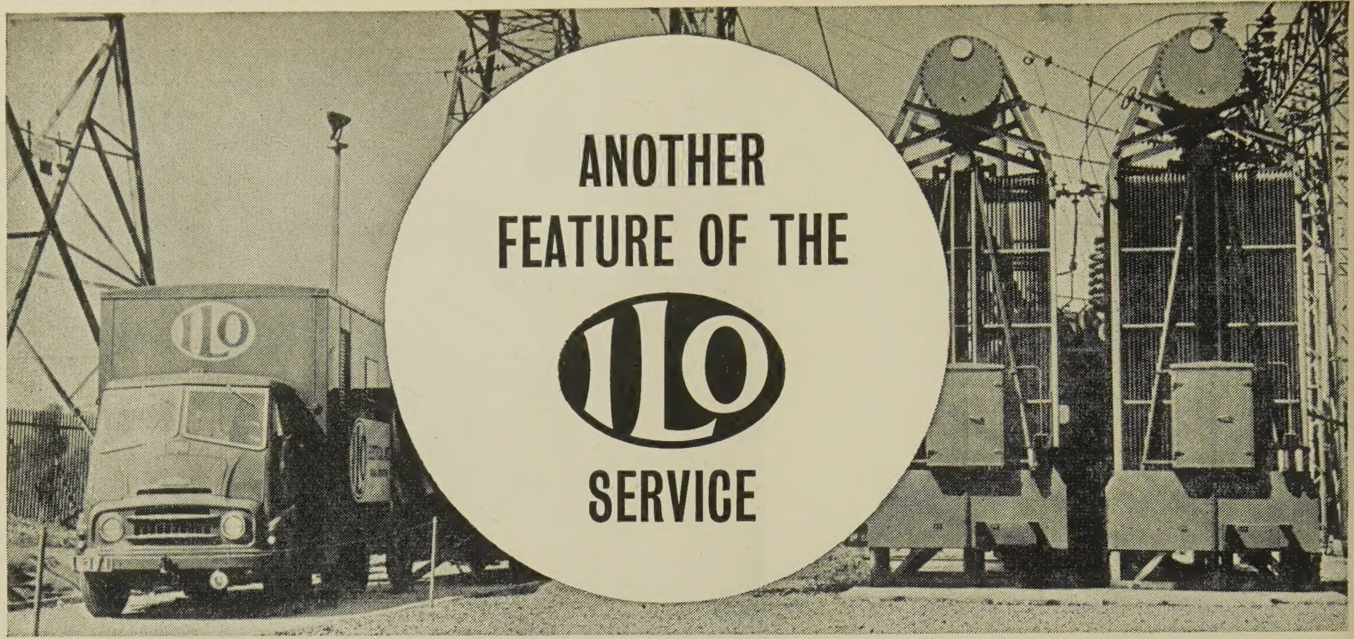
Deputy Secretary
F. JERVIS SMITH, M.I.E.E.

Principal Assistant Secretary
F. C. HARRIS.

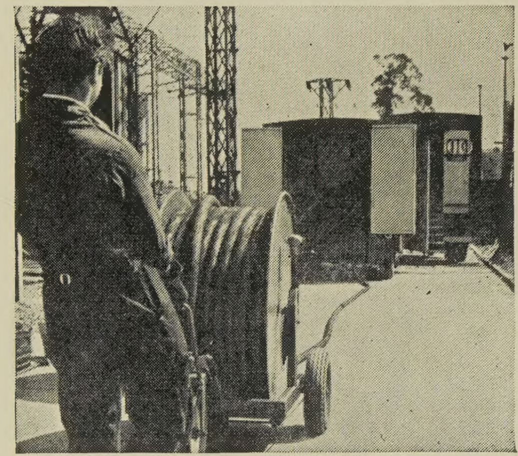
Editor-in-Chief
G. E. WILLIAMS, B.Sc.(Eng.), M.I.E.E.



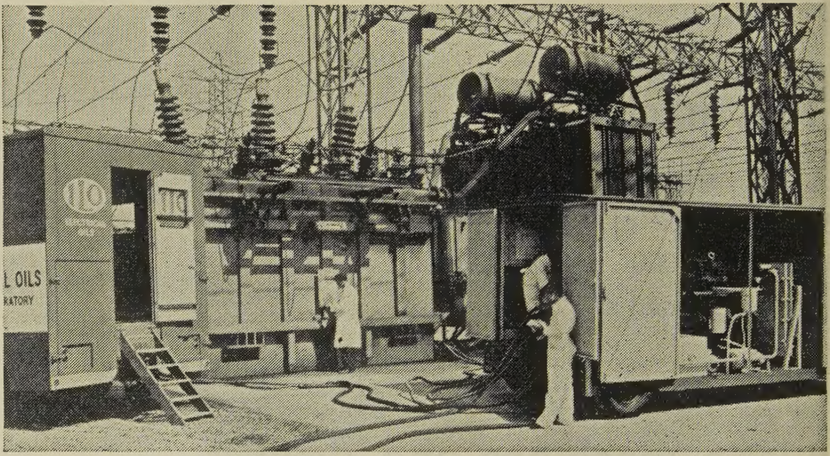




Treatment of electrical oils on site



1 Arrival on site of the ILOVAC and the laying out of its power cable for connection to supply.



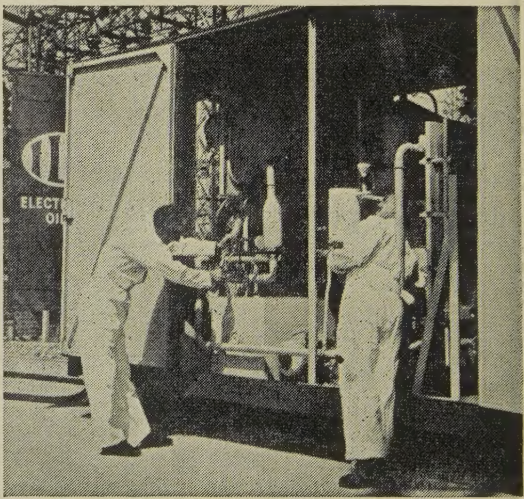
2 The ILOVAC at work, drying the transformer by circulating super dried oil. The Mobile Laboratory, which controls the operation, is seen in the left foreground.

The I L O Service is a complete and comprehensive one. It will guarantee the safe and efficient running of your transformers by keeping their insulating oil always in perfect physical and chemical condition. The Wakefield-Dick electrical engineers are always at your service. Their advice can be invaluable.

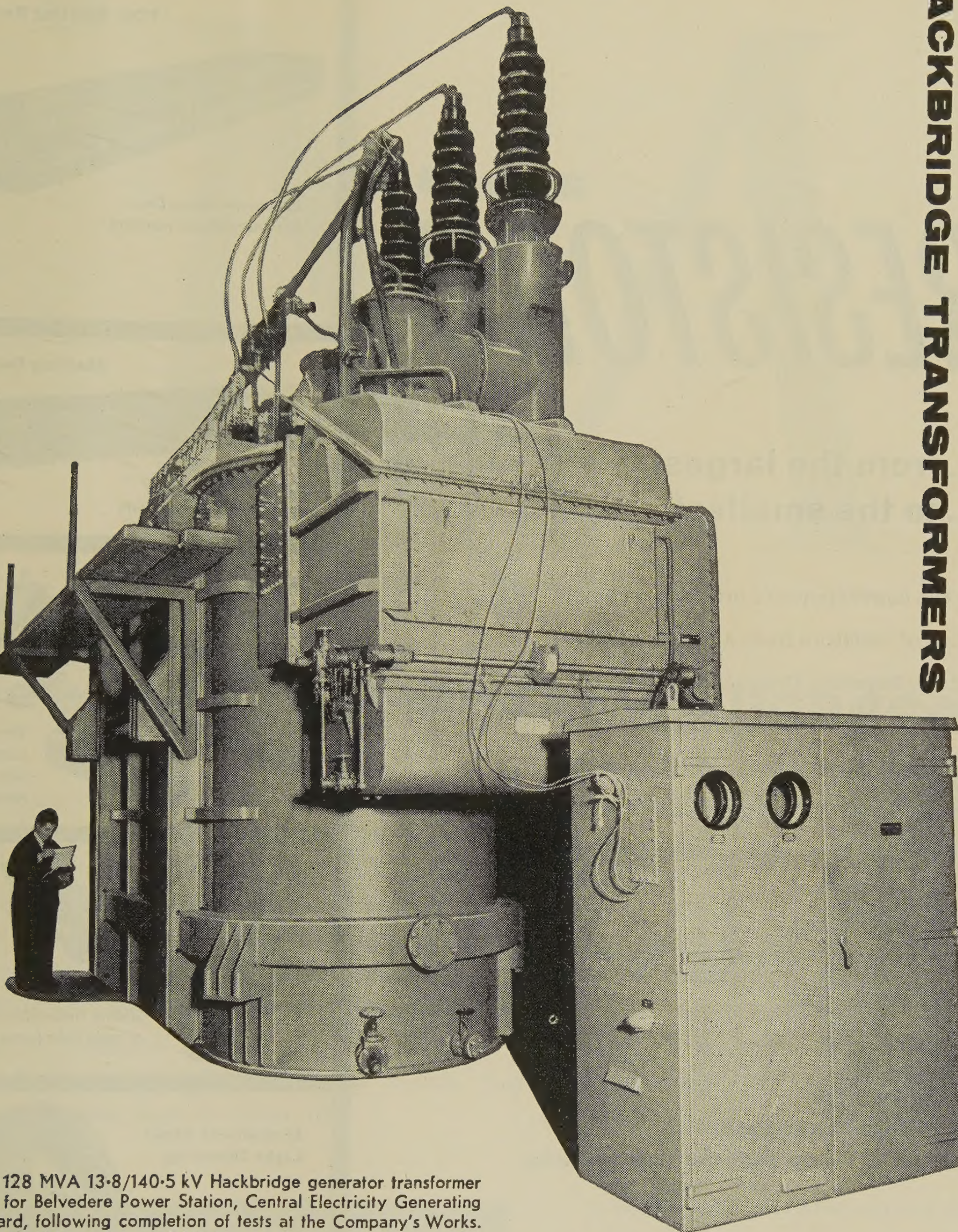
ELECTRICAL OILS

WAKEFIELD-DICK INDUSTRIAL OILS LIMITED
Castrol House, Marylebone Rd, London, N.W.1 Telephone: HUNter 4455

Member of the Castrol Group of Companies



3 Two of the ILOVAC operators drawing samples of the treated oil for check tests in the Mobile Laboratory during treatment.



RESISTORS

...from the largest ✳ ...to the smallest

To the convenience of ordering a complete range of resistors from a single reliable source, Expamet Cressall are able to add the advantages of technical experience unequalled in this field. Highly qualified Expamet Cressall engineers are freely available to translate this experience into practice.

✳ *From over 20,000 Kilowatts to as low as 4 watts.*

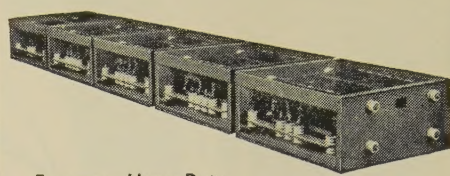
EXPAMET CRESSALL
—go together all the way

Expamet Cressall

The Electrical Division of The Expanded Metal Company Ltd.

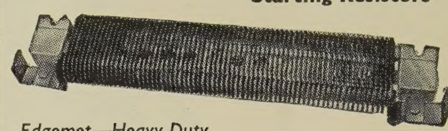
London Office: 16 Caxton Street, London, S.W.1
Telephone: ABBey 7766
Works: Stranton Works, West Hartlepool.
Telephone: Hartlepools 5531
The Cressall Manufacturing Company Ltd., Eclipse Works, Tower Street, Birmingham 19. Telephone: Aston Cross 2666

Main Starting Resistors



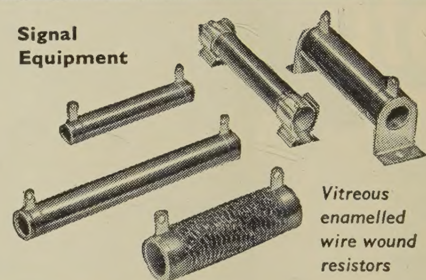
Expamet—Heavy Duty Expanded Metal resistors

Starting Resistors

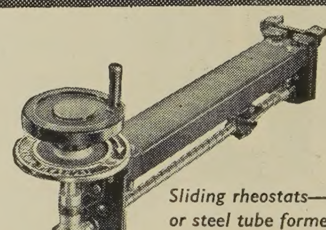


Edgemet—Heavy Duty edge wound strip resistors

Signal Equipment

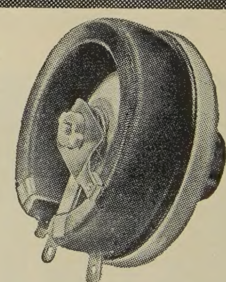


Vitreous enamelled wire wound resistors

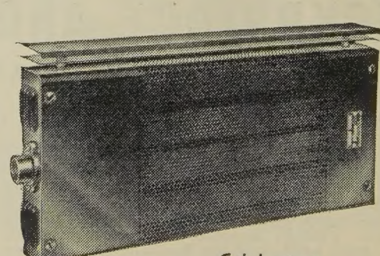


Sliding rheostats—slate or steel tube formers

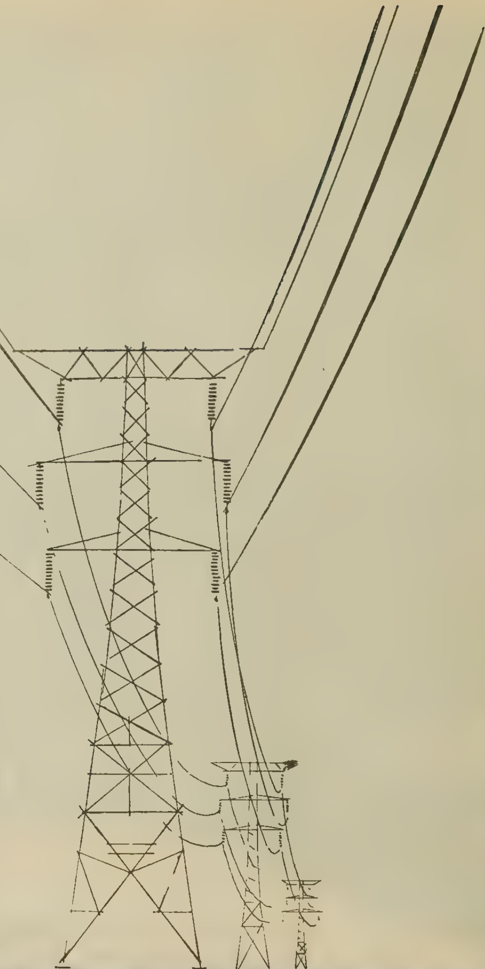
Instrument Panel Light Dimming



Toroidally wound rheostats—vitreous enamelled or open wound

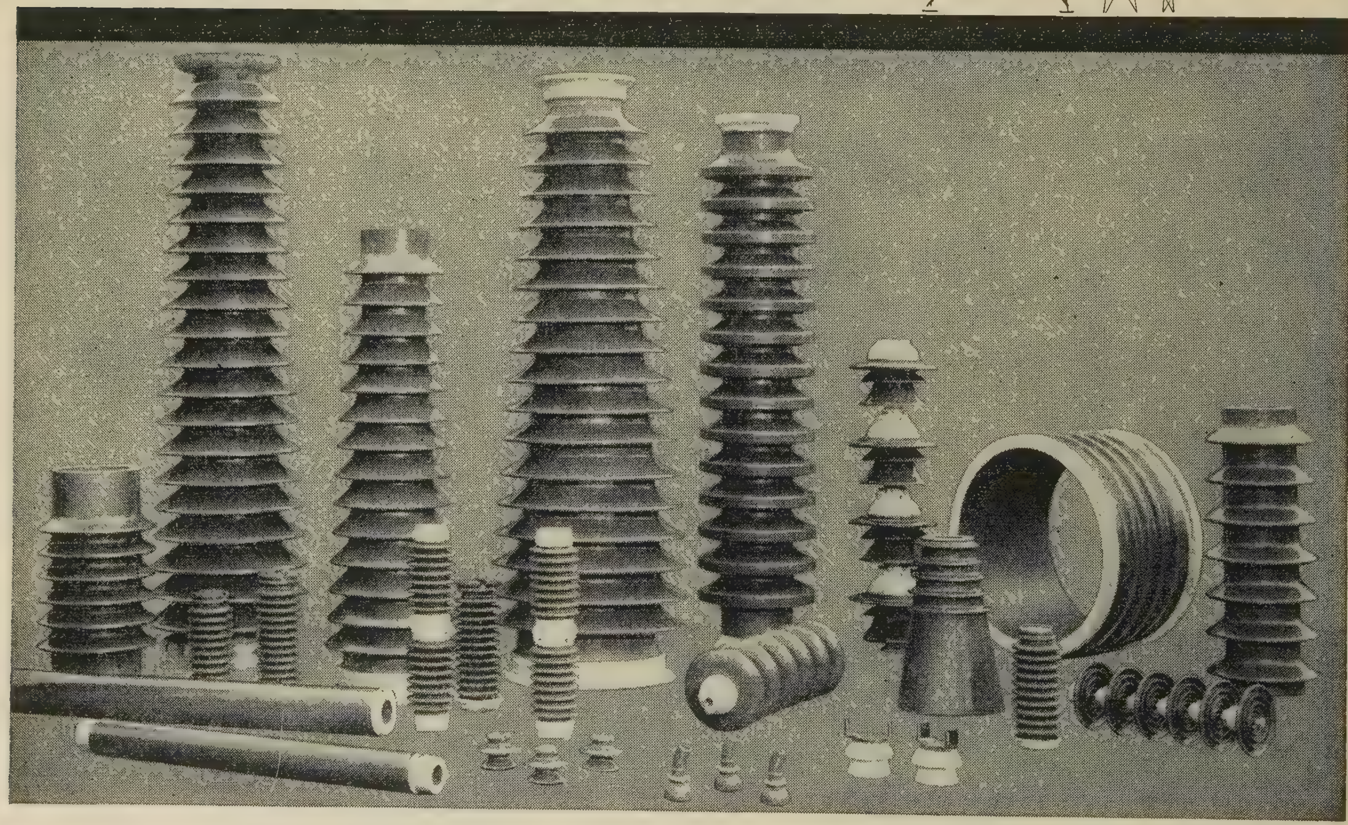


Cab heaters



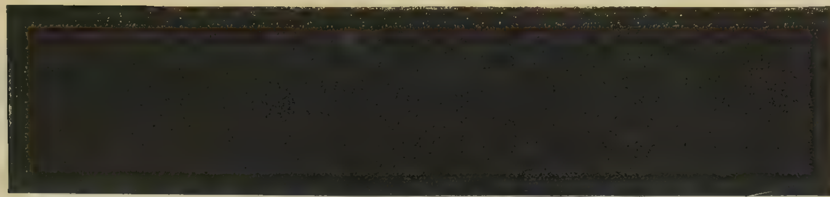
Positive proof all over the world

Wherever the giant transmission lines stride across the land,
porcelain insulation is likely to be in evidence.
And the chances are that this porcelain insulation is by Taylor, Tunnicliff.
A few of the large variety of insulators we have
the capacity and knowledge to produce are shown in the illustration below.
No insulation problems are too difficult for us
to consider. Most of them we can solve.



TAYLOR TUNNICLIFF & CO. LTD

Head Office: EASTWOOD · HANLEY · STOKE-ON-TRENT · Tel: STOKE-ON-TRENT 25272-5
London Office: 125 HIGH HOLBORN · LONDON W.C.1 · Tel: HOLBORN 1951-2



THEY ARE OFTEN CHEAPER

First consider all the expenditure incumbent upon the installation of oil-filled transformers. Special bunkers and fireproof vaults are usually needed—and special fire-fighting equipment. The installation is often located a considerable distance from the load centre—which implies expensive low-voltage cable runs.

Now consider the very considerable cost-cutting advantages of Class C and Class H transformers as demonstrated for instance at the Kent factory of Medway Paper Sacks Ltd, a member of the Reed Paper Group. Here a 750 kVA 3-phase air natural cooled transformer, built by Ferranti Ltd, has been neatly mounted within the roof truss space. Space limitations —making it undesirable to build an adjoining substation for a Class A unit—together, of course, with freedom from fire hazard, were the major considerations. As a Group technician pointed out, the transformer's low weight enabled it to be sited thus, on a moderately-sized platform, making it possible to run 'a very nice low-voltage distribution' to individual machines without floor excavations to accommodate long, costly cable runs.
And so, in simple indisputable terms, it often costs less to have all the advantages of a Class C or a Class H installation.

Midland Silicones Ltd supply the silicone resins and elastomers used in the manufacture of Class C and Class H transformers. Here is a list of well-known British manufacturers producing silicone-insulated dry-type transformers for the United Kingdom and overseas.

Associated Electrical Industries (Manchester) Ltd
Bonar, Long & Co Ltd
Brentford Transformers Ltd
Brush Electrical Engineering Co Ltd
Bryce Electric Construction Co Ltd
Crompton Parkinson Ltd
The English Electric Co Ltd
Ferranti Ltd
Foster Transformers Ltd
The General Electric Co Ltd
Gresham Transformers Ltd
Hackbridge & Hewittic Electric Co Ltd
London Transformer Products Ltd
Bruce Peebles & Co Ltd
South Wales Switchgear Ltd
Transformers (Watford) Ltd
Woden Transformer Co Ltd
The Yorkshire Electric Transformer Co Ltd

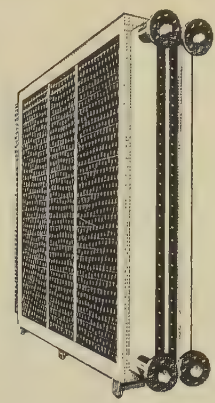
MIDLAND SILICONES LTD

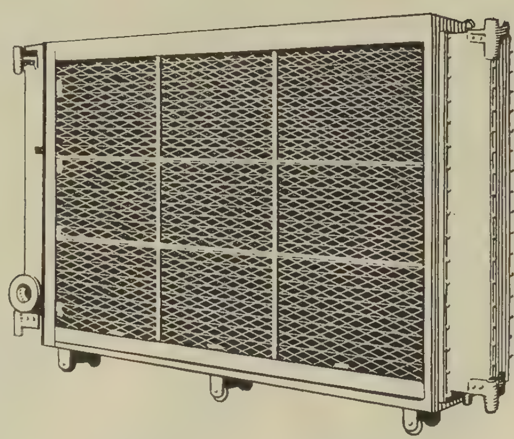
(Associated with Albright & Wilson Ltd and Dow Corning Corporation)

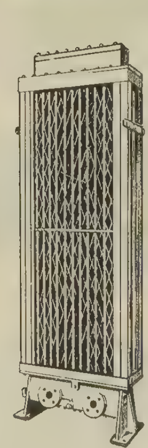
first *in British Silicones*

68 KNIGHTSBRIDGE · LONDON · SW1 · TELEPHONE: KNIGHTSBRIDGE 7801

Area Sales Offices: Birmingham, Glasgow, Leeds, London, Manchester · Agents in many countries MSEI a







for cables in a

hot spot silastomer*

provides the best protection

IN MANY CABLE INSTALLATIONS, the ability to withstand high temperatures is a consideration of prime importance.

Silastomer* silicone rubber insulation permits continuous long-term operation in ambient temperatures up to 200°C. Glass braided silicone rubber cables will continue to operate through and after a fire.

Silastomer insulated cables are resistant to corona discharge, weathering, low pressure steam, water and many chemicals and are flexible at temperatures as low as —70°C.

Silastomer insulated cables are manufactured by:

Associated Electrical Industries (Woolwich) Ltd
British Insulated Callender's Cables Ltd
Enfield Standard Power Cable Co Ltd
W. T. Glover & Co Ltd
Greengate and Irwell Rubber Co Ltd
Johnson & Phillips Ltd
Pirelli-General Cable Works Ltd
Rist's Wire & Cables Ltd
Sterling Cable Co Ltd
St. Helens Cable & Rubber Co Ltd

* *Silastomer is the registered trade name of a comprehensive range of silicone rubbers manufactured and marketed by*

(MS) MIDLAND SILICONES LTD

(Associated with Albright & Wilson Ltd and Dow Corning Corporation)

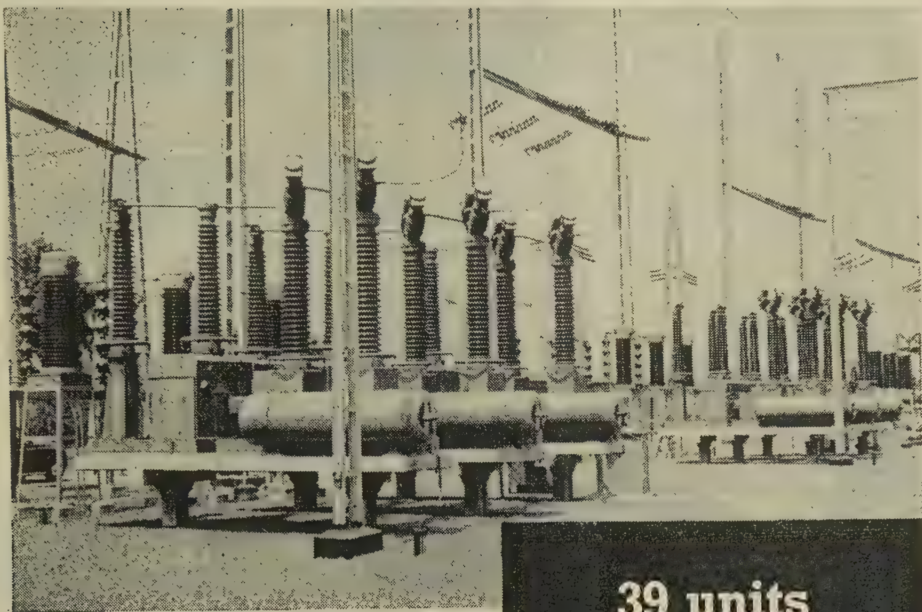
first *in British Silicones*

68 KNIGHTSBRIDGE · LONDON · SW1 · TELEPHONE: KNIGHTSBRIDGE 7801

Area Sales Offices: Birmingham, Glasgow, Leeds, London, Manchester
Agents in many countries

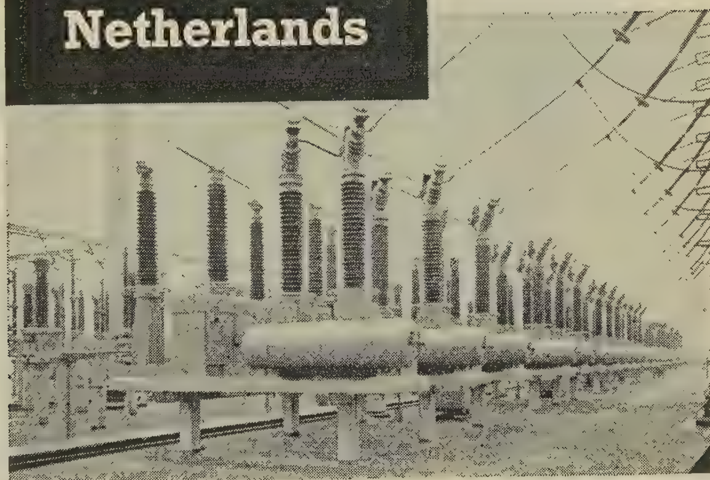
MSR2a

Buggenum power-station N.V. Provinciale Limburgsche Electriciteits-Maatschappij



39 units in the Netherlands

Centrale Hemweg Gemeentelijk Electriciteitsbedrijf Amsterdam



150-kV 7500 MVA TYPE-OB30

AIR-BLAST CIRCUIT-BREAKERS

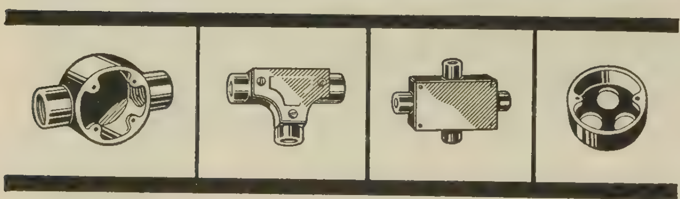
Reyrolle

A. Reyrolle & Co. Ltd - Hebburn - County Durham - England

"All round" cable protection

METALLIC Conduit Tubes and Fittings give "all round" protection to electric cables. They prevent damage from impact or abrasion... and there is a finish and size to meet every need.

Choose METALLIC for dependability.

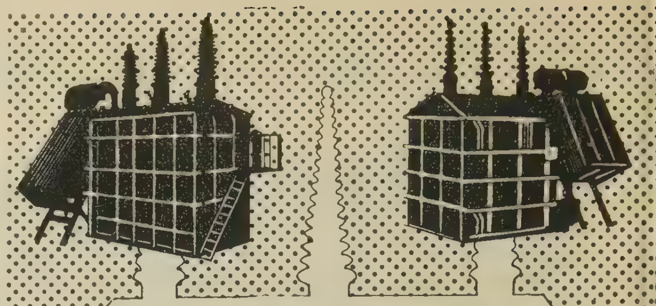


METALLIC

CONDUIT TUBES & FITTINGS

A TI COMPANY

THE METALLIC SEAMLESS TUBE CO. LTD,
LUDGATE HILL, BIRMINGHAM 3
Also at London, Newcastle-on-Tyne, Leeds, Swansea and Glasgow



LOBITOL

TRANSFORMER OIL

Dussek Bros. are the distributors of "Lobitol" Transformer Oil for Lobitos Oilfields Ltd., the pioneer producers of transformer oil in Great Britain. "Lobitol" quality complies in full measure with B.S. requirements and satisfies those of many foreign countries. Quality control guards "Lobitol" from the oil wells to transformer installations wherever they may be.

You can depend on Dussek

DB ESTD 1850

Please write to Advisory Dept. P.E.4.

DUSSEK BROTHERS & CO., LTD.

THAMES ROAD, CRAYFORD, KENT
Telephone: CRAYFORD 22322

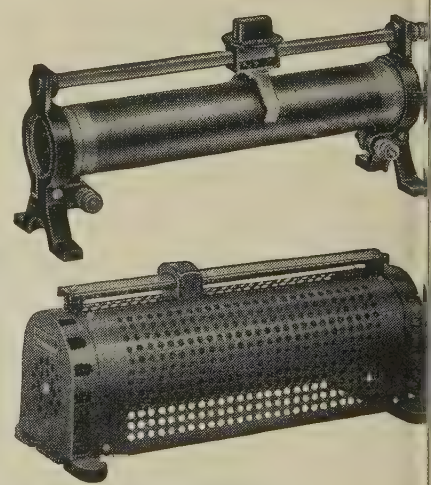
ZENITH

(REGD. TRADE-MARK)

TUBULAR SLIDING RESISTANCES

Wound on vitreous enamelled steel tubes with oxidised wire having a negligible temperature co-efficient. Non-inductive windings on refractory formers are also available in many sizes.

Illustrated catalogue of all types free on request



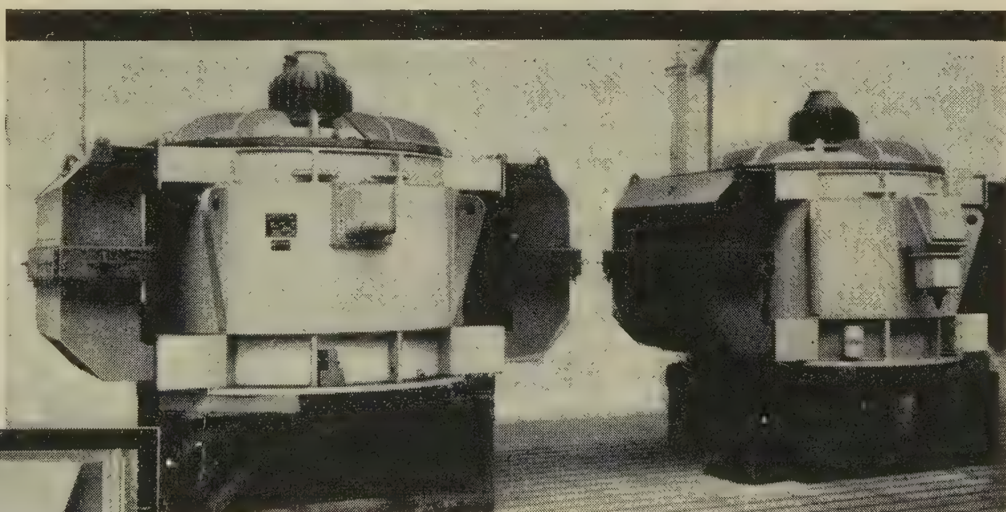
The ZENITH ELECTRIC CO. Ltd

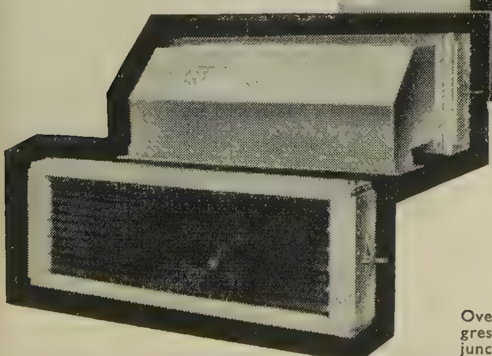
ZENITH WORKS, VILLIERS ROAD, WILLESDEN GREEN
LONDON, N.W.2

Telephone: WILlesden 6581–5 *Telegrams:* Voltaohm, Norphone, London

MANUFACTURERS OF ELECTRICAL EQUIPMENT INCLUDING RADIO AND TELEVISION COMPONENTS

INDEX OF ADVERTISERS





The Institution is not, as a body, responsible for the opinions expressed by individual authors or speakers.
An example of the preferred form of bibliographical references will be found beneath the list of contents.

THE PROCEEDINGS OF THE INSTITUTION OF ELECTRICAL ENGINEERS

EDITED UNDER THE SUPERINTENDENCE OF W. K. BRASHER, C.B.E., M.A., M.I.E.E., SECRETARY

VOL. 107. PART A. No. 35. OCTOBER 1960

621.317.785

The Institution of Electrical Engineers
Paper No. 3343
Oct. 1960

ELECTRICITY METERS

A Review of Progress

By G. F. SHOTTER, Member.

(1) INTRODUCTION

It is now twenty-three years since the last Progress Review on electricity meters was published by The Institution,* and consequently it might be expected that some changes and progress had been made in the design and performance of such instruments.

Apart from any electrical and mechanical improvement in the actual meter which has taken place, the publication in 1952 of Parts 1 and 2 of the revised British Standard No. 37, 'Electricity Meters', has led to the standardization of fixing and terminal dimensions and also to the reduction in the number of sizes of single-phase meters. This was made possible by the extension of the current range to 60/1. A review of the various changes will be given in the next Section.

It should be noted that, whereas the previous editions of B.S. 37 applied to all types of meter, the various Parts of the new edition deal with particular classes, the exception being Part 1, which consists of general clauses applicable to all types. This is a more convenient and logical arrangement than having a single Standard covering all classes.

(2) CHANGES DUE TO THE REVISED BRITISH STANDARD FOR SINGLE-PHASE METERS

(2.1) Extension of Accuracy and Load Range

In the previous specification two accuracy load ranges were provided, i.e. 1/20 to 125% of the marked current, and the so-called 'long range' from 1/20 to 200%. This was very inconvenient, particularly from the users' point of view. It led to difficulties in choosing the right size of meter for any given consumer load conditions, and where the two different ranges were in use, led to a more frequent change of meter with expanding loads and consequently to increased costs.

The new accuracy range of 60/1 allows a greater flexibility in the choice of a meter for any given load condition. For those not too familiar with the problem, one may compare the new ranges with the old ones: a marked 5 amp meter, for instance, under the old specification but having a load range of 60/1, would give an accuracy range from 1/20 of marked current to three times marked current, i.e. 15 amp, and would be capable of carrying this current continuously. A certain number of meters manufactured under the old specification were capable of such performance.

(2.2) Reduction in Number of Sizes

The three sizes laid down in the new specification are 10, 40 and 80 amp, which are maximum current ratings, compared with the old marked ratings of 2½, 5, 10, 20/25, 50 and 100 amp.

The choice of the three new ratings was agreed upon between manufacturers and users and was based on some preliminary investigation into the characteristics of the various types of consumer loads. Knowledge of these characteristics prior to the investigation was very meagre, and the six sizes previously made were to a certain extent decided upon without any real knowledge of the various types of load conditions.

Much more comprehensive investigations on the subject have been made—and are being made—to get an even greater knowledge of the facts. Further investigation may lead to some modification in the sizes in some future specification.

(2.3) Name-Plate Markings

In consequence of the above alteration in load range, single-phase meters are now marked with their maximum continuous current rating which is, in many respects, a more logical marking than that in the previous specification. This means that 1/60 of this marked current at unity power factor is the lowest point of the meter accuracy range.

(2.4) Standardization of Fixing Dimensions and Terminal Spacings

Another important point in the new specification for single-phase whole-current meters is the inclusion of standard fixing dimensions and terminal spacings. This was necessary owing to the development of the 'service unit' for use in domestic consumers' premises, comprising main switch and fuses, upon which the meter could be readily mounted. It has proved invaluable in reducing the time necessary for connecting meters for test purposes.

(2.5) Standardization of Terminal Holes and Clamping Screws

A further advance in the new specification was the standardization of the terminal holes, the bore being the same for all sizes of meter, i.e. 10, 40 and 80 amp; in addition, the 'nose' of the

* SHOTTER, G. F.: 'Integrating Electricity Meters', *Journal I.E.E.*, 1937, 80, p. 202.

clamping screw is so shaped that any wire from $1/\cdot048$ in to $19/\cdot052$ in can be effectively secured.

There is no doubt of the benefits which have been brought about by the new specification for the single-phase meter, not only to the manufacturer but certainly to the meter departments of the various Electricity Boards.

(3) ELECTRICAL PERFORMANCE CHARACTERISTICS OF SINGLE-PHASE METERS: THE NEW SPECIFICATION COMPARED WITH THE OLD

The increase of the range of the single-phase instrument has extended the error bands of $\pm2\%$ at unity power factor and $+2\%$ and $-2\frac{1}{2}\%$ at $0\cdot5$ power factor to the 60/1 load range from the corresponding figures for the previous ranges in the old specification of 40/1 and 25/1.

Although a few makes of meter manufactured under the old specification were capable of giving an accuracy within $\pm2\%$ up to the 60/1 load range, the majority were not. Some of the meters now manufactured are capable of being calibrated to this accuracy, or even better, up to the 120/1 load range, but this does not necessarily mean that they can carry the maximum current of that range continuously. In fairness to the old type of 'long-range' meter, it may be added that the error band up to 200% load, i.e. load range of 40/1, was better than $\pm2\%$.

The limits imposed by the new specification for voltage, frequency and temperature variation are similar to those in the old one.

It will be seen from the above analysis that the electrical quality of the meters manufactured under the new specification has not had to be lowered to attain the longer range, and it is to be noted that most, if not all, of the modern instruments are capable of being adjusted to closer limits than those demanded in the specification.

(4) SINGLE-PHASE TWO-WIRE PREPAYMENT METERS

Part 3 of the revised Standard covering the above type of meter includes the flat-rate meter, the multi- or 'load-rate' instrument and the two-part-tariff meter. The first is the simple prepayment meter; in the second, the price charged is determined by the load (above a certain load the price is reduced); and the third has, in addition, a time-controlled release mechanism, generally a synchronous motor, to take care of the fixed charge.

The specifications for the credit and prepayment meters differ in two main points: the number of sizes and the accuracy limits. There are two marked current ratings, i.e. 10 and 40 amp, these being the maximum currents that the meter shall be required to carry. The error limits are slightly different from those chosen for the credit meters, which are $\pm2\%$ from 1/60 of marked current to marked current at $0\cdot5$ power factor. The figures for the prepayment meter are $+2\%$ and -3% from 1/30 of marked current at unity power factor and $+2\%$ and $-3\frac{1}{2}\%$ from 1/15 of marked current to marked current for $0\cdot5$ power factor. This increase of the accuracy limit at the low-load end is necessary owing to the extra friction loading imposed on the meter by the prepayment mechanism.

It is interesting to note that, whereas prior to the new specification the multi-coin prepayment meter was designed to take 1d., 6d. and 1s. coins, a 2s. model is now available. It would appear that the 1d. coin pattern has now been generally dropped.

(5) POLYPHASE AND SINGLE-PHASE THREE-WIRE WHOLE-CURRENT AND TRANSFORMER-OPERATED METERS: ALSO SINGLE-PHASE TWO-WIRE TRANSFORMER-OPERATED METERS

The above types of instrument are covered by Part 4 of the new Standard, in which no alteration has been made to the range of sizes from those in the 1937 edition, and, in general, little change has been made in the limits.

The development of the single-disc type of multi-element meter for polyphase 3- and 4-wire circuits has been extended, more manufacturers having taken it up. This type of instrument requires very careful design owing to the difficulty of getting rid of, or compensating for, the interaction between the electro-magnetic systems. Where this is accomplished, the smaller size and weight of the instrument compared with some of the multiple-disc types presents certain advantages to the user.

(6) PRECISION-TYPE METERS

Another important change in the Standard is the issue of two separate specifications for these instruments, namely Part 7 for precision a.c. instruments for accounting and statistical purposes and Part 8 for precision instruments for testing purposes. These are not concerned with ratings, which are not specified, but they deal definitely with performance.

In the specification for accounting meters, the limits of error are taken down to 1/20 of full load and are considerably reduced compared with those of commercial instruments. Other limits are also tightened.

In the precision meter for testing purposes, the limits of error are again much closer; also they are specified only down from 5/4 to 1/5 of full load. Closer limits are also set for other factors.

It will be appreciated that for these important instruments it is essential to have specifications which cover the special features of each one.

(7) METERS WITH MAXIMUM-DEMAND INDICATORS

The above instruments are covered by Part 5 of the new Standard.

An interesting development by one maker is the cumulative maximum-demand recorder, in which, when the maximum-demand pointer is reset to zero, its recording is transferred to a cumulative register. This ensures that the previous recording is not lost.

Under this heading should be included the addition to the plain Merz-type maximum-demand indicator of alarm devices for indicating when a demand is being extended beyond a predetermined value, and also an attachment to limit the load. The last two additions to the maximum-demand attachment are primarily for the use of consumers, but they also help the supply undertaking with its bulk-supply problems.

(8) DIRECT-CURRENT TWO-WIRE METERS

With the exception of heavy-current instruments for traction purposes, the use of d.c. consumers' meters is rapidly declining in this country, if not already finished, but a specification dealing with these instruments covers both ampere-hour and watt-hour types. This specification (Part 6) differs very little from that in the old edition of B.S. 37.

(9) FACTORS GOVERNING THE STABILITY OF A METER

As will be seen from the above, the British Standard for electricity meters covers the electrical characteristics of new meters in a comprehensive manner. What is of the greatest importance to the supply undertaking is, however, the sustained accuracy of meters over as long a period as possible, and a low cost of testing and overhauling.

Sustained accuracy is dependent upon two factors: (*a*) the stability of the braking magnet, and (*b*) the stability of the friction at the various wearing points, namely the top bearing, the register train and the bottom bearing.

With regard to (*a*), the greater use of the very-high-energy anisotropic alloys for these magnets, combining as they do high coercivity and high remanence, and the extended experience gained by manufacturers in their correct use will have led to a greater stability than obtained previously. It will be appreciated that any variation in the strength of the braking magnet, which in general results in a weakening of the magnet, means that a meter will show a fast error over the whole of its load/accuracy curve.

With regard to (*b*), apart from good design, the friction increase with time will depend to a large extent on the finish and polish of the wearing parts.

The friction of a well-designed and finished pointer-type register is very low, about 0·5 dyne/cm. It is interesting to note, however, that the Ministry of Power has decided to approve the use of the cyclometer register. This was for many years opposed by meter experts on the ground that the friction imposed by such a mechanism was much higher than that of the pointer type. The manufacturers have therefore produced, by the use of light alloys, cyclometer registers which give less friction than the older types, and which, with larger figures, will enable the consumers to read their own meters and forward the readings on a suitable form to the Electricity Board. This should lead to a reduction of meter reading costs, particularly in rural areas.

The greatest cause of the increase of friction in the past has been the bottom bearings. Friction increase in these is dependent upon two factors: the number of revolutions performed by the disc and the extent of the so-called 'parasitic' forces which exist in all induction meters in varying degrees, depending upon the design, and which can be the cause of excessive wear on the bottom bearing.

This, however, has been the subject of considerable research by the Electrical Research Association over the past twenty-two years and has led to a greater clarification of the subject, making possible an important increase in the life of such a bearing, i.e. an extension of the service period for a given increase of friction. This also applies to the parasitic forces, which have been classified and measured, and their effects on the life of the bottom bearing estimated.

Use has been made by both manufacturers and users of the information given in the reports issued from time to time by the E.R.A. to improve the performance of the bottom bearing, and at least one manufacturer has made a considerable reduction in the parasitic forces, thereby minimizing the wear on the bottom bearing.

Whereas the service life at the time of the previous review was of the order of seven years, it is now possible to extend this to fifteen years or more. This has led to the saving of a considerable sum of money by the supply undertakings.

It will be appreciated that an increase of friction with time affects the low load of a meter to the greatest extent, and therefore an economic balance has to be struck between the loss of registration and the cost of servicing. This is governed by a knowledge of the relationship between the load characteristics of a consumer and the size of meter used.

With regard to the cost of testing and overhauling, it is essential that it should be easy to dismantle and clean a meter, and to replace worn parts and reassemble with the minimum of disturbance to the original calibration. The manufacturers appear to have recognized this in their latest designs, which should lead to a saving in servicing time and cost to the users.

It was thought necessary to give the foregoing summary of the various facets of the problems for those who are not meter specialists to enable them to see upon what basis the comparison between the previous review and the present one has been made.

From an investigation which has been made there is no doubt that in general the standard of the modern meter is above that of those under review in 1937. Precision methods of manufacture have led to very fine instruments which will compare with the products of any other country.

(10) SUMMATION METERING

Summation metering has shown further development since the last review, particularly in instruments using the impulse method. The older methods employing mechanical and electrical summation—parallel current transformer, multi-coil meter and multi-unit meter—are still used and available, but their utility is somewhat circumscribed by the distance between the current transformer and the meter. Consequently the impulse method has been further developed to meet modern supply conditions, particularly where centralization of readings is required. The principle is not new, one method having been described and used by the author before 1936.

Many ingenious electrical and mechanical designs have been devised, covering integration, maximum-demand apparent power and reactive power. Up to 24 channels are listed by the manufacturers.

(11) THE MEASUREMENT OF APPARENT-POWER MAXIMUM DEMAND

A new and ingenious instrument for the evaluation of apparent-power maximum demand has been marketed by one manufacturer within the last few years. The measurement of this quantity has always confronted the meter designer with many problems, but it would seem that the new instrument should prove of value. It is of interest that this instrument measures the arithmetic sum in a polyphase circuit and not the vector sum.

(12) CONCLUSION

Summing up this review: it can be seen that the biggest steps forward have been due to the effect of the new British Standard specifications relating particularly to the single-phase house-service meter, to the application of principles based upon research on the bottom bearings of meters, and also to the study of the effect of wear due to parasitic forces. The mechanical quality of meters has certainly shown great improvement since the last review.

The period between the reviews has not produced knowledge of any new underlying principles, but there has been an extension of older methods—for instance in impulse summation—to meet the present-day requirements of the supply industry.

(13) BIBLIOGRAPHY

(1) SHOTTER, G. F.: 'Meter and Instrument Jewels and Pivots' (E.R.A. Report Ref. T/T39; 1944).
(2) SHOTTER, G. F.: 'Parasitic Forces existing in Induction Watthour Meters' (E.R.A. Report Ref. T/T46; 1949).
(3) TAGG, G. F.: 'Mathematical Notes on the Parasitic Forces in Induction-pattern Watthour Meters' (E.R.A. Report Ref. T/T47; 1949).
(4) SHOTTER, G. F.: 'Meter and Instrument Jewels and Pivots' (E.R.A. Report Ref. T/T48; 1953. Continuation of E.R.A. Report Ref. T/T39; 1953).
(5) SHOTTER, G. F.: 'Phenomena associated with the Debris formed in a Meter-type Bearing' (E.R.A. Report Ref. T/T49; 1956).
(6) SHOTTER, G. F.: 'The Polishing of Meter Pivots, with special reference to Steel' (description of apparatus developed for re-radiusing and polishing pivots) (E.R.A. Report Ref. T/T50; 1954).

(7) Shotter, G. F.: 'Further Researches on the Phenomenon of Debris formed in a Meter-type Bearing, with special reference to Osmium-Rhodium/Sapphire and Diamond Combinations' (E.R.A. Report Ref. T/T51; 1954).
(8) Tagg, G. F., and Shotter, G. F.: 'Investigation into the Laws relating to the Rate of Wear and Change of Friction with the Number of Revolutions performed in Meter-type Bearings' (E.R.A. Report Ref. T/T52; 1956).
(9) Tagg, G. F., and Shotter, G. F.: 'Analysis of the Problems relating to the Protection of Meter Jewels and Pivots from Damage due to Shock' (E.R.A. Report Ref. T/T53; 1956).
(10) Tagg, G. F.: 'Nomographic Representation of Elastic Contact Conditions between Steel Pivots and Sapphire or Diamond Jewels' (E.R.A. Report Ref. T/T54; 1956).
(11) Shotter, G. F.: 'The Steel 90° Sapphire Meter Bearing. Further Research on Life and Debris formed' (E.R.A. Report Ref. T/T55; 1958).
(12) Shotter, G. F.: 'Experience with, and Problems relating to, Bottom Bearings of Electricity Meters', *Journal I.E.E.*, 1934, **75**, p. 755.
(13) Shotter, G. F.: 'Meter and Instrument Jewels and Pivots', *ibid.*, 1946, **93**, Part II, p. 15.
(14) Shotter, G. F.: 'Parasitic Forces existing in Induction Watt-hour Meters', *Proceedings I.E.E.*, Paper No. 832 M, March, 1949 (**96**, Part II, p. 729).
(15) Schiller, P.: 'The Load of the Individual Domestic Consumer—First Report: Methods of Analysis and Preliminary Results' (E.R.A. Report Ref. K/T122; 1947).
(16) Schiller, P.: 'The Application of Statistical Methods to Electricity Supply Problems', *Journal I.E.E.*, 1948, **95**, Part II, p. 161.
(17) Rowson, R. B.: 'Electricity Supply—A Statistical Approach to some Particular Problems', *Proceedings I.E.E.*, Paper No. 1195 S, November, 1951 (**99**, Part II, p. 151).
(18) 'A Large-Scale Sampling Survey of Domestic Consumers' (E.R.A. Report Ref. K/T125; 1948, and Supplement, 1949).
(19) Schiller, P., and Pridmore, W. A.: 'Investigation of a Sample of Domestic Two-Part-Tariff Consumer' (E.R.A. Report Ref. K/T127; 1949).
(20) Schiller, P.: 'New Meter Standards: their Bearing on Present Distribution of Domestic Consumers' Loads', *Electrical Review*, 1949, **145**, p. 1231.
(21) Golds, L. B. S., and Schiller, P.: 'Meter Problems and Consumers' Load Characteristics', *Proceedings I.E.E.*, Paper No. 1483 M, April, 1953 (**100**, Part II, p. 619).

DISCUSSION ON 'AN ELECTROSTATIC DUST MONITOR'*

Dr. F. H. Last (*communicated*): When developing any type of instrument it is essential to determine its purpose and retain this as the objective. It may be that other uses will result from the final product.

The author refers to the 'toxicity' of chimney discharge. It must be categorically stated that chimney discharge from a pulverized-fuel-fired boiler is not toxic, although it is a nuisance.

I do not agree with the statement that the offensiveness of a chimney discharge would be better expressed in terms of the total surface area of solids entering the atmosphere. Atmospheric conditions, gas exit temperature and particle size are conditions which control the concentration of deposition at ground level—the more realistic measure of offensiveness. Measurement based on the specific surface of the dust particles is misleading. The greater the specific surface area, the smaller the particle and the less likely is an offensive concentration of dust at ground level.

I believe that the instrument does not achieve the author's main purpose in giving a true indication of the offensiveness of chimney discharge. I agree with the first conclusion that the instrument could be used as an effective monitor of the performance of dust-precipitation plant.

Dr. D. H. Grindell (*in reply*): The electrostatic monitor is inherently sensitive to smokes and all gas-entrained dust particles, so that the introductory remarks in my paper are not meant to refer specifically to emissions from pulverised-fuel-fired boilers, but rather concern sources of air pollution in general. The word 'toxic' was chosen to imply that air polluted by solid particles constitutes a potential danger when inhaled, owing to its adverse effects upon the respiratory system. Moreover, dusts and smokes, besides being generally unhealthy and psychologically depressing, can seriously aggravate an industrial fog. For these reasons, I believe that a chimney discharge is always to some extent offensive, whether or not it causes an obvious nuisance by depositing dirt. However, it cannot be denied that modern power stations with efficient combustion processes and chimneys discharging small particles at a high level, often above a fog layer, have gone far in reducing their offensiveness.

The electrostatic instrument was developed to meet the need for a robust and maintenance-free dust or smoke recorder which would provide both additional information about the solid content of flue gases and a more significant measurement than is at present obtainable by gravimetric gas-sampling or the use of Ringelmann charts. I agree that it cannot indicate true offensiveness; that is a complex quality which no simple instrument fitted in a boiler flue duct can truthfully measure, since it is neither defined by any one parameter nor determined solely by flue-duct conditions. Apparent or potential offensiveness is measureable at source, but actual offensiveness at ground level must ultimately remain dependent upon the weather.

It is encouraging to note that Dr. Last agrees the equipment would be useful for monitoring dust precipitators.

* Grindell, D. H.: Paper No. 3184, January, 1960 (see 107 A, p. 353).

621.315.211.048 : 621.317.333.8

The Institution of Electrical Engineers
Paper No. 3143 S
Dec. 1959

THE IMPULSE STRENGTH OF FULLY-IMPREGNATED-PAPER DIELECTRICS AS USED IN HIGH-VOLTAGE CABLES

By B. SALVAGE, B.Sc.(Eng.), Ph.D., Member, and J. A. M. GIBBONS, B.Sc.

(*The paper was first received* 18*th March, and in revised form* 14*th August*, 1959. *It was published in December*, 1959, *and was read before the* SUPPLY SECTION 9*th March, and the* NORTH MIDLAND CENTRE 5*th April*, 1960.)

SUMMARY

An investigation into the impulse strengths of model cables and cable-impregnating oils and compounds is described. In particular, the effects of the density, air impermeability and thickness of the paper and the dielectric thickness and temperature of the impregnant have been studied.

Special papers possessing a combination of properties designed to give a high impulse strength have been incorporated in an experimental oil-filled cable and the improvements obtained closely approach those indicated by measurements on model cables.

An impulse breakdown mechanism in a fully-impregnated-paper cable dielectric is suggested whereby the impulse strength, although it may be varied by changing the properties of the paper, is shown to be primarily dependent on the impulse strength of the impregnant.

(1) INTRODUCTION

Since the end of the war there has been a rapid growth in the installation of various types of pressure cable, designed to operate at voltages of 33 kV and above, in order to meet the increasing demands for the transmission of electrical energy. Pressure cables have therefore been the subject of intensive development, and the behaviour of their dielectrics at high electric stresses has received special attention. Impregnated paper still remains practically unrivalled as a dielectric for pressure cables in spite of the increasing use of plastics at low voltages, and it is now recognized that the insulation thickness required in a pressure cable, of either the oil-filled or one of the various gas-pressure types, is governed primarily by the impulse electric strength of its dielectric. Accordingly, in recent years, impulse-breakdown phenomena in impregnated-paper dielectrics have been extensively studied, and accounts of research on the subject have been published by Gazzana Priaroggia and Palandri[1] and Hall and Skipper.[2]

Following initial experiments described previously,[3] the present authors have made a systematic investigation into the breakdown of model cables and cable-impregnating oils and compounds under impulse voltages. Although model cables can be used for studying a wide range of problems associated with the performance of actual cables under impulse conditions, it is probably in the investigation of the effects of variations in the physical properties of the paper and in the nature of the impregnant on the impulse strength of the composite dielectric that the model technique is most rewarding, and this aspect receives particular attention in the present paper. It is to be expected that the impulse strength of impregnated-paper dielectric should be closely related to that of its impregnant—the main reason for impregnating the paper is to increase its electric strength—and hence an investigation has been made into the impulse strengths of cable-impregnating oils and compounds and the various factors on which they are dependent. The work is described in detail, since no account of similar research appears to have been published previously.

In addition, impulse tests have been made on a length of experimental oil-filled cable having a dielectric incorporating special papers. These tests show the relevance of the experiments on model cables to the behaviour of actual cables and demonstrate the important advances in cable performance which have been achieved as the result of laboratory studies.

Finally, the impulse breakdown mechanism in lapped impregnated-paper dielectric is discussed. It is considered that the investigation into the impulse strengths of cable-impregnating oils and compounds is of particular value in helping towards an understanding of impulse breakdown phenomena in high-voltage cables.

(2) MODEL CABLES

One of the main aims of the experiments on model cables has been the study of the effects of the density, air impermeability and thickness of the paper on the impulse strength of impregnated-paper cable dielectrics. For this purpose the thickness of the paper has been measured using a dial micrometer, the density has been deduced from the mass of $100\,cm^2$ of the paper after drying and the measured thickness, and the air impermeability has been determined using an Emanueli porosimeter.[4] All the papers examined were wood-pulp papers of British manufacture.

Papers having densities ranging from approximately $0{\cdot}75$ to $1{\cdot}00\,g/cm^3$ have been used. There is a tendency for the air impermeability to be higher the denser the paper, but a much more important factor controlling the air impermeability is the degree of beating to which the pulp is subjected during manufacture, and by suitable control of this process a very wide range of air impermeabilities has been produced in the papers employed in the present experiments (approximately $0{\cdot}2$–260×10^6 Emanueli units). The effect of the paper density has been studied by testing model cables made from two papers having different densities but practically the same air impermeability and thickness, and similarly the effect of the air impermeability has been investigated with model cables made from two papers having different air impermeabilities but approximately the same density and thickness; all the papers used for these particular experiments were made by one manufacturer from the same wood pulp. This procedure is more direct than the statistical analysis covering a large number of papers possessing a wide range of physical characteristics, which has been adopted by Hall and Skipper[2] to assess the effect of each variable, but it can be criticized on the ground that the conclusions are based on the study of far fewer papers. Unfortunately it is difficult to vary the properties of paper independently during manufacture. Therefore, to check these conclusions, certain special papers combining values of the properties designed to give a high impulse strength have been obtained and have been the subject of further experiments.

The effect of the paper thickness has been studied by making experiments on model cables composed of papers having thick-

Dr. Salvage is in the Electrical Engineering Department, Leeds University, and Mr. Gibbons is with the Central Electricity Generating Board. They were both formerly with W. T. Henley's Telegraph Works Co. Ltd.

nesses ranging from approximately 1 to 6 mils; although the densities and air impermeabilities of these papers varied appreciably and the papers were made by different manufacturers from different wood pulp, the important influence of the thickness, by itself, has been apparent.

Two impregnants have been used for the experiments on model cables, namely a compound consisting of a refined mineral oil with approximately 15% of refined rosin as used in solid-type and gas-cushion cables, and a light mineral oil having a so-called 'normal' viscosity as used in oil-filled cables. During the investigation into the impulse strengths of cable-impregnating oils and compounds to be described in Section 3, experiments have been made on both these impregnants together with a compound containing approximately 30% of the hydrocarbon polymer polyisobutylene, as used in gas-pressure cables, and another light mineral oil used in oil-filled cables having a 'low' viscosity. The details of the viscosity, density and permittivity of all these impregnants are given in Table 1. In the text 'oil-filled cable oil' has been abbreviated to o.f.c. oil.

Table 1

Impregnant	Viscosity		Density		Permittivity	
	20° C	85° C	20° C	85° C	20° C	85° C
	centistokes	centistokes	g/cm³	g/cm³		
Rosin-containing compound	26 000	98	0·949	0·908	2·40	2·32
Polyisobutylene-containing compound	14 500	113	0·916	0·877	2·29	2·23
Oil-filled cable oil (normal viscosity) ..	29	3·6	0·891	0·850	2·24	2·18
Oil-filled cable oil (low viscosity) ..	14	2·5	0·878	0·838	2·22	2·16

(2.1) Construction

Each model cable is made by lapping ¾ in-wide paper tapes on to a smooth cylindrical copper tube 1 in in diameter and 8½ in long. Six tapes are applied helically under a small constant tension with a butt-gap width of $\frac{1}{16}$ in and a 75/25% registration. The lapping conditions thus approximate to those in an actual cable. A paper cone, 2 in long, is wound around each end of the model to eliminate end faults during testing, leaving an effective cylindrical length of dielectric of 4½ in. The outer electrode consists of metallized paper applied with the metallizing inwards.

The models are dried for 48 hours at 100° C under a vacuum of approximately 0·2 mm Hg in a specially designed glass apparatus. They are subsequently impregnated with oil or compound which has been degassed at 100° C under a vacuum of 0·2 mm Hg, the models being allowed to cool to ambient temperature under vacuum after 3 hours' impregnation at 100° C. About 15 models at a time can be prepared in this way.

(2.2) Test Procedure

When testing, each model is immersed in a bath containing the same oil or compound which was used for its impregnation. Check tests made on models impregnated with o.f.c. oil have shown that the transfer of the models from the impregnating vessel to the bath results in no significant change in impulse strength. Experiments can be made at elevated temperatures by heating the bath electrically, the temperature being controlled to an accuracy of within ±1° C. The impulse strength of impregnated-paper cable dielectric decreases slightly with increase in temperature, the magnitude of the decrease depending primarily on the nature of the impregnant. Hence it has become the normal practice to make impulse tests on cables at elevated temperatures. Most of the present experiments have therefore been made at 85° C, which is the maximum conductor temperature at present permitted in pressure cables.

A 4-stage Marx-type impulse generator having a capacitance of 0·04 μF per stage and capable of supplying an output voltage up to about 160 kV has been used for the tests. A 1/5 microsec wave is usually employed, the short wavetail being desirable in order to minimize the energy discharged through the sample on breakdown. Negative impulses are applied to the inner electrode of the model, the outer electrode being earthed. The voltage is increased from about 70% of the expected breakdown voltage in steps of approximately 3%, with one impulse at each voltage until failure occurs. A simple breakdown detector incorporating a gas-filled triode is used in conjunction with the impulse generator to indicate a breakdown.

After the completion of the tests each model is examined and the position of the breakdown path noted. The dielectric thickness is measured with a micrometer as near the breakdown position as is practicable, and the impulse strength is calculated as the quotient of the breakdown voltage and thickness.

Owing to the inherent scatter in impulse-strength measurements, generally about ten models have been tested for each particular variable being studied. Breakdown in or adjacent to an end cone occurred only rarely, and in all such cases the results have been ignored.

(2.3) Experimental Results

(2.3.1) Effect of Paper Density.

Fig. 1 shows the effect at 85° C of the paper density. The arithmetic-mean impulse strength in kilovolts per centimetre is stated for each test and the range is indicated. Beneath the mean value are given the standard deviation in kilovolts per centimetre and the number of individual measurements on which

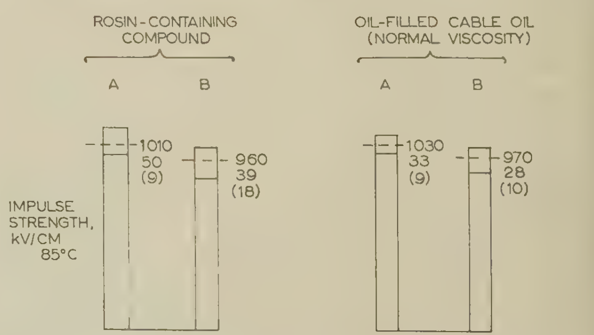


Fig. 1.—Effect of paper density on the impulse strength of model cables.

Paper	*Thickness* mils	*Density* g/cm³	*Air impermeability* *Emanueli units* × 10⁶
A	5·6	0·67	1·4
B	5·4	0·80	1·4

the mean is based. With both rosin-containing compound and o.f.c. oil (normal viscosity) the paper having the lower density gives a slightly higher impulse strength; the difference is shown by the *t*-test to be statistically significant at the 1% level.

(2.3.2) Effect of Paper Impermeability.

Fig. 2 shows the effect at 85° C of the paper air impermeability. For both impregnants the impulse strength is slightly higher

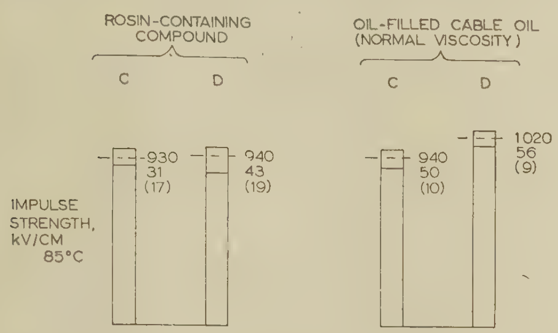


Fig. 2.—Effect of paper air impermeability on the impulse strength of model cables.

Paper	Thickness mils	Density g/cm³	Air impermeability Emanueli units $\times 10^6$
C	4·7	0·89	1·5
D	4·6	0·88	30

with the paper having the higher air impermeability, the difference being statistically significant at the 1% level with o.f.c. oil (normal viscosity), but not, however, with rosin-containing compound.

(2.3.3) Effect of Paper Thickness.

Fig. 3 shows the effect at 85° C of the paper thickness. Since the densities and the air impermeabilities of the papers used varied appreciably and the papers were made by different manufacturers from different wood pulp, no attempt has been made to determine a precise relationship between the impulse strength and the thickness, but for both impregnants it is clear that the effect of the paper thickness is important, the impulse strength increasing as the paper thickness is decreased.

(2.3.4) Special Papers.

The experiments described in the three previous Sections indicate that the impulse strength of lapped impregnated paper is increased when the air impermeability of the paper is increased and the density and thickness are decreased. The special papers which have been chosen to check these conclusions, and especially those relating to the density and the air impermeability of the paper, may be classed in three groups as follows:

(*a*) A 3-mil paper having a low density and a high air impermeability (Paper F in Fig. 4).

(*b*) A 2½-mil paper, G, having a normal density and a high air impermeability, and the same paper, H, super-calendered to a thickness of 2 mils resulting in a high density and an even higher air impermeability.

(*c*) A 3-mil paper, J, having a normal density and a very high air impermeability, and the same paper, K, super-calendered to a thickness of 2½ mils resulting in a high density and a still higher air impermeability.

Both rosin-containing compound and o.f.c. oil (normal viscosity) have been used as impregnants for (*a*), and tests have been made at both 20° C and 85° C. O.F.C. oil (normal viscosity) has been employed for (*b*) and (*c*), and testing has been confined to 20° C. In each case a comparison has been made with a paper E having

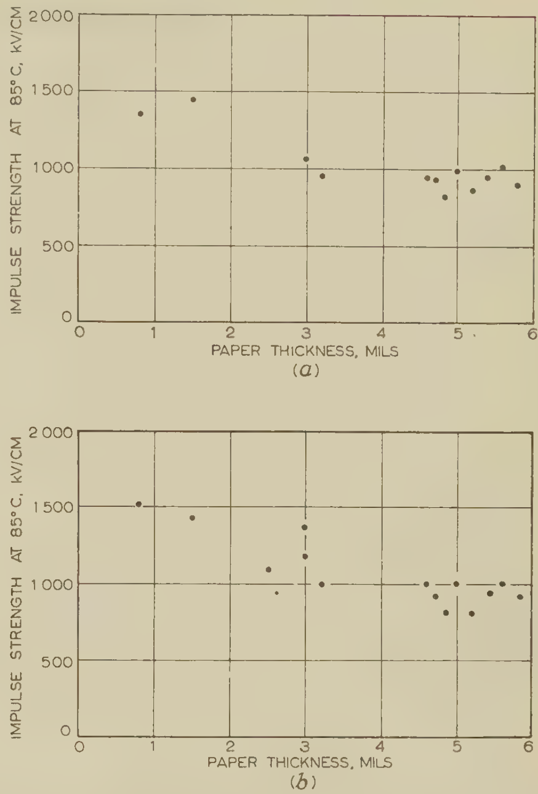


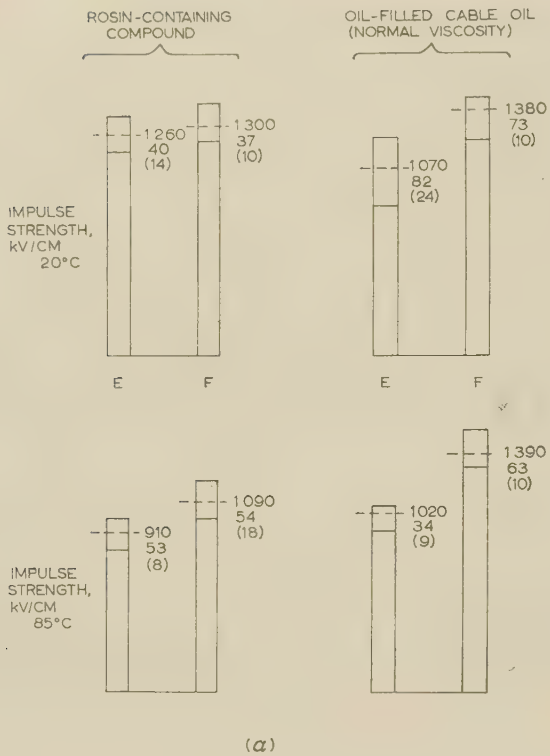
Fig. 3.—Effect of paper thickness on the impulse strength of model cables, using the following impregnants:

(*a*) Rosin-containing compound.
(*b*) Oil-filled cable (normal viscosity).

more normal characteristics. The results of the experiments are given in Figs. 4(*a*), (*b*) and (*c*).

Referring first to Fig. 4(*a*), the results support the earlier conclusions regarding the effects of the density and the air impermeability of the paper, but they indicate that the effect of the air impermeability is related to the viscosity of the impregnant, an increase in the air impermeability causing no significant change with viscous impregnants but resulting in a considerable improvement in the impulse strength with impregnants having a low viscosity. Increasing the temperature from 20° C to 85° C results in very little, if any, decrease in the impulse strength of the dielectric impregnated with o.f.c. oil (normal viscosity), but it causes an appreciable reduction in the impulse strength of the dielectric impregnated with rosin-containing compound. With the normal paper E, the dielectric impregnated with o.f.c. oil (normal viscosity) has a lower impulse strength than that impregnated with rosin-containing compound at 20° C but a higher impulse strength at 85° C. With the paper F, having a low density and a high air impermeability, the impulse strength of the dielectric impregnated with o.f.c. oil (normal viscosity) is higher than that of the dielectric impregnated with rosin-containing compound at both 20° C and especially 85° C.

Considering now Fig. 4(*b*), it is clear that the 2½-mil paper G, having a normal density and a high air impermeability, gives an appreciably higher impulse strength than the normal 3½-mil paper E, in accordance with the earlier conclusions regarding the effects of the thickness and the air impermeability of the paper, but that the 2-mil super-calendered paper, H, results in only a small further increase in impulse strength. This small increase is consistent with the reduction in the paper thickness. It would appear that the effects of the calendering process in increasing



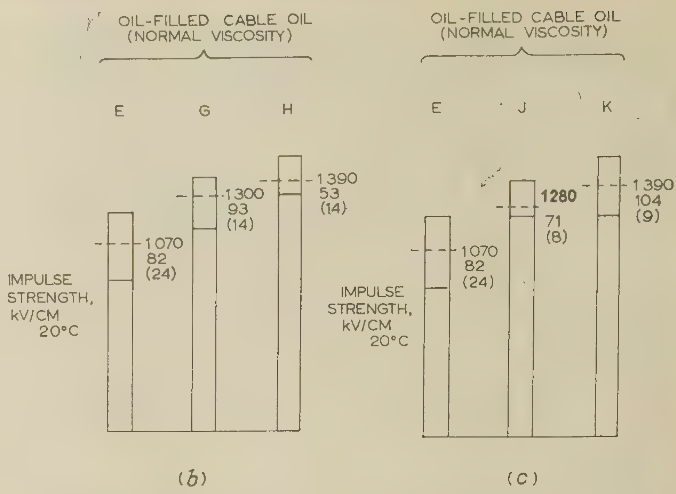


Fig. 4.—Impulse strength of model cables using special papers.

	Paper	*Thickness* mils	*Density* g/cm³	*Air impermeability* *Emanueli units* $\times 10^6$
(*a*)	E	3·2	0·82	1·3
	F	3·0	0·76	60
(*b*)	E	3·2	0·82	1·3
	G	2·5	0·82	17
	H	2·0	1·02	35
(*c*)	E	3·2	0·82	1·3
	J	2·8	0·86	145
	K	2·5	0·98	260

the density and the air impermeability of the paper tend to oppose one another, i.e. the experiments support the earlier conclusion that an increase in the paper density, by itself, results in a reduction in the impulse strength of the dielectric.

Finally, the results given in Fig. 4(*c*) support those in Fig. 4(*b*). It will be noticed that the special papers referred to in Fig. 4(*c*), which are only slightly thicker than those described in Fig. 4(*b*) but have a very much higher air impermeability, give practically the same impulse strength as those of Fig. 4(*b*). This indicates that the effect of the air impermeability of the paper in increasing the impulse strength becomes progressively less as the air impermeability is increased to very high values.

(3) IMPREGNATING OILS AND COMPOUNDS

Very little information has been published on the impulse strengths of cable impregnants. Davis[5] has reported an impulse strength of 1 900 kV/cm for cable compound using 6 mm-diameter spheres with an electrode gap of 0·5 mm, presumably at ambient temperature. More recently Bennett[6] described measurements using 13 mm-diameter spheres with a gap of 5 mils; a compound containing 20% of refined rosin had an impulse strength of 2175 kV/cm at 20° C and 1 410 kV/cm at 60° C, whereas o.f.c. oil had an impulse strength of approximately 1 250 kV/cm at 20° C, the value decreasing only very slightly with increase in temperature up to 85° C. Mention may also be made of the impulse-strength measurements on transformer oil reported by Toriyama,[7] Watson and Higham,[8] Zein El-Dine and Tropper[9] and Hancox and Tropper,[10] which are relevant more especially to investigations on o.f.c. oil than cable compounds.

(3.1) Test Cell

The test cell, which is shown in Fig. 5, has been designed especially for the measurement of the impulse strengths of cable-impregnating oils and compounds, and is a slightly modified form of those previously described.[11, 12] The cell requires a volume of 50 cm³ of oil, which is contained in a vertical glass tube of length 15 cm and internal diameter 2·5 cm. Flanges which support the electrode assemblies are attached to the top and bottom of the glass tube by means of copper-to-glass seals. Chromium-plated phosphor-bronze spherical electrodes of

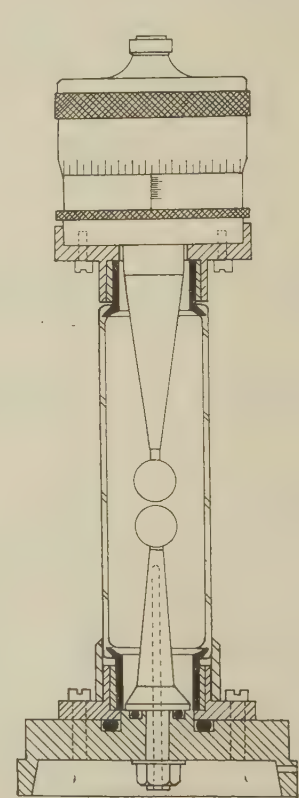


Fig. 5.—Oil test cell.

13 mm diameter are screwed to the electrode shanks, and the spheres are drilled and tapped in four positions to allow new electrode surfaces to be readily available. The lower electrode shank is drilled to permit the insertion of a thermocouple for the measurement of the gap temperature. The micrometer head used for the measurement of the electrode gap is capable of measuring a gap of 0·005 in to an accuracy of within $\pm 2\%$. Leakage of oil from the cell is prevented by the use of a Neoprene washer.

(3.2) Test Procedure

Before each test the electrodes are polished with a soft cloth using jeweller's rouge, as described by Goodwin and Macfadyen.[13] The test cell is subsequently cleaned in trichlorethylene and ether. In the light of the work of Hancox,[14] who has shown that surface layers on the electrodes can affect the impulse strength of transformer oil, the precaution is taken to cover the electrodes with the test liquid immediately after cleaning. Each oil or compound is carefully filtered through a sintered glass filter having an average pore size of 20–30 microns and is degassed, employing a final vacuum of 10 mm Hg. The test cell is rinsed twice with the test liquid before filling with the test sample.

In order to make measurements on oils and compounds at elevated temperatures the test cell is immersed to the upper flange in a specially constructed oil bath whose temperature is controlled to an accuracy of within $\pm 1°$ C. (This bath has also been employed to test model cables at elevated temperatures.)

A single-stage impulse generator which can produce 1/5 or 1/50 microsec waves having peak values of up to 40 kV has been built for the measurements on impregnating oils and compounds. Since it is very desirable to keep the amount of energy which is fed into the test-cell on the occurrence of breakdown as small as possible the source capacitance of the generator has been limited to 500 pF. For voltages higher than those given by this equipment the 4-stage generator previously mentioned has been used. As with the tests on model cables a simple breakdown detector incorporating a gas-filled triode is used in conjunction with the impulse generator to indicate a breakdown.

For all tests a 1/5 microsec wave of negative polarity is used, the short wavetail again being desirable in order to minimize the energy discharged through the sample on breakdown. The voltage is increased from about 70% of the expected breakdown voltage in steps of approximately 5%, with one impulse at each voltage until breakdown occurs. Experiments on o.f.c. oil (normal viscosity) at 20° C over a range of dielectric thicknesses have shown that there is no significant difference in the mean impulse strength when a 1/50 microsec wave is employed (see Fig. 6). Further tests on rosin- and polyisobutylene-containing compounds at 20° C with a dielectric thickness of 3 mils have confirmed this conclusion.

When testing o.f.c. oil it was found possible to make at least 50 measurements on each test sample, allowing a minimum 'resting' period of one minute between breakdowns, before the impulse strength decreased significantly. With the more viscous rosin- and polyisobutylene-containing compounds a new sample of the liquid had to be used for each measurement at 20° C, but at 85° C it was found possible to make approximately ten measurements on each sample with a resting period of 3 min between successive measurements. The magnitude of the resting period may be associated with the diffusion of the breakdown products into the bulk of the liquid.

For each measurement the impulse strength has been calculated as the quotient of the breakdown voltage and the dielectric thickness. The errors introduced by neglecting the non-uniformity of the field between the spherical electrodes are very small. For example, the factor given by Russell[15] for the largest gap employed, namely 20 mils, is 1·027.

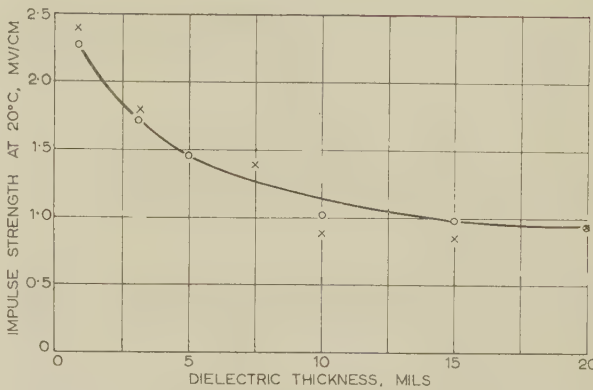


Fig. 6.—Effect of waveform on impulse strength of oil-filled cable oil (normal viscosity).

○ 1/5 microsec wave
× 1/50 microsec wave.

During the investigation an appreciable scatter in the impulse-strength measurements was obtained. Fig. 7 shows typical histograms for o.f.c. oil (normal viscosity) at 85° C for various dielectric thicknesses from 1·0 to 10·0 mils. Throughout the investigation the arithmetic mean value has been taken as the

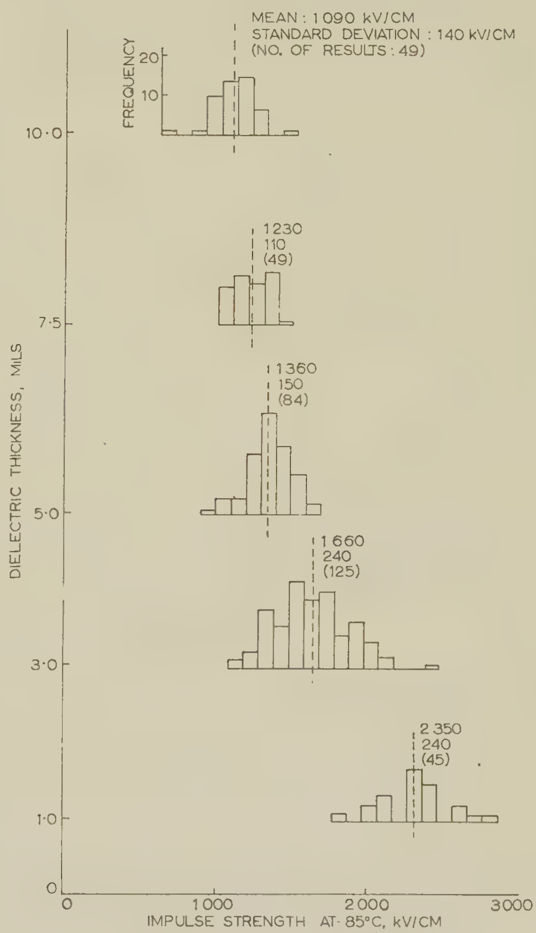


Fig. 7.—Oil-filled cable oil (normal viscosity).

Impulse-strength histograms.

representative impulse strength; each point shown in Figs. 6, 8 and 10 is the mean value of at least 20 individual measurements for o.f.c. oil and of at least ten individual measurements for the rosin- and polyisobutylene-containing compounds.

(3.3) Experimental Results

Experiments have been made on the four impregnating oils and compounds already listed in Table 1.

(3.3.1) Effect of Dielectric Thickness.

Measurements of the impulse strengths of the four impregnants have been made over a wide range of dielectric thickness from 1·0 to 15·0 mils at both 20° C and 85° C. The results are shown in Fig. 8.

It will be seen that the impulse strength is approximately constant for dielectric thicknesses greater than a certain value, but it increases rapidly as the thickness is reduced below this value. For the rosin- and polyisobutylene-containing compounds this thickness is approximately 5 mils at 20° C and 10 mils at 85° C. For o.f.c. oil the thickness is approximately 10 mils at both 20° C and 85° C.

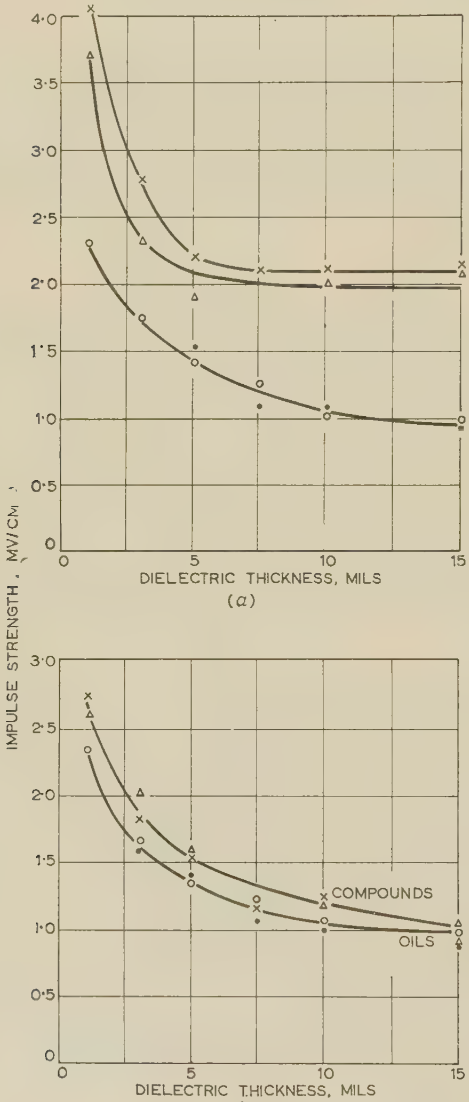


Fig. 8.—Cable-impregnating oils and compounds.
Effects of dielectric thickness and temperature on impulse strength.
× Rosin-containing compound.
△ Polyisobutylene-containing compound.
○ Oil-filled cable oil (normal viscosity).
● Oil-filled cable oil (low viscosity).
(a) Test temperature 20° C.
(b) Test temperature 85° C.

(3.3.2) Effect of Temperature.

It will be seen from Fig. 8 that increasing the temperature from 20°C to 85°C decreases the impulse strengths of both the rosin- and polyisobutylene-containing compounds but has no significant effect on the impulse strength of o.f.c. oil. The decrease of viscosity with increase in temperature is relatively much greater for both the rosin- and polyisobutylene-containing compounds than for o.f.c. oil, as can be seen from Table 1, and this appears to be associated with the different impulse-strength/temperature dependence. It will be noticed that, whereas at 20° C the compounds have appreciably higher impulse strengths than o.f.c. oil, at 85°C the impulse strengths are similar.

(4) EXPERIMENTAL OIL-FILLED CABLE. EFFECT OF SPECIAL PAPERS

The results of the experiments on model cables made with the special papers described in Section 2.3.4 have offered the promise of a considerable improvement in the impulse strength of oil-filled cables. To investigate this possibility an experimental length of oil-filled cable has been manufactured consisting of four sections incorporating near the conductor the normal 3½-mil paper E, the 2½-mil paper having a normal density and a high air impermeability, G, and the latter paper super-calendered to a thickness of 2 mils to give a high density and an even higher air impermeability, H. The cable was manufactured as a single length so that any difference in the impulse strengths of the various sections would be attributable to the different properties of the papers and not to processing variations. The cable was a $0\cdot2\,\text{in}^2$ single-core cable having a nominal insulation thickness of 0·345 in. The normal stranded conductor was screened with semiconducting carbon-black paper, the cable was impregnated with o.f.c. oil (low viscosity) and the lead sheath was fluted to facilitate oil transference. The dielectrics of the four sections were composed of the following papers:

(i) 3½-mil normal paper, E. 5-mil paper having a normal density and air impermeability.
(ii) 2½-mil paper, G. 5-mil normal paper.
(iii) 2-mil paper, H. 5-mil paper having a high density and a high air impermeability. 5-mil normal paper.
(iv) 2-mil paper, H. 5-mil paper having a high density and a high air impermeability.

Two 8 yd samples of each section of the cable were prepared for testing in the normal way, the oil pressure being adjusted to $20\,\text{lb/in}^2$, and impulse tested to breakdown at ambient temperature using 1/50 microsec waves. The voltage was increased from 490 kV, in steps of approximately 20 kV, ten negative and ten positive impulses being applied at each voltage.

The results of the experiments are given in Table 2. In each case, the impulse strength has been calculated from measurements of the insulation thicknesses made as close to the breakdown as possible. For all the sections of the cable calculations have been based on the assumption of a uniform dielectric, and for sections (iii) and (iv) further calculations have been made allowing for the permittivity grading arising from the use of highly dense papers, the permittivities of the dielectric when impregnated with oil-filled cable oil being as follows:

3½-mil and 5-mil normal paper	3·5
2½-mil paper, G	3·5
2-mil paper, H	3·9
5-mil paper having a high density and a high air impermeability	3·8

Table 2

Section of cable	Impulse strength	
	Assuming uniform dielectric	Allowing for permittivity grading
	kV/cm	kV/cm
(i)	980, 1 010; Mean 990	—
(ii)	1 170, 1 180; Mean 1 180	—
(iii)	1 270, 1 340; Mean 1 300	1 190, 1 250
(iv)	1 230, 1 270; Mean 1 250	1 200, 1 240; Mean 1 220 (for 1 190, 1 250, 1 200, 1 240)

It will be seen that the special papers result in an important increase in the impulse strength of the cable dielectric, the improvements, when due allowance is made for permittivity grading, amounting to 19 and 23% for the $2\frac{1}{2}$-mil paper G and 2-mil paper H, respectively, compared with normal $3\frac{1}{2}$-mil paper E. These values closely approach the increases of 21 and 30% obtained in the experiments on cable models.

(5) DISCUSSION

It is now intended to discuss the impulse breakdown mechanism in fully-impregnated-paper cable dielectrics in the light of the experiments on model cables, cables and impregnants which have been described in the preceding Sections.

(5.1) Impulse Strength of Impregnated-Paper Cable Dielectrics. Effects of Paper Density, Air Impermeability and Thickness

The present investigation confirms the conclusions of Gazzana Priaroggia and Palandri[1] and Hall and Skipper[2] on the dependence of the impulse strength of impregnated-paper cable dielectrics on the air impermeability of the paper. An increase in the air impermeability has no significant effect with viscous impregnants but causes an appreciable increase in the impulse strength with impregnants having low viscosity.

With regard to the effect of the paper density, it is considered that the experiments, and particularly those on the supercalendered papers, provide evidence that an increase in the paper density, by itself, results in a decrease in the impulse strength; this conclusion applies both to model cables and to cables in which allowance has been made for any permittivity grading in the dielectric. The decrease in the impulse strength is consistent with the increased permittivity of the dielectric and hence higher butt-gap stresses. This conclusion is not in agreement with that of Hall and Skipper, who have found that the impulse strength increases with paper density, more markedly with low- than with high-viscosity impregnants. However, they state that, for the particular papers they examined, air impermeability and density were to a certain extent interrelated, and thus it may be questioned whether they have succeeded in determining the separate effects of these properties. Gazzana Priaroggia and Palandri consider that density and air impermeability, while being inseparably associated, produce directly opposite effects—an increase in the paper density, by itself, decreases the impulse strength as a result of the higher permittivity of the dielectric and hence greater stresses in the oil in the butt gaps. The conclusions of the present experiments are in accordance with this view. It should be noted that, when a paper having a high density and a high air impermeability is impregnated with an oil or heated compound of low viscosity, the effect of the air impermeability generally predominates over that of the density. It must also be added that, when a paper having a high density and hence a high permittivity, is used in a cable dielectric near the conductor the resultant permittivity grading of the dielectric can give rise to a valuable increase in the impulse breakdown voltage; if the grading is ignored and the dielectric is assumed to be uniform there is a corresponding apparent increase in impulse strength. This effect of using highly dense papers is quite distinct from the phenomena forming the main subject of the present discussion and will not be considered further.

The increase in the impulse strength of lapped impregnated-paper dielectrics which can be achieved by reducing the paper thickness has previously been demonstrated at ambient temperature by Domenach[16] in tests on cables, and by Gazzana Priaroggia and Palandri,[1] and Hall and Skipper,[2] in investigations on model cables; the present experiments show that the effect is also important at elevated temperatures.

(5.2) Impulse Strength of Cable Impregnants. Effects of Dielectric Thickness and Temperature

It is evident that the effects of the dielectric thickness and temperature are of special interest and importance. The appreciable increase observed in the impulse strength of a cable impregnant as the dielectric thickness is decreased is clearly related to that obtained in lapped impregnated-paper as the paper-tape thickness and hence butt-gap thickness is reduced; both Gazzana Priaroggia and Palandri, and Hall and Skipper have concluded that the variation in the impulse strength of model cables with paper thickness is primarily due to the resulting changes in the butt-gap thickness. In Fig. 9 the

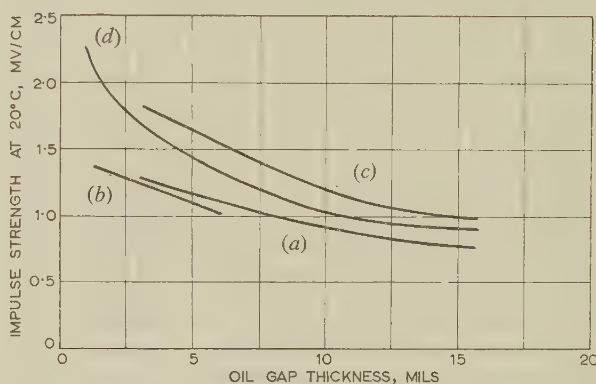


Fig. 9.—Variation of impulse strength of model cables and oil-filled cable oil (normal viscosity) with oil-gap thickness.

(a) Model cables. Gazzana Priaroggia and Palandri.
(b) Model cables. Hall and Skipper.
(c) Oil calculated. Gazzana Priaroggia and Palandri.
(d) Oil measured.

relationships between the impulse strength of model cables impregnated with o.f.c. oil and the oil-gap thickness as given by Gazzana Priaroggia and Palandri, and Hall and Skipper, have been reproduced. Gazzana Priaroggia and Palandri have also calculated the stress in the oil gap at breakdown, and its variation with the oil-gap thickness is also shown, together with the impulse strength measurements made on o.f.c. oil over a range of dielectric thicknesses by the present authors. It will be noticed that there is fair agreement between the measured and calculated impulse strengths of the oil, but the close similarity between the ways in which both vary with the oil-gap thickness is of special note. The increase obtained in the electric strength of liquid dielectrics when the dielectric thickness is reduced has

been observed previously by Goodwin and Macfadyen[13] and Lewis[17] in investigations on simple organic liquids, and by Watson and Higham[8] and Zein El-Dine and Tropper[9] in measurements on transformer oil, and it is considered that it is due to the restriction of the dielectric thickness below that required for the adequate spatial development of the electron avalanche, the multiplication of electrons by impact ionization being the principal process in the breakdown of liquid dielectrics. The effects of temperature on the impulse strengths of model cables impregnated with rosin-containing compound and o.f.c. oil found by the present authors are in good agreement with those given by Gazzana Priaroggia and Palandri, and Hall and Skipper. The present experiments show that the temperature effect is practically the same in a model cable as in its impregnant.

Finally, it may be noted that the impulse-strength measurements made on oils and compounds are in satisfactory agreement with those of Davis[5] and Bennett[6] which were quoted in Section 3.

(5.3) Impulse Breakdown Processes in Impregnated-Paper Cable Dielectrics

Very little has been published on the impulse breakdown mechanism in impregnated-paper cable dielectrics. There can be little doubt that breakdown is initiated in the impregnant as distinct from the cellulose fibres, since the permittivity of the cellulose fibres, which is about 6, is much higher than that of the impregnating oil or compound, which is normally between $2\cdot 2$ and $2\cdot 4$, so that much higher electric stresses are set up in the impregnant than in the fibres during the application of impulse voltages. One of the present authors[3] has suggested that, in a solid-type or gas-cushion cable at ambient temperature, the breakdown process is initiated in a butt gap at the conductor and that, after the local failure of the impregnating compound in the gap, which causes a field distortion, complete breakdown is rapidly propagated throughout the whole cable insulation. It is considered that the experiments, which have demonstrated that varying the air impermeability of the paper over wide limits has no significant effect on the impulse strength of model cables impregnated with a viscous impregnant, are consistent with these ideas. However, as the air impermeability of the paper has been shown to exert a considerable influence on the impulse strengths of model cables impregnated with oils or heated compounds having a low viscosity, it must be concluded that, in an oil-filled cable or a solid-type or gas-cushion cable tested at elevated temperatures, the impulse-breakdown process is not initiated solely in the impregnant in a butt gap.

Reference was made earlier to the principal electronic process in the breakdown of liquid dielectrics. In addition, it is thought[13, 17] that there are further processes involving the formation of negative and positive ions and their motion in the electric field to produce field distortion and instability. Whitehead and Minor[18] have reported a value of $1\cdot 0 \times 10^{-4}$ cm/sec per volt/cm for the ionic mobility in transformer oil at 22° C, the viscosity being very similar to that of o.f.c. oil (normal viscosity) and this value has recently been confirmed.[19] Considering an oil-filled cable dielectric and bearing in mind the enhanced electric stress in the butt gaps, this mobility corresponds to an ionic velocity in the butt gaps of roughly $0\cdot 07$ mil/microsec for stresses of the order of the impulse breakdown strength of the cable, i.e. about 1 000 kV/cm. These measurements of the mobility were made at low stresses, and the mobility may be considerably higher at stresses approaching the breakdown strength. It would seem, therefore, that in oil-filled cable dielectric the ionic mobility is such that some of the ions produced in the butt gaps during the application of high-voltage impulses will be accelerated into the structure of the paper, and some correlation between the resistance of the paper fibres to their motion or the 'barrier effect' and the air impermeability of the paper seems reasonable. Some evidence for the formation of ions in lapped oil-impregnated-paper dielectric during the application of high impulse stresses is provided by the charged globules of oil that are clearly visible in an oil-filled cable model during dissection after several impulses just below the breakdown voltage have been applied.

No measurements of ionic mobility appear to have been made on the viscous compounds used in solid-type and gas-pressure cables, but it has been shown[20] that the ionic mobility in liquid paraffins is inversely proportional to (viscosity)$^{3/2}$, and therefore it is reasonable to assume that the ionic mobility in a rosin-containing compound at ambient temperature is very much smaller than that in o.f.c. oil. Accordingly, in dielectric impregnated with a rosin-containing compound any motion of ions from the butt gaps into the paper will be very small indeed at ambient temperature during the application of high impulse stresses. Moreover, since the ionic velocities will be very much lower, the paper fibres would be expected to provide a much more effective barrier; little or no dependence of the impulse strength of the dielectric on the air impermeability of the paper is therefore to be expected. However, when the dielectric is heated to 85° C the viscosity of the impregnating compound is reduced to the same order as that of o.f.c. oil, and hence an effect of the air impermeability of the paper on the impulse strength of the dielectric at this temperature is not surprising.

An interesting relevant practical matter may be mentioned. It is well known that creasing of the paper dielectric, which results in local damage to the fibrous structure of the paper, appears to have no effect on the impulse strength of solid-type or gas-pressure cables when tested at ambient temperature,[21, 22] whereas it can cause a significant reduction in the impulse strength of these cables at elevated temperatures and of oil-filled cables at both ambient and elevated temperatures. This behaviour is consistent with the above ideas on the impulse breakdown mechanism.

Hall and Skipper have suggested that the impulse breakdown of a lapped impregnated-paper dielectric is a two-phase process consisting of failure of the impregnant in a butt gap (by itself not necessarily leading to complete breakdown) followed by failure of the impregnated paper in series. In their examination of model cables after impulse testing, the present authors have never observed any local carbonization in the dielectric such as a partial breakdown would be expected to cause, the dielectric having a perfectly normal appearance apart from the actual breakdown path. For this reason the present authors consider it most unlikely that the impulse breakdown of lapped impregnated-paper dielectric is, in fact, a two-stage process. Local failure of the impregnant will undoubtedly cause a distortion of the field in the cable dielectric at the extremity of the discharge, but lack of information about the radii of discharge channels in cable impregnants prevents any precise calculation of its magnitude. However, the information given by Papadoyannis and Higham[23] on spark channels in transformer oil is of interest. In their experiments the radius of the spark channel rapidly increased after its initiation, presumably from molecular dimensions, reaching a value of $0\cdot 2$ mm after 1 microsec. The magnitude of the field existing at the extremity of a conducting discharge channel in a homogeneous solid dielectric has been considered by Mason.[24] He calculates that, if the dielectric is subjected to a uniform stress E_0 and it is assumed that the discharge channel forms a hemi-spheroid projecting perpendicularly from one of the electrodes, the length of the discharge l being much greater than the base radius r but much smaller

than the dielectric thickness, the stress at the extremity of the discharge channel l is given by

$$E = E_0 \frac{l^2/r^2}{\log_\varepsilon 2l/r - 1}$$

On this basis it is evident that the stress concentration produced under impulse conditions at the extremity of a discharge in the impregnant of an impregnated-paper dielectric will be very severe. This very severe stress concentration will last for only a fraction of a microsecond owing to the growth of the spark channel, and will exist over only a very small volume of dielectric. Nevertheless, the large magnitude of the stress concentration confirms the lack of evidence of partial breakdowns in indicating that breakdown of lapped fully-impregnated-paper dielectric is, in effect, a single-phase process. Accordingly, once local breakdown of the impregnant in a butt gap (if the impregnant has a high viscosity) or of the impregnant in a butt gap and in the adjacent paper (if the impregnant has a low viscosity) has occurred complete failure of the whole dielectric will ensue. The impulse strength of a cable will thus be determined primarily by the impulse strength of the impregnant, due allowance being made, where appropriate, for any influence the structure of the paper may have on the breakdown process in the impregnant.

Gazzana Priaroggia and Palandri, and Hall and Skipper have reported that the impulse strength of model cables impregnated with o.f.c. oil is reduced when a 50/50% registration is used instead of a 70/30% registration, although one would expect the electric stresses in the butt gaps to be lower in the former instance. Hall and Skipper have used this as evidence to conclude that the impulse strength of lapped oil-impregnated paper dielectric cannot be primarily dependent on the impulse strength of the impregnant in a butt gap. When discussing the breakdown process in a model cable, the effects of the space charges produced by the negative and positive ions formed in the impregnant in the butt gaps must be considered; with a 50/50% registration the space charges in any one butt gap will cause an increase in the actual electric stresses in adjacent butt gaps and thereby tend to reduce the breakdown voltage. The effect of the space charges on the electric stresses in adjacent butt gaps will clearly be more important with a 50/50% registration than with a 70/30% registration, and their action may well explain the lower impulse strength in the former case. One would expect that the effect of the space charges would be more pronounced the higher the ionic mobility, i.e. the lower the viscosity of the impregnant. This is in agreement with the observation[25] that the dependence of the impulse strength of a model cable on the registration is much less marked with a viscous compound than with o.f.c. oil.

(5.4) Impulse Strength of Cable Impregnants. Effect of Additives

It would appear that, whereas the impulse strength of impregnated-paper cable dielectric can be increased by suitably changing the characteristics of the paper, no means has yet been found of improving the impulse strength of the composite dielectric by appropriate modifications to the impregnant. Booth and Johnson[12] have demonstrated that the impulse strengths of oils can be raised by the addition of certain organic substances in small quantities, and this held out the prospect of significant increases in the impulse strength of oil-insulated electrical equipment. One of the most effective additives in o.f.c. oil is 8-hydroxy-quinoline, and the impulse strength of the oil when containing 2% of this additive is shown in Fig. 10 as a function of the dielectric thickness. It will be seen that, whereas for comparatively large dielectric thicknesses the impulse strength is improved by about 29% by the incorporation of the additive, at smaller thicknesses, where the impulse strength is dependent on the thickness, the effectiveness of the additive becomes less marked, and for thicknesses equal to the butt-gap thicknesses in cables the additive produces no improvement at all in the impulse strength. It has been verified that there is no significant increase in the impulse strength of model cables impregnated with o.f.c. oil containing this additive.

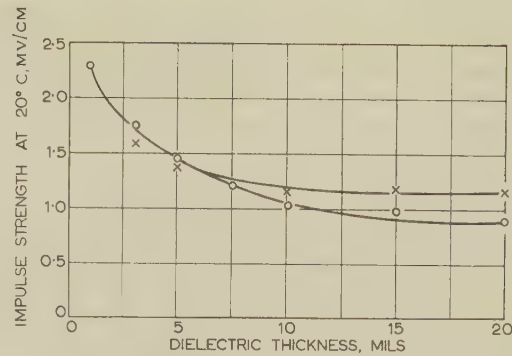


Fig. 10.—Effect of additive on impulse strength of oil-filled cable oil (normal viscosity).

○ Oil-filled cable oil.
× Oil-filled cable oil containing 2% 8-hydroxy-quinoline.

The reasons for these phenomena are not yet clear, but the interesting behaviour of certain additives underlines the need for more fundamental research into the breakdown of liquid dielectrics; this is clearly warranted by the importance of insulating oils and compounds in high-voltage apparatus.

(5.5) Relationship between Impulse Strengths of Cables, Model Cables and Cable Impregnants

Although the important relationship between the impulse strengths of cables, model cables and cable impregnants has been discussed, no attempt has been made to relate the impulse strengths quantitatively. The impulse strength of a cable or a model cable is lower than that of an impregnant, the principal reason being the stress concentration produced in the butt gaps of the former during the application of impulse voltages. Although the calculated stress in the butt gap of a model cable at breakdown is in fair agreement with the measured impulse strength of the impregnant, close agreement is not to be expected at this stage. It must be remembered, for example, that an 'open' test cell has been used for the impulse-strength measurements on impregnants, so that the measured values may have been affected by the presence of dissolved air; this matter is being investigated by the use of a 'closed' test cell which has been developed to enable measurements to be made on degassed impregnants. The somewhat higher impulse strength of a model cable compared with a cable is probably due, primarily, to the absence of stranding in a model and, on the basis of the statistical theory of extreme values[26] and the dispersion in the individual impulse strength measurements, to the much larger electrode area in a cable sample than in a model cable; this lack of agreement is not considered important, since it has been demonstrated that cable models provide a reliable means of comparing the behaviour of different materials when used in a cable against any given standard.

(6) CONCLUSIONS

The experiments on model cables have demonstrated the manner in which the impulse strength of fully-impregnated-paper cable dielectric is dependent on the density, air impermeability and thickness of the paper. It is concluded that the impulse strength is increased when the density of the paper, by itself, is

reduced. The effect of the air impermeability of the paper is related to the viscosity of the impregnant, an increase in the air-impermeability producing no significant change in the impulse strength of a dielectric impregnated with a viscous compound but an appreciable increase with an oil or heated compound having a low viscosity. The impulse strength is increased when the paper thickness is reduced.

The impulse strength of a cable-impregnating oil or compound varies with the dielectric thickness and the temperature. The variation with the dielectric thickness follows closely the dependence of the impulse strength of model cables on the paper thickness. Increase in temperature causes an appreciable reduction in the impulse strength of a rosin- or polyisobutylene-containing compound but results in no significant change in the impulse strength of o.f.c. oil. These temperature effects are practically the same as those observed in model cables.

Special papers have been obtained possessing a combination of properties designed to give a high impulse strength. These papers have been employed in an experimental oil-filled cable, the improvements in impulse strength closely approaching those indicated by measurements on model cables.

An impulse breakdown mechanism in a fully-impregnated-paper cable dielectric is suggested, the process being initiated in the impregnant at or near the conductor. If the dielectric is impregnated with a viscous compound the initial process is confined to the impregnant in a butt gap, but if the dielectric is impregnated with an oil or compound having a low viscosity the initial process takes place in the impregnant in a butt-gap and in the adjacent paper. After the local failure of the impregnant, which causes field distortion, complete breakdown of the dielectric rapidly ensues. The impulse strength of the cable is thus primarily dependent on the impulse strength of the impregnant, and the improvements which may be produced by varying the properties of the paper are due to the modifications in the breakdown processes in the impregnant which the changes in the paper cause.

(7) ACKNOWLEDGMENTS

The authors wish to record their thanks to Mr. W. C. Barry, Manager of the Research Laboratories, to Mr. S. E. Goodall, Director and Chief Engineer, and to W. T. Henley's Telegraph Works Co., Ltd., for permission to publish the paper. They also gratefully acknowledge the assistance of their colleagues in the company throughout the course of the investigations.

(8) REFERENCES

(1) GAZZANA PRIAROGGIA, P., and PALANDRI, G.: 'Research on the Electric Breakdown of Fully Impregnated Paper Insulation for High-Voltage Cables', *Transactions of the American I.E.E.*, 1956, **74**, Part III, p. 1343.

(2) HALL, H. C., and SKIPPER, D. J.: 'The Impulse Strength of Lapped Impregnated Paper Dielectric', *Proceedings I.E.E.*, Paper No. 2025 S, April, 1956 (**103 A**, p. 571).

(3) SALVAGE, B.: 'The Impulse Breakdown of High-Voltage Cables of the Solid and Gas-Cushion Types', *ibid.*, Paper No. 1468 M, February, 1953 (**100**, Part IIA, p. 163).

(4) EMANUELI, L.: 'The Emanueli Porosity Tester', *Paper Trade Journal*, 1927, **85**, No. 10, p. 48.

(5) DAVIS, R.: 'Impulse Electric Strength of High-Voltage Cables', *Journal I.E.E.*, 1942, **89**, Part II, p. 52.

(6) BENNETT, G. E.: 'Paper Cable Saturants: European Preferences and Selection', *Transactions of the American I.E.E.*, 1957, **76**, Part III, p. 687.

(7) TORIYAMA, Y.: 'Untersuchungen an Transformatorenöl', *Archiv für Elektrotechnik*, 1927, **19**, p. 31.

(8) WATSON, P. K., and HIGHAM, J. B.: 'Electric Breakdown of Transformer Oil', *Proceedings I.E.E.*, Paper No. 1501 M, March, 1953 (**100**, Part IIA, p. 168).

(9) ZEIN EL-DINE, M. E., and TROPPER, H.: 'The Electric Strength of Transformer Oil', *ibid.*, Monograph No. 135 S, June, 1955 (**103 C**, p. 35).

(10) HANCOX, R., and TROPPER, H.: 'The Breakdown of Transformer Oil under Impulse Voltages', *ibid.*, Paper No. 2408 M, September, 1957 (**105 A**, p. 250).

(11) SALVAGE, B.: 'The Dielectric Breakdown of Some Simple Organic Liquids', *ibid.*, Monograph No. 2, May, 1951 (**98**, Part IV, p. 15).

(12) BOOTH, D. H., and JOHNSON, O. S.: 'The Influence of Additives on the Breakdown Strength of Insulating Oils and Compounds', C.I.G.R.É., Paris, 1954, Paper No. 122.

(13) GOODWIN, D. W., and MACFADYEN, K. A.: 'Electrical Conduction and Breakdown in Liquid Dielectrics', *Proceedings of the Physical Society*, B, 1953, **66**, p. 85.

(14) HANCOX, R.: 'Dependence of the Dielectric Breakdown of Transformer Oil on Electrode Conditions', *Nature*, 1956, **178**, p. 1305.

(15) RUSSELL, A.: 'The Dielectric Strength of Air', *Philosophical Magazine*, 1906, **11**, p. 237.

(16) DOMENACH, M.: 'Cables with Reduced Thickness of Insulant under Pressure', C.I.G.R.É., Paris, 1948, Paper No. 214.

(17) LEWIS, T. J.: 'Electrical Breakdown in Organic Liquids', *Proceedings I.E.E.*, Paper No. 1488 M, March, 1953 (**100**, Part IIA, p. 141).

(18) WHITEHEAD, J. B., and MINOR, E. E.: 'Space Charge in Liquid Dielectrics', *Physics*, 1935, **6**, p. 380.

(19) SATO, T., NAGAO, S., and TORIYAMA, Y.: 'Space-Charge Limited Current in Insulating Oil', *British Journal of Applied Physics*, 1956, **7**, p. 297.

(20) ADAMCZEWSKI, I.: 'Mobilités des ions dans la série des carbures d'hydrogène liquides et leur rapport avec le coefficient de viscosité', *Annales de Physique*, 1937, **8**, p. 309.

(21) BRAZIER, L. G., HOLLINGSWORTH, D. T., and WILLIAMS, A. L.: 'An Assessment of the Impregnated Pressure Cable', *Proceedings I.E.E.*, Paper No. 1484 S, April, 1953 (**100**, Part II, p. 641).

(22) GAZZANA PRIAROGGIA, P.: 'Prove ad Impulso su Cavi ad Alta Tensione', *L'Elettrotecnica*, 1954, **41**, p. 289.

(23) PAPADOYANNIS, M., and HIGHAM, J. B.: 'Spark Channels in Liquids—Interim Report', E.R.A. Technical Report Ref. E/T68, 1956.

(24) MASON, J. H.: 'The Deterioration and Breakdown of Dielectrics resulting from Internal Discharges', *Proceedings I.E.E.*, Paper No. 1053, July, 1950 (**98**, Part I, p. 44).

(25) SYMONS, A. R. W.: Contribution to the Discussion on Reference 2.

(26) GUMBEL, E. J.: 'Statistical Theory of Extreme Values and some Practical Applications', U.S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series No. 33, 1951.

DISCUSSION BEFORE THE SUPPLY SECTION, 9TH MARCH, 1960

Dr. A. L. Williams: Two major and important papers on this subject* were published about four years ago reporting research work carried out contemporaneously, but independently, in two laboratories. Therefore, it is appropriate to inquire in what way this present paper advances knowledge, and it seems that a large part of it is a confirmation of results previously published.

There is, however, one point of controversy, namely whether an increase in the density of the insulating paper increases impulse strength or reduces it. This question is of great importance to the cable maker, especially at the present time. In an earlier phase of development of high-voltage cables, emphasis was laid on reducing capital cost by increasing the electric stress in the dielectric to the maximum, while still complying with impulse-strength requirements laid down by the user. The present phase is that of reducing revenue costs by reducing dielectric losses while, at the same time, trying to maintain the established levels of electric stress and impulse strength. Losses may be reduced by using paper of reduced density and it would be a happy circumstance for the cable maker if this also increased the impulse strength.

The authors state that Gazzana Priaroggia and Palandri (Reference 1 of the paper) consider that this is the case. This statement is not wholly justified because, although they did express this as an opinion in replying to the discussion, they made it very clear in the paper itself that they did not have samples of sufficiently widely varying characteristics to enable them to draw a conclusion. Hall and Skipper (see Reference 2 of the paper) were quite definite; after the statistical analysis of a vast number of experiments with a very wide range of papers, they concluded that increased density gives an increased impulse strength. In discussing this latter paper, Dr. Salvage claimed that the opposite is true.

In 1958, Thornton and Booth† produced a paper on the gas-filled cable system, and they suggested an empirical formula to relate impulse strength to paper impermeability and paper density; they agreed with Hall and Skipper regarding the effect of the latter. Now, Dr. Salvage and Mr. Gibbons again say categorically that increasing density reduces impulse strength. The evidence given on the matter in Section 2.3.1 is very weak, and is, in fact, the same as that given by Dr. Salvage in 1956 in the discussion on the paper by Hall and Skipper. Only two test samples of paper are mentioned, one of them having a density below the range used in high-voltage cable manufacture. The difference in the result was small, and although I do not question its reality, I do question whether there is any justification for attributing it to density alone.

The authors have disregarded another important paper‡ which drew attention to the paramount importance of considering the *quality* and *uniformity* of insulating paper before attempting to draw conclusions regarding the effects of the more easily measured properties of density, impermeability, etc. Had the authors taken this into consideration, I think that they would have found themselves in agreement with earlier workers on this point also.

Mr. C. C. Barnes: Section 4.3 of the paper by Irving§ read before The Institution in 1944 contained these observations.

> With the development in recent years of the various types of 132 kV pressure cables, the C.E.B. has now introduced impulse tests into its cable specification. The method of testing is that set out in B.S. 923.

* See References 1 and 2 of the paper.
† THORNTON, E. P. G., and BOOTH, D. H.: 'The Design and Performance of the Gas-Filled Cable System', *Proceedings I.E.E.*, Paper No. 2754 S, October, 1958 (106 A, p. 207).
‡ HALL, H. C., and KELK, E.: 'Physical Properties and Impulse Strength of Paper', *ibid.*, Paper No. 2024 S, April, 1956 (103 A, p. 564).
§ IRVING, D. B.: 'Cable Terminations', *Journal I.E.E.*, 1945, 92, Part II, p. 73.

Impulse tests made in 1944 were infrequent and confined to short cable lengths and terminations. Four important changes have been introduced more recently by the C.E.G.B. as follows:

(*a*) All cable accessories and terminations used with e.h.v. pressure cable systems are impulse tested in one assembly. This necessitates the circuit values of the impulse generator being adjusted to produce an impulse wave conforming to the requirements of B.S. 923: 1940, except that the wavefront may have any duration from 0·5 to 5·0 microsec. Oscillograms are made on short- and long-time sweeps to record the wavefront and wavetail durations of the test impulse wave.

(*b*) Before cable is included in the impulse test assembly it is subjected to a bending test.

(*c*) During impulse tests the conductor is maintained at a point between the maximum operating temperature and +5°C. This is essential because of the increased drop in impulse strength with temperature for dielectrics using thick compound compared with those using a low-viscosity oil.

(*d*) After having successfully completed tests at the specified impulse withstand level, the test assembly is then impulse tested to breakdown and a careful examination is made of the assembly.

In the Introduction it is stated that the main reason for impregnating the paper is to increase its electric strength. Other important reasons are to lower the thermal resistivity of the dielectric and to aid handling during processing.

It has been normal practice for some years with oil-filled cables to use the so-called 'low-viscosity oil'. Why is the information in Fig. 7 based on 'normal-viscosity oil'?

The modern trend in the British cable industry is the improved processing techniques used for e.h.v. cables. Can the authors state to what extent improved handling of the cable, better drying and impregnating techniques and precision application of the insulating tapes have increased the impulse strength of e.h.v. cables, as distinct from other factors such as the use of thin tapes, special papers, etc.?

Does 'conditioning' of a test assembly improve the impulse level of oil-filled or gas-pressure cable systems? If so, to what extent does it do this, and why?

Mr. D. H. Booth: From the title of the paper I had expected to obtain comparative impulse breakdown data on different types of high-voltage cable dielectric. I feel that such comparative data are of great value, and as the authors have confined themselves to the low-pressure oil-filled dielectric, I propose to compare their results with similar data we have obtained on the gas-filled dielectric.

On the question of the effect of paper characteristics we have found that for papers having moderate impermeabilities, say up to 1 000 Gurley sec, impermeability is of prime importance, but when the impermeability is increased above that figure, density becomes of particular significance. These results are in general agreement with those published by Hall and Skipper (Reference 2 of the paper) for the low-pressure oil-filled dielectric, and we are surprised to learn that the authors are at variance with these views.

We have made a particular study of the effect of the paper impregnant on the impulse breakdown strength of the dielectric, and typical results are shown in Fig. A. It will be seen that the physical nature of the impregnant appears to have an important effect on the nature of the impulse-strength/temperature characteristics.

The effect of gas pressure is also significant, and with nitrogen we find that an increase in pressure from atmospheric to 200 lb/in^2 results in about a 40% increase in impulse strength. It would be helpful if the authors could comment on the effect of pressure in their own type of dielectric.

Finally, we have shown that at 50 lb/in^2 the impulse strength of the composite dielectric filled with sulphur hexafluoride is

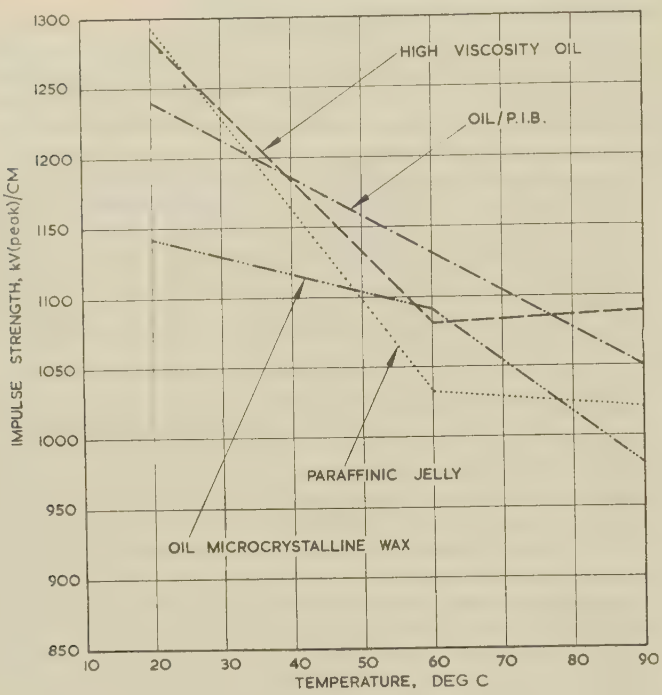


Fig. A

approximately 20% higher than that of the same dielectric filled with nitrogen.

In conclusion, therefore, we can state that with the gas-filled dielectric, changes of any one component can have quite significant effects on the composite impulse strength without there being any evidence, as suggested by the authors, that these effects are due to the resultant change on only one of the components.

Mr. D. J. Skipper: There appears to be no justification for assuming, as is done in the last paragraph of Section 5.3, that, with 50/50 registration, space charges in any one butt gap will cause an increase in stress in adjacent butt gaps. Any charge separation in the impregnant must reduce the stress in all the butt gaps and produce an increased stress in the radially adjacent paper. The authors state (without any experimental support) that space-charge effects 'will clearly be more important' with 50/50 than with 70/30 registration. However, our further work extending data given in Reference 2 of the paper shows that the effect of polarity reversal, which is determined by space-charge fields, is to reduce impulse strength by 18% with 70/30 registration but by only 12% with 50/50 registration. Thus the only relevant experimental evidence indicates that the authors' assumption is quite unwarranted.

Why did the authors use six paper tapes lapped with 75/25 registration in the construction of the cable models? With this arrangement the most probable breakdown path traverses two butt gaps and four impregnated paper tapes, and hence the results obtained are not characteristic of 75/25 or of any other actual cable registration.

In Section 5.1 it is stated that permittivity grading of the dielectric can give rise to a valuable increase in impulse breakdown voltage. Whilst this construction leads to a more uniform stress distribution in the paper, the stress in the impregnant (of a given permittivity) in a conductor-adjacent butt gap is determined solely by the overall capacitance of the cable. How do the authors reconcile the view that permittivity grading is beneficial with their postulated mechanism of breakdown?

Mr. E. Kelk: I am glad the authors recognize that the air impermeability of a paper is mainly controlled by the degree of beating of the pulp, because they are indirectly admitting the presence of another variable—formation or uniformity of fibre dispersion. Not only has the high-impermeability paper D been obtained by increased beating, but the low-density paper A also has had its impermeability brought up to that of paper B by increased beating. The main variable, therefore, is degree of beating, which has such a pronounced effect on the formation of the paper.

Difference in formation is largely responsible for some of the discrepancies in this paper. Considering Fig. 3, we reach the conclusion that we can expect a decrease in impulse strength of at least 120 kV/cm for each 1 mil increase in thickness. Applying this to papers B and E, which have almost identical densities and air impermeabilities, we would expect B to have an impulse strength at least 260 kV/cm lower than that of E, instead of 50 kV/cm lower as found by the authors.

Similarly, if we consider papers A and D we should be able to correct their impulse strengths to allow for differing physical properties. Assuming that A is correct, we then get an impulse strength of 930 kV/cm for D, instead of the 1 020 kV/cm of the authors. This difference is more marked than either of those shown in Figs. 1 and 2.

These two examples show how misleading it can be to try to establish the effect of one variable by taking two papers, especially when another variable, formation, has been ignored. When a larger number of papers are taken it will always be found that the highest impulse strengths are only obtainable with the highest-density papers.

Finally, I should like to know why paper F has been ignored in the full-scale cable experiments.

Dr. J. H. Mason: With the possible exception of Mr. Skipper, recent authors are agreed that impulse failure in oil-impregnated cables is initiated in the butt gaps, so that any factors which affect the inception voltage and magnitude of discharges in the butt gaps are likely also to affect the impulse strength.

Church and Krasucki* have shown that the presence of even 0·1% of residual moisture reduces the discharge inception stress in oil-impregnated paper capacitors, and I wonder whether the drying techniques used by the present and previous authors would ensure this degree of dryness.

Constandinou at the E.R.A. has investigated the effect of dryness on the electrical properties of small unimpregnated paper capacitors consisting of 5 mm-wide paper and 3 mm-wide metal foil. Even with such small capacitors he has found that at least 5 hours was required to obtain equilibrium dryness at 100° C and 0·01 mm Hg. In fact, he allowed 20 hours before making electrical measurements. In cable models, water must diffuse along a much longer path than in these capacitors, and I wonder whether 48 hours was adequate to attain complete and uniform dryness.

Have the authors used any electrical tests to assess dryness? Previous authors used loss-angle measurements, but the inherent losses in cellulose make such tests insensitive to small quantities of moisture. The E.R.A. dispersion meter† is much more sensitive and gives a linear variation of dispersion with moisture content. Constandinou's results show that moisture contents as low as 0·1% are measurable at 100° C.

Mr. A. M. Morgan: The theory advanced in Section 5.3 that the strength of oil-filled cable dielectric is enhanced by an increase in impermeability of the paper resulting in an improved barrier action does not correspond to the results of tests on models and oils. With infinitely impermeable paper the breakdown process in an oil-filled cable would be identical with that in a cable impregnated with a more viscous compound, and the

* *Journal of the Electrochemical Society*, June, 1960.
† E.R.A. Report Ref. V/T135; 1957.

relative breakdown strength of the models would depend only upon the strength of the impregnants.

In Fig. 8, for butt gaps of $2\frac{1}{2}$ and 5 mils, respectively, the electric strength of a viscous compound at 85° C is 14–18% higher than for an oil-filled cable model. Therefore, a cable model using a viscous compound should have a strength at least 14–18% higher than an oil-filled cable model. The results shown in Figs. 1, 2 and 4(*a*) give a higher strength for oil-filled cable models. Either the theory proposed by the authors is not valid, or the models using viscous compound have not been properly prepared.

The statement in the fourth paragraph of Section 5.3 regarding the deleterious effect of creasing on the impulse strength of a cable with viscous impregnant when tested at elevated temperature is not confirmed by published data. References 21 and 22 do not give any data in support of this opinion.

Tests carried out in our own laboratory on two compression cables, which were manufactured with identical construction, but one cable was lapped very tightly to produce heavy creasing and splitting of the papers close to the conductor whilst the second cable was lapped to give a crease-free dielectric, showed a 7% higher impulse strength for the cable with splits and creases.

Dr. F. J. Miranda: The authors' results are generally in agreement with those of Hall and Skipper and of Gazzana Priaroggia and Palandri, and, within the limits of Section 2, with the conclusions of our own investigation, so that I shall discuss mainly their testing technique, which is open to some criticism.

Standardization on six lapped layers of paper may introduce two possible sources of error. First, the variation of model thickness in the ratio 2 : 5 (2- and 5-mil papers were used) may give a misleading interpretation of the results. Within the range of thicknesses used I would expect the breakdown voltage to decrease as the thickness of the model is increased. Do the authors find that samples made with the same thin paper but with a greater number of layers give the same breakdown strength? A possible alternative would have been to standardize the thickness of the model rather than the number of layers.

The second possible source of error is the screen. With very thin insulation any field distortion such as that due to the sharp edge of metallized paper is of paramount importance. In our experience, to obtain results which genuinely represent the strength of an actual cable, considerably greater insulation thicknesses are necessary. Will the authors describe the means adopted to avoid sharp edge effects?

In Section 3.2 the authors state that as many as 50 breakdowns were obtainable with one filling of the oil test cell. In each discharge a fair amount of gas will be produced. Am I correct in assuming that this was relatively unimportant because of the poor degasification standard adopted? In any case, is it the authors' view that results obtained with a residual pressure of the order of 10 mm Hg are valid also for the highly degasified oils used in cables?

With regard to the suggested mechanism of breakdown in Section 5.3, there is no suggestion in the literature that ion mobility is a non-linear function of gradient. Much greater mobility would result if the breakdown phenomenon had sufficiently advanced to cause gas production, but in this case, the discharge would cause a field distortion, which, by itself, is adequate to cause total breakdown.

However, even if the ion mobility is one order of magnitude greater than that quoted, since with a 1/5 microsec wave the voltage is held at or near the crest for only a small fraction of a microsecond, the effect of ion motion should be small. I should like to have the authors' opinion on this point.

Mr. G. E. Bennett: Although it would be expected from the theory of breakdown advanced by the authors that oil-rosin-compound cable models would have a higher impulse strength at 85° C than cable models impregnated with o.f.c. oil, in all the cases shown [see Figs. 1, 2 and 4(*a*)] the reverse is found. The case of paper F in Fig. 4(*a*) should be specially noted.

In Section 5.3 it is stated that 'it must be concluded that, in oil-filled cable or a solid-type or gas-cushion cable tested at elevated temperatures, the impulse-breakdown process is not initiated solely in the impregnant in a butt gap'. I am puzzled by this explanation. If breakdown is initiated under impulse conditions in a viscous saturant at ambient temperature, i.e. if the saturant is the weak link in the chain, one would expect it to be an even weaker link in the chain at 85° C, because both the viscosity and the impulse strength are greatly reduced, and because the permittivity is reduced; and yet this is apparently not the case.

On the question of testing technique, it is clear that the authors have gone to great trouble and care in preparing the electrodes when testing cable saturants. They state that 'before each test the electrodes are polished with a soft cloth using jeweller's rouge', and also 'the precaution is taken to cover the electrodes with the test liquid immediately after cleaning', etc.; but apparently when testing cable models no care or precautions are taken with the inner electrode. This seems illogical, especially as the electrode in this case is a copper tube. Copper is a relatively soft metal easily pitted by electrical discharge. Experiments in our laboratories have shown that pitting which can be seen with the naked eye is produced by impulse waves both of 1/5 and 1/50 shape, even with a generator of small capacity if copper is used. This is one of the reasons why we have adopted electrodes of stainless steel for our cable model tests. I should like to know whether the copper electrodes used by the authors were polished prior to each test or whether an investigation was made to ascertain that the irregularities produced on the copper tube by previous breakdowns did not, in fact, influence subsequent breakdown tests.

Mr. E. H. Ball: The main conclusion of the paper is that there are considerable benefits to be gained by using thin highly impermeable papers in the highly stressed parts of the dielectric. In general, we can confirm this statement from tests on actual cables made in this way.

The use of metallized paper as a screen has already been criticized. I am surprised that the authors, having stated that a smooth copper cylinder is used for the inner electrode, should use metallized paper for the outer electrode, since the stress is almost the same with the dielectric thickness used, and close examination of metallized paper will show how irregular the edges are. I feel that stress concentrations at those sharp edges must be a source of error in the results.

With regard to Section 5.3, describing the suggested breakdown mechanism, there has already been discussion of the part which butt gaps play in the breakdown process. The authors emphasize their importance, and it may be of interest to consider the case of models made without gaps. Though not, of course, possible in practical cables, one can eliminate the gaps in cable models. In Reference 1 of the paper, using plane condenser samples, results are reported of tests without gaps, and it is of interest that the influence of impermeability and the viscosity of the impregnant is very similar to that reported in the present paper in a dielectric with gaps.

With regard to tests on actual cables, we have found in the examination of samples after test that the breakdown path is not necessarily through the gap adjacent to the conductor but may sometimes be directly through the first paper tape. We can confirm that we have never found any visible evidence of a secondary breakdown, showing that breakdown in all probability

takes place completely during the application of a single impulse.

Mr. W. Holttum: The paper suggests the need for some fundamental consideration as a guide to what may be expected to effect improvement in impulse strength. An impulse is like a blow. There is no time for deterioration; it is a question of instantaneous strength. It seems probable that impulse strength is closely related to strength under a mechanical blow.

If that connection is kept in mind it gives a good deal of guidance on the directions in which improvement in impulse strength may be effected. It indicates that density, viscosity of compound and freedom from voids should help, but pressure should have effect only in so far as it eliminates voids. This shows how the compression cable gains. I understand that if oil-filled cable is operated under pressure there is little advantage in impulse strength, and that fits in with my analogy. Higher temperature reduces viscosity, and that fits in also.

I suggest, therefore, that any variations which affect impulse strength in a way which does not correspond with the mechanical analogy should be looked at with suspicion, as there may be side effects.

In the Table appended to Fig. 2 it is astonishing to find that, with a density change of 0·89 to 0·88, impermeability is 20 times greater. Can the authors give some indication of the relation between impermeability and density, which one would expect to run together?

I am not sure to what extent the authors suggest that breakdowns occur in butt gaps at the conductor, but I would expect a butt gap to have little effect unless it were fairly thick.

Mr. R. E. James: For detecting failure during breakdown tests the authors state in Section 2.2 that 'a simple breakdown detector incorporating a gas-filled triode is used'. Can we have more details of this device? On the question of discharge inception during application of impulse voltages, have the authors used any sensitive method of detection in an attempt to determine whether discharges take place in the dielectric before breakdown? It seems possible to develop such methods, and sensitive techniques, for example a bridge circuit,* have been used in transformer testing for many years, but I do not know whether such methods are employed in the testing of cables.

I wish to comment on the practice of impregnating samples and then taking them out of the tank in which they were impregnated and putting them in another for testing. The authors have checked that this makes no difference, but inherently it seems wrong that, in the testing of insulation, the samples should be changed from one tank to another after they have been impregnated.

In measuring practical insulation dried by normal industrial procedures we find that on the dispersion meter the reading can be 1 or 2%.

Dr. T. J. Lewis: The problem of the relative importance of the oil and paper is far too complex for any conclusion to be reached about the influence of oil viscosity, paper density and air impermeability. In describing the system in terms of these quantities we are almost certainly excluding others which may be more important. All the experiments performed so far on these impregnated-paper dielectrics are so lacking in precise scientific control as to make the controversy indulged in by several speakers very futile.

In our laboratory we have tried to deal with one aspect of the breakdown of a composite dielectric by considering the liquid alone and simplifying the situation by studying these in clean systems. We are at present investigating a carefully prepared oil, a well purified paraffin *n*-hexane and an extremely simple liquid, namely argon. In all these cases the experimental control is much better than that used in any of the studies referred to in the discussion. Yet we are forced to conclude that the mechanism of breakdown is highly complex even in liquid argon. Three factors influence our current thinking. First, microscopic particles on the cathode can initiate the breakdown. Secondly, the residual gas content (especially oxygen), which in our work is far less than that claimed in the present studies, can change the strength by as much as 50%. Thirdly, the method of impulse testing significantly alters the ultimate strength. It is a serious omission that the present and earlier authors have ignored the time lag of breakdown. Our present picture is that dense streams of electrons burst out from sites on the cathode, and in the neighbourhood of the anode initiate a breakdown. This process is vitally dependent on conditions (e.g. gas content) in the anode region.

Translating this picture to the butt gap of a cable we have to pay attention to microscopic particles in the oil and to the oil-paper interface on the positive side of the gap. The residual gas on and between the paper fibres in this region then becomes extremely important and deserves controlled scientific study. Until such control is exercised, results will continue to reflect the particular testing techniques of each laboratory. It is naïve to think that the breakdown process in impregnated paper has been established, and it is a pity that the discussion could not have reflected a more earnest desire to understand the complex processes so crudely investigated.

[The authors' reply to the above discussion is on the next page.]

DISCUSSION BEFORE THE NORTH-MIDLAND CENTRE, AT YORK, 5TH APRIL, 1960

Mr. T. Wilson: With regard to the effect of space-charges in the butt gaps, the suggested mechanism is consistent with a voltage conditioning effect which would raise the impulse breakdown voltage of the model cables as measured by the authors. Have the authors observed any such effect, and if so, do they believe that more representative results might have been obtained using the procedure laid down in the C.E.G.B. type test for cables, or alternatively by reversing the polarity of alternate impulses as the voltage was increased?

The authors state that a valuable increase in impulse breakdown voltage can be obtained as a result of permittivity grading, by using a high-density high-permittivity paper around the central conductor, the rest of the dielectric consisting of low-density low-permittivity paper. Whether or not any gain is achieved must surely depend on the basis for comparison. If one accepts the authors' view that a low-density dielectric has a higher impulse strength than a high-density one, we would expect permittivity grading to result in an improvement when comparison is made with a cable made up entirely of high-density paper, but when compared with a low-density dielectric the gain might be small or non-existent. This would follow from the effect of paper permittivity on the butt-gap stresses, which feature very prominently in the authors' suggested breakdown mechanism.

Mr. C. A. Arkell: In Section 2.3.1, the effect of paper density has been investigated by using two papers with densities of 0·67 and 0·8 g/cm^3. The densities of papers used for high-voltage cables, however, are normally in the range from just below 0·8 to above 1·0 g/cm^3. Therefore I do not think that, from the results on these two papers, it is correct to conclude that impulse strength decreases with increase in density for the whole range

* WADLAND, D.: C.I.G.R.É., Paris, 1952, Vol. 1, p. 165.

of densities. In fact, the evidence obtained by my company from tests on a wide range of papers points in the opposite direction. While low-density high-impermeability papers can give impulse strengths approaching that obtained with high-density paper, the highest impulse strengths have been obtained on papers with a density greater than unity. As an example of this, two 3-mil papers were tested recently in models constructed in a similar manner to that described in the paper and impregnated with oil-filled cable oil. One had a density of $1 \cdot 05\,g/cm^3$ and an impermeability 60×10^6 Emanueli units and the other a density of $0 \cdot 86\,g/cm^3$ and an impermeability of 220×10^6 Emanueli units. Although the air impermeability of the high-density papers was lower than that of the low-density paper, its impulse strength was 5% higher. I think it would be wrong to conclude from our work that high density of itself is necessarily responsible for high impulse strengths. Another variable that must be taken into account in assessing a paper is its structural uniformity. The high-density papers, i.e. papers with a density of unity and above, have a better structural uniformity, probably due to the additional calendering operations in manufacture. To summarize the position, I feel that the paper characteristics necessary to obtain the highest impulse strength are high air impermeability and good structural uniformity. To achieve good structural uniformity means at present going to high density.

In trying to relate the physical properties of papers to impulse strength we must remember that these properties are not necessarily the same when wound on to a model or cable as that obtained from flat-sheet testing. For instance, when the thickness of the individual papers is determined from measurements of the insulation thickness of models, it is found that the value is less than that obtained from the standard method of measuring four flat sheets. This is due to shrinkage during drying and the comparably rough surfaces of the paper which permits fitting of the papers together during lapping. The reduced thickness produces an insulation with a higher density than that deduced from sheet testing. It has been found that this difference in densities increases as paper density is reduced. Have the authors considered this aspect and have they any values for the models insulated with the $0 \cdot 67$ and $0 \cdot 8\,g/cm^3$ density papers? I suspect that the difference in the insulation densities would be appreciably less than the difference in paper densities.

THE AUTHORS' REPLY TO THE ABOVE DISCUSSIONS

Dr. B. Salvage and **Mr. J. A. M. Gibbons** (*in reply*): We cannot agree with Dr. Williams that a large part of our paper is 'a confirmation of results previously published'. No detailed account of research into the impulse strength of cable impregnants has previously been given, and this is the first occasion on which impulse-strength measurements on an oil-filled cable have been reported in an Institution paper. Our experiments on model cables provide evidence on the effect of the paper density leading us to a different conclusion from that previously accepted (Reference 2 of the paper), and our view on this point is not based solely on the information given in Section 2.3.1, as Dr. Williams and Mr. Arkell infer, but also on the new work on super-calendered papers described in Sections 2.3.4 and 4.

Our conclusions regarding the dependence of the impulse strength of fully-impregnated paper on the paper density and the impulse strength of the impregnant appear to have aroused some controversy. We would make two comments:

(i) Those who believe that an increase in the paper density causes an increase in the impulse strength have not, so far, suggested any breakdown process which appears to be in satisfactory accordance with experimental observations.

(ii) Since the paper was written, de Vos and Vermeer* have described impulse-strength experiments on a dielectric comprising oil-impregnated plastic tapes, and, while recognizing the differences between this insulation and fully-impregnated paper, it is significant that they have come to the same conclusion as our own on the importance of the impulse strength of the impregnant.

Our reason for not mentioning the surface uniformity of the paper, to which Dr. Williams, Mr. Kelk and Mr. Arkell refer, is that our experiments on model cables have shown that this property is relatively unimportant compared with density, air impermeability and thickness.

Important differences in breakdown phenomena are to be expected in gas-filled cable dielectrics, which Dr. Williams and Mr. Booth mention, as distinct from fully-impregnated paper dielectrics. We presume that Mr. Booth's data in Fig. A refer to models and not actual cables.

* DE VOS, J. C., and VERMEER, J.: 'On the Impulse Strength of a Composite Insulation of Synthetic High Polymer Materials and Oil as a Dielectric for High and Extra-High Voltage Cables', C.I.G.R.É., Paris, 1960, Paper No. 226.

In reply to Mr. Barnes, there was no special reason for selecting normal-viscosity oil for the tests reported in Fig. 7, and we would expect similar results with low-viscosity oil. It is not possible to generalize on the precise improvements which better manufacturing techniques and, in certain circumstances, voltage conditioning cause; the increased impulse strength of the oil-filled cable reported in Section 4 is directly attributable to the use of special papers.

We think that the effects of space charges in butt gaps, raised by Mr. Skipper, will become clearer by referring to Fig. B. In

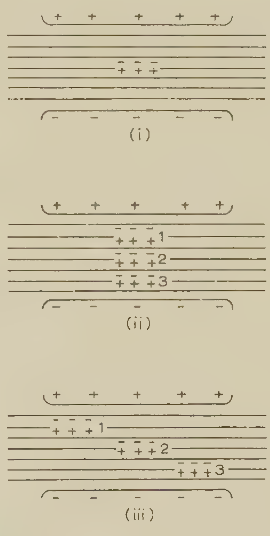


Fig. B.—Effect of space charges in butt gaps.

Fig. B(i) the charge separation in the impregnant in an isolated butt gap will reduce the stress in the butt gap, but in Fig. B(ii) the charge separation in the adjacent butt gaps 1 and 3 will tend to increase the stress in butt gap 2; in Fig. B(iii), where the butt gaps are not in line, the charge separation in butt gaps 1 and 3 will have much less effect on the stress in butt gap 2.

The available evidence regarding voltage conditioning by the application of repeated impulses and a polarity reversal effect,

which are mentioned by Messrs. Skipper and Wilson, is by no means unanimous. We believe that any testing procedure other than the one we have adopted would not have resulted in any more representative results.

Mr. Kelk's attempt to find discrepancies in the model cable results is unsound, since it is incorrect to assume that there is a linear dependence of the impulse strength on the paper thickness which is independent of the other properties of the paper. The full-scale cable experiments were designed primarily to show the effects of super-calendering the paper and there was no special reason for omitting paper F.

In reply to Dr. Mason, the moisture content of the paper in our model cable experiments was approximately 0·2% of the dry paper weight.

We agree with Messrs. Morgan and Bennett that the impulse strength of o.f.c. oil appears low compared with that of the compounds, and, as suggested in Section 5.5, this is possibly associated with the use of an 'open' test cell. This point is also relevant to Dr. Miranda's question regarding the degasification standard of our impregnants. However, we consider that this does not invalidate the essential features of our suggested breakdown mechanism in fully-impregnated paper. The impulse strength of our model cables was only very slightly dependent on the number of tapes, and standardization of the model cable thickness, as mentioned by Dr. Miranda, was not adopted since it would have resulted in an appreciably greater time for their construction. We consider that, for the mobility quoted in the paper, ionic motion will be significant even with 1/5 microsec waves; in such waves the voltage will be greater than 90% of the peak voltage for about a microsecond.

In reply to Mr. Bennett, we consider that in a dielectric impregnated with an oil or heated compound the breakdown process is initiated in the impregnant located not only in the butt gap but also in the adjacent paper. The tubes for our model cables were buffed and cleaned with trichloroethylene before applying the paper tapes.

The screen of each model, discussed by Dr. Miranda and Mr. Ball, was made from a single sheet of metallized paper, and not from tape, to avoid edge effects.

The mechanical strength of a dielectric, to which Mr. Holttum refers, and the impulse electric strength are quite different physical properties, and any argument by analogy is of doubtful value.

Mr. James will find details of the breakdown detector in Reference 12 of the paper. We are unaware of any discharge detection methods similar to those he mentions being used in impulse tests on cables.

Dr. Lewis's remarks on the breakdown of liquid dielectrics are in accordance with our reference in Section 5.4 to the need for more fundamental research into this subject. However, his speculations are at variance with much sound cable-testing experience. Dr. Lewis appears to have misunderstood the purpose of our experiments on model cables. They were governed by the urgent need for information on the best possible materials to use in cable manufacture, and accordingly the construction, processing and testing of the models followed as closely as possible techniques adopted with actual cables. We hope that in his enthusiasm for a more fundamental long-term approach he will not overlook the success we have achieved in increasing the impulse strength of cable dielectrics.

Our mention of permittivity grading, referred to by Messrs. Wilson and Skipper, related to the benefit it effects when a high-density paper is used next to the conductor.

It is difficult to comment on Mr. Arkell's results in the absence of precise details of the paper thicknesses and the number of tests. We are well aware of the dimensional changes of the paper during the processing of the model cables, but they do not affect our conclusion regarding paper density.

621.314.2.045.15 : 621.3.015.3

The Institution of Electrical Engineers
Paper No. 3286 S
Oct. 1960

AN EXPERIMENTAL STUDY OF SURGES AND OSCILLATIONS IN WINDINGS OF CORE-TYPE TRANSFORMERS

By E. L. WHITE, B.Sc.(Eng.), Associate Member.

(*The paper was first received* 29*th April*, 1959, *and in revised form* 13*th April*, 1960.)

SUMMARY

Measurements on two experimental core-type transformers, one with two limbs and the other with three limbs, were used to test the validity of theories of transient oscillations in windings with particular reference to Coleman's theoretical treatment of multi-limb transformers. Wherever practicable, 'equivalent single-limb' parameters were measured in accordance with this theory. Spectral distributions of oscillation frequencies and variations of mutual leakage inductance between winding elements with separation between elements were adopted as the main criteria in comparisons of experiment and theory.

These comparisons showed the superiority of an exponential distribution of mutual inductance over other approximations assumed in theories. No critical oscillation frequency as predicted by Rüdenberg was reached or approached, but some support was found for assuming the velocity of travelling waves to be independent of frequency when calculating inter-section voltages.

Measured responses in terms of spatial distribution, amplitude and waveform showed reasonable agreement with theory. The analytical treatment by Coleman of the transient behaviour of multi-limb transformers was verified by numerous tests.

LIST OF SYMBOLS

l = Axial length of a limb winding from neutral end to line end.

x, s = Axial distances along a limb winding from neutral end.

$C_G = lC_g$ = Capacitance to core and tank of a single-limb winding.

C_{G0}, C_{G1}, C_{G2} = Equivalent single-limb capacitances to core and tank.

$C_K = lC_k$ = Capacitance between adjacent limb windings.

$C_H = lC_h$ = Capacitance between limb windings A and C on a 3-limb transformer.

C_S = Series capacitance between line and neutral ends of a limb winding.

$\alpha = (C_G/C_S)^{1/2}$ = Initial distribution constant.

m = Number of voltage antinodes in length l for a single-frequency oscillation.

$f_m = \omega_m/2\pi$ = Frequency of mth oscillation mode.

t = Time.

$M(x, s)$ = Mutual leakage inductance between unit lengths of a limb winding at distances x and s from neutral end. (Unit length = length of winding element = one disc pitch).

M_0 = Self-leakage inductance of a unit length of winding (winding element, disc).

L = Effective inductance per disc.

ψ = Spatial decrement factor in the expression for $M(x, s)$.

μ_m, ν_m = Constant factors in relation to the mth oscillation mode.

v_m = Velocity of wave along conductor at the mth oscillation frequency.

l_w = Length of conductor in one limb winding.

V_T = Applied peak impulse voltage.

A, B = Constants defining linear distribution of mutual inductance.

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

Mr. White is with the British Electrical and Allied Industries Research Association.

(1) INTRODUCTION

In the design of power transformers consideration must be given to the characteristics of the transient oscillations which can occur within the windings due to surge voltages applied to the terminals. The possibility of computing the surge performance of new designs of winding has encouraged the formulation of numerous theories[1–9] to account for the phenomena which can be observed experimentally in existing transformers. A survey of published theories is given in Reference 10.

It is the principal object of the paper to examine the validity of existing theories of transient oscillations in transformer windings in the light of experimental evidence, with particular reference to Coleman's theory.[1]

Published accounts of experiments[5, 8, 10, 11, 13, 15, 16] aimed at verifying the elementary theories of oscillations in iron-cored windings have not shown beyond doubt that any one treatment of the problem is markedly superior to the others, yet the assumptions upon which the various theories are based show considerable diversity, especially as regards the treatment of mutual inductances between sections of the windings. For practical applications it is necessary to extend and elaborate theories derived from considerations of quite simple types of winding, to take into account such common complicating features as voltage-regulating tappings, non-uniform spacing of turns and series connection of coaxial windings. It is not unlikely that such calculations of the transient response of windings could lead to tolerably correct results although founded on unsound principles.

In these circumstances critical re-examination of simple types of iron-cored winding was felt to be justified. The experiments described in the paper were arranged with particular regard to the theory developed by Coleman.[1] In view of the prevalence of core-type transformers in this country, only this type is considered, and the behaviour of transformers having more than one limb wound (multi-limb transformers) is therefore regarded as a matter of major importance. As Coleman's theory does not cover oscillatory transients involving secondary windings, steps were taken to avoid as far as possible the occurrence of such effects in the experiments. Measurements were made on the higher-voltage windings of 2-winding transformers having large turns ratios, with the lower-voltage windings open-circuited and earthed. Under these conditions induced voltages and currents in the lower-voltage windings were small compared with those in the higher-voltage windings and their effects were neglected. Radial voltage gradients within the windings were not measured, nor are they considered in the theory. It should not be inferred that secondary oscillations and radial voltage gradients are necessarily negligible in practical designs, but to investigate

these phenomena thoroughly would involve much additional theoretical and experimental work.

The investigation is confined to those modes of oscillation usually associated with the term 'earthed neutral' (though a neutral-earthing connection is not always an essential feature). Isolated-neutral conditions occurring in practice are invariably found in conjunction with delta or parallel windings on the same core, and in these circumstances the effects of these other windings are too large to be ignored.

(2) GLOSSARY OF SPECIAL TERMS

Special terms used in the paper are defined as follows.

Limb Winding.—A winding on one limb. Limb windings are designated A and B in a single-phase 2-limb transformer and A, B and C in a 3-phase 3-limb transformer. The ends of each limb winding are designated line end and neutral end respectively. Series or star connections are assumed.

Tandem Condition.—A symmetrical condition in which equal voltages are applied to the line ends of all limb windings.

Single-Opposition Condition.—A symmetrical condition in which equal and opposite voltages are applied to the line ends of two limb windings (A and C in the case of a 3-phase transformer).

Double-Opposition Condition.—A symmetrical condition in which equal voltages are applied to the line ends of limb windings A and C and a voltage of twice the amplitude and of opposite polarity is applied to the line end of limb winding B.

Equivalent Single-Limb Winding.—A fictitious winding on an isolated single limb, whose parameters are the effective parameters per limb of a multi-limb transformer subjected to one of the three conditions already defined.

Symmetrical Mode.—An oscillation mode in which the voltages at any two points at equal distances from the line and neutral ends are equal and in phase.

Antisymmetrical Mode.—An oscillation mode in which the voltages at any two points at equal distances from the line and neutral ends are equal but in phase opposition.

(3) EXPERIMENTAL TRANSFORMERS

The tests to be described were performed on two experimental transformers, one single-phase (2-limb)[11] and one 3-phase (3-limb),[12] each having two concentric windings. All results, except those of Fig. 15, relate to the outer windings.

As the inner winding of the 2-limb transformer has few turns, the conditions for negligible secondary oscillations were achieved by earthing it at one point. The turns ratio of the 3-limb transformer is nominally unity, but to comply with these same conditions the winding not under test was subdivided into many small sections (single discs) and one end of each section was earthed.

Both transformers have laminated cores with interleaved joints and substantially uniform windings of paper-insulated copper strip wound in single discs, closely resembling commercial transformers. Exceptional features which deserve mention here are:

(*a*) The outer winding of the 2-limb transformer is rated at 12·5 kV but is insulated for 33 kV. This arises from the fact that the transformer was intended solely for surge investigations on a typical coil structure and that, in order to keep its weight within the lifting capacity available at the time, the cross-section of the core was reduced to a fraction of its normal area. There are minor differences between the limbs in the matters of insulation and numbers of turns per disc.

(*b*) Each end of each inner disc of the 3-limb transformer projects through the radial ducts of the outer winding, and the usual insulating cylinder between windings is omitted. In both windings, discs are interconnected only through removable links.

Other details can be found in Section 14.

Throughout the investigation a single disc was regarded as the smallest winding element to be considered.

(4) CAPACITANCES AND INITIAL DISTRIBUTION CONSTANTS

Direct measurement of series capacitance in a continuous winding is impracticable. Following a previously described practice,[13] the series capacitance was estimated from a measurement between two adjacent discs, with the interconnection between discs removed, dividing the result by the number of radial ducts.

Capacitances between limb windings and capacitances from limb windings to core and tank are of a distributed nature but can be effectively lumped for measurement with an a.f. bridge by joining the terminals and any available tappings in each limb winding. From the measurements obtained, the equivalent single-limb capacitances to earth for the three symmetrical conditions defined in Section 2 were deduced from expressions given by Coleman as follows:

Tandem condition (2- or 3-limb) $C_{G0} = C_G = lC_g$

Single-opposition condition (2-limb) $C_{G1} = C_G + 2C_K$
$= l(C_g + 2C_k$

Single-opposition condition (3-limb) $C_{G1} = C_G + C_K + 2C$
$= l(C_g + C_k + 2C_h$

Double-opposition condition (3-limb) $C_{G2} = C_G + 3C_K$
$= l(C_g + 3C_k$

Impulse voltages complying with these same terminal conditions were applied to the transformers and instantaneous voltage distributions for several short intervals of time from the start of the impulses were plotted from oscillograms with the object of determining the equivalent single-limb values of α, the initial distribution constant of theory. Assuming that any of these instantaneous distributions corresponds to the initial distribution, a value of α can be obtained from each distribution curve by a graphical method described by Selig.[11] There is found to be a variation of α with time interval, as illustrated by Fig. 1.

It is necessary to select a value of α, for each terminal condition

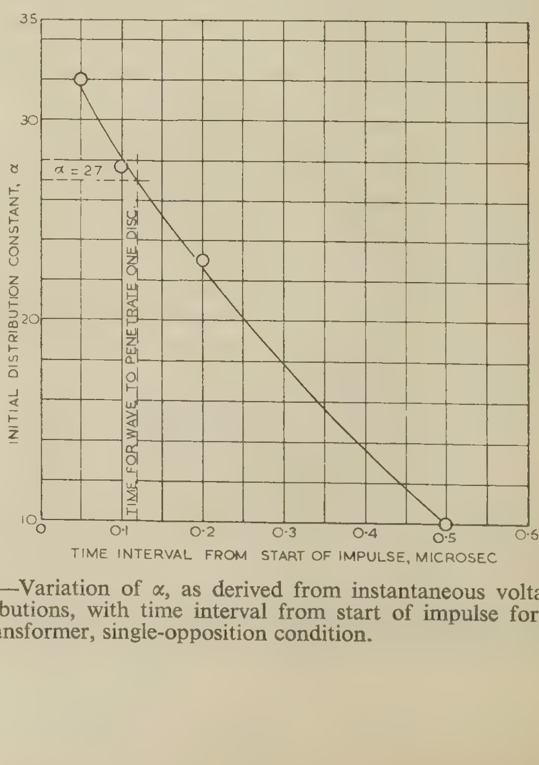


Fig. 1.—Variation of α, as derived from instantaneous voltage distributions, with time interval from start of impulse for 2-limb transformer, single-opposition condition.

which is appropriate to the present analysis. Distributions measured at intervals much less than 10^{-7} sec from the start cannot be regarded as appropriate, for two reasons:

(*a*) The capacitance network of the whole winding cannot all be charged simultaneously, as assumed in the theory. In fact, propagation of the field along the winding length would require a time of the order of 5×10^{-9} sec.

(*b*) For such short intervals, a single disc would need to be represented as a distributed circuit.

On the other hand, for intervals much greater than 10^{-7} sec from the start there is the likelihood of appreciable sharing of charge between the capacitances of adjacent discs due to the flow of current through the inductances. The movement of a charge between corresponding points on adjacent discs cannot, however, take place in less time than the time of propagation of the charge through the whole length of conductor in one disc. This is estimated from the velocity of waves in windings determined by Norris[14] and is found to be approximately 0·12 microsec for the 2-limb transformer and approximately 0·07 microsec for the 3-limb transformer.

A further consideration is the finite duration of the wavefront of the impulses applied in the tests, on account of which the average time available for movement of charge is less than the duration of the interval. Taking into account these various considerations, it was decided to choose intervals equal to the wave-propagation times through single discs to obtain appropriate values of α.

The results of the measurements of capacitances and estimates of initial distribution constants are grouped together in Table 1.

Table 1

Equivalent Single-Limb Capacitance Parameters

1	2	3	4	5
Terminal condition	Measured series capacitance, C_S	Measured shunt capacitance	Initial distribution constant, α	[Col. 3/ Col. 2]$^{1/2}$ $= \alpha$
	pF	pF		
2-Limb transformer:				
Tandem	3·4	1 520	23	21
Single-opposition	3·4	1 850	27	23
3-Limb transformer:				
Tandem	3·6	2 730	25	28
Single-opposition	3·6	2 830	26	28
Double-opposition	3·6	2 910	27	28

There are discrepancies between the values of α in column 4 obtained from voltage distributions and those in column 5 calculated from measured capacitances, indicating experimental errors. It is believed that the measurements of series capacitance are the least reliable, and for this reason the results in columns 3 and 4 of the Table were used in subsequent considerations. These equivalent single-limb parameters have distinctly different values for the different terminal voltage conditions, although the differences are not very great.

(5) OSCILLATION FREQUENCIES

Oscillation frequencies for each symmetrical terminal condition were measured by detecting resonances with an applied sinusoidal voltage. A variable-frequency oscillator used as the source was connected between the line terminals and core for tandem conditions, and for opposition conditions it was coupled to the terminals via a small transformer having appropriate ratios.

Resonances were indicated as voltage maxima on a valve voltmeter connected between winding and core. For symmetrical modes the voltmeter was connected to the mid-point, whilst for antisymmetrical modes the connection was moved to a point approximately $\lambda/4$ from the mid-point. The measured frequencies for both transformers are given in Table 2. The ratio

Table 2

Measured Oscillation Frequencies

Oscillation mode, m	Frequency				
	2-Limb transformer		3-Limb transformer		
	Tandem	Single-opposition	Tandem	Single-opposition	Double-opposition
	kc/s	kc/s	kc/s	kc/s	kc/s
1	31·9	21·1	43·8	27·2	23·3
2	78	58·5	115	94	84
3	138	116	209	185	172
4	196	170	312	284	270
5	255	230	418	387	375
6	309	281	527	495	481
7	361	337	632	605	590
8	409	383	740	713	700
9	462	433	841	820	805
10	509	486	—	910	—
11	558	—	—	—	—
12	605	—	—	—	—

of the mth oscillation frequency, f_m, to the fundamental frequency, f_1, affords a useful criterion upon which to assess the merits of theories of oscillations in windings.

Theoretical frequency ratios according to Wagner[2] and Rüdenberg[3] are given by

$$f_m/f_1 = m(\pi^2 + \alpha^2)^{1/2}(m^2\pi^2 + \alpha^2)^{-1/2} \quad . \quad . \quad (1)$$

whereas according to Blume and Boyajian[4] and Bewley[6] the ratios should be

$$f_m/f_1 = m^2(\pi^2 + \alpha^2)^{1/2}(m^2\pi^2 + \alpha^2)^{-1/2} \quad . \quad . \quad (2)$$

These are plotted, together with ratios determined experimentally, in Figs. 2–5. Only for the double-opposition condition of the 3-limb transformer do the experimental ratios approximate to either of the above expressions; in that case the results are fairly near to Bewley's frequency ratios.

Coleman gives the frequency ratio as

$$f_m/f_1 = \mu_m\nu_m/\mu_1\nu_1 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where
$$\mu_m^2 = \alpha^2(\nu_m^2 + \psi^2)/(\nu_m^2 + \alpha^2) \quad . \quad . \quad . \quad . \quad (4)$$

and
$$\nu_m \tan \tfrac{1}{2}\nu_m + \mu_m \tanh \tfrac{1}{2}\mu_m + (\mu_m^2 + \nu_m^2)/\psi = 0 \quad . \quad (5)$$

when m is odd (i.e. symmetrical modes) for tandem and opposition conditions, or

$$\mu_m \coth \tfrac{1}{2}\mu_m - \nu_m \cot \tfrac{1}{2}\nu_m + (\mu_m^2 + \nu_m^2)(2 + \psi)/\psi^2 = 0 \quad . \quad (6)$$

when m is even (i.e. antisymmetrical modes) for opposition conditions only, or

$$\mu_m \coth \tfrac{1}{2}\mu_m - \nu_m \cot \tfrac{1}{2}\nu_m + (\mu_m^2 + \nu_m^2)/\psi = 0 \quad . \quad (7)$$

when m is even, for tandem condition only.

By assuming values for ψ and using conditions (4), (5) and (7), the ratios given by eqn. (3) were calculated and plotted as in Figs. 2 and 4. These are nearer to the experimental results

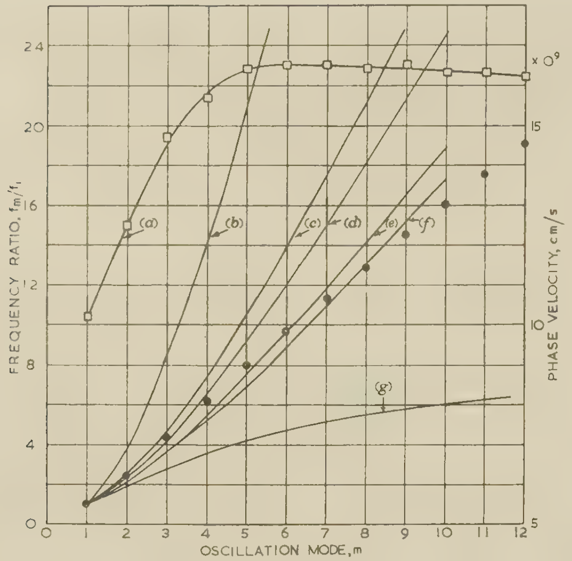


Fig. 2.—Frequency ratios and phase velocities for 2-limb transformer, tandem condition.

(a) Phase velocity deduced from measured frequencies.
(b) Frequency ratio curve computed from eqn. (2) (Blume and Boyajian, Bewley).
(c) Frequency-ratio curve computed from eqn. (8) for $\psi = 6$ (Coleman, approximate).
(d) Frequency-ratio curve computed from eqns. (3), (4), (5) and (7) for $\psi = 6$ (Coleman).
(e) Frequency-ratio curve computed from eqn. (8) for $\psi = 10$ (Coleman, approximate).
(f) Frequency-ratio curve computed from eqns. (3), (4), (5) and (7) for $\psi = 10$ (Coleman).
(g) Frequency-ratio curve computed from eqn. (1) (Wagner, Rüdenberg).
● Measured frequency ratios.

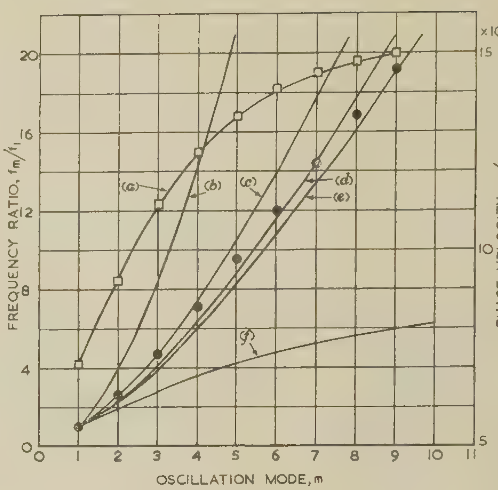


Fig. 4.—Frequency ratios and phase velocities for 3-limb transformer, tandem condition.

(a) Phase velocity deduced from measured frequencies.
(b) Frequency-ratio curve computed from eqn. (2) (Blume and Boyajian, Bewley).
(c) Frequency-ratio curve computed from eqn. (8) for $\psi = 6$ (Coleman, approximate).
(d) Frequency-ratio curve computed from eqn. (8) for $\psi = 8$ (Coleman, approximate).
(e) Frequency-ratio curve computed from eqns. (3), (4), (5) and (7) for $\psi = 7 \cdot 7$ (Coleman).
(f) Frequency-ratio curve computed from eqn. (1) (Wagner, Rüdenberg).
● Measured frequency ratios.

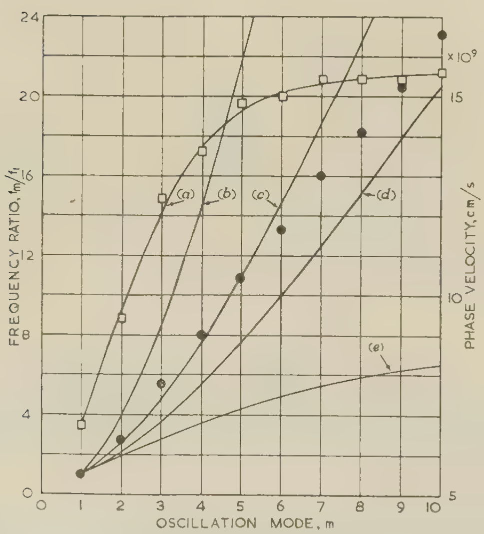


Fig. 3.—Frequency ratios and phase velocities for 2-limb transformer, single-opposition condition.

(a) Phase velocity, deduced from measured frequencies.
(b) Frequency-ratio curve computed from eqn. (2) (Blume and Boyajian, Bewley).
(c) Frequency-ratio curve computed from eqn. (8) for $\psi = 6$ (Coleman, approximate).
(d) Frequency-ratio curve computed from eqn. (8) for $\psi = 10$ (Coleman, approximate).
(e) Frequency-ratio curve computed from eqn. (1) (Wagner, Rüdenberg).
● Measured frequency ratios.

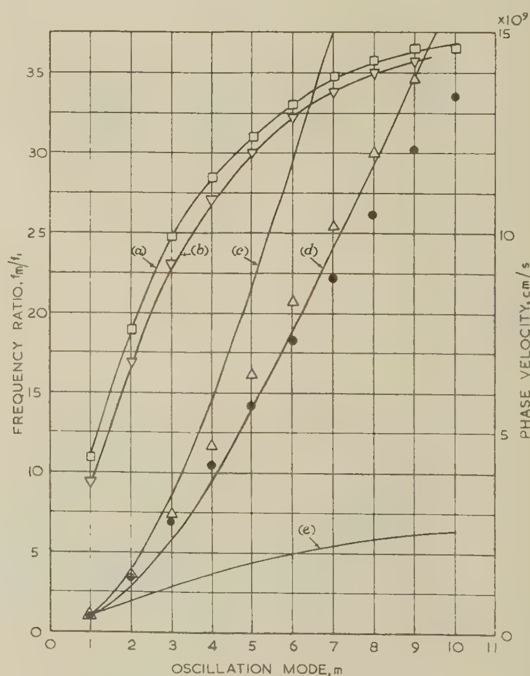


Fig. 5.—Frequency ratios and wave velocities for 3-limb transformer, single-opposition and double-opposition conditions.

(a) Single-opposition phase velocity, deduced from measured frequencies.
(b) Double-opposition phase velocity, deduced from measured frequencies.
(c) Frequency-ratio curve computed from eqn. (2) (Blume and Boyajian, Bewley).
(d) Frequency-ratio curve computed from eqn. (8) for $\psi = 4$ (Coleman, approximate).
(e) Frequency-ratio curve computed from eqn. (1) (Wagner, Rüdenberg).
Curves (c), (d) and (e) are for single-opposition condition.
● Measured frequency-ratios, single-opposition condition.
△ Measured frequency-ratios, double-opposition condition.

than the ratios predicted by Bewley or Rüdenberg, though the choice of a suitable value for ψ appears to depend on the range of frequencies it is desired to represent. Similar remarks apply to ratios plotted in Figs. 2, 3, 4 and 5 from the expression

$$f_m/f_1 = m[(\psi^2 + m^2\pi^2)(\alpha^2 + \pi^2)]^{1/2}[(\psi^2 + \pi^2)(\alpha^2 + m^2\pi^2)]^{-1/2} \quad . \quad . \quad . \quad . \quad (8)$$

which can be obtained from eqns. (3) and (4) by the approximation

$$\nu_m \simeq m\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It is interesting to compare oscillations of the same mode for different terminal conditions. For example, in the 2-limb transformer the fundamental frequency ($m = 1$) for opposition condition is only 66% of the frequency for tandem condition, and in the 3-limb transformer the single- and double-opposition fundamental frequencies are 62% and 53% respectively of the tandem frequency. As m is increased the ratio of frequencies of corresponding oscillation modes approaches unity. These large differences in frequency are not accounted for by the capacitance parameters as determined in the previous Section, and hence large differences in the self- and mutual-inductance parameters can be expected. On the other hand, there are relatively small differences between single-opposition and double-opposition frequencies, which suggests that a reasonable approximation can be made for the 3-limb transformer by assuming equal parameters for these two conditions, i.e. assuming 'triangular' symmetry.

The frequency ratios for the 2-limb transformer, tandem condition, are numerically quite close to those obtainable from experimental results given in Reference 15 for an iron-cored coil. Calculations of the phase velocity in this same coil from the field theory of wave propagation are published in Reference 8 and from these can be deduced the frequency ratios, which are found to be from 20 to 40% higher than the measured ratios up to $m = 9$. Other calculations of frequencies given in Reference 15 based on an equivalent air-cored coil show much larger divergence from the experimental ratios. It appears from the various results that for two iron-cored windings having comparable frequency characteristics, the computations of the frequency ratios by Coleman's theory and by the theory of Poritski, Abetti and Jerrard are about equally successful.

(6) MUTUAL LEAKAGE INDUCTANCES

The mutual inductance between two winding elements is due to linkages with a common flux which can be divided into a core flux which is wholly in iron, and a leakage flux which is only partly in iron or does not enter the iron at all.

That part of the mutual inductance due to leakage-flux linkages is defined in the theory as mutual leakage inductance. The mutual leakage inductance $M(x, s)$ between elements at distances x and s from the neutral end is assumed by Coleman to obey the law

$$M(x, s) = M_0\varepsilon^{-\psi|x-s|/l} \quad . \quad . \quad . \quad . \quad (10)$$

where M_0 is the limiting value of $M(x, s)$ when $x = s$, i.e. it is identified with the self leakage inductance of a single element. M_0 and ψ are assumed to be independent of frequency.

It is also assumed that the core flux induced by a current in a single turn (or single element) is independent of the position of the turn (or element) relative to the yokes. From this it follows that a current flowing through two discs connected in series and in magnetic opposition produces no resultant core flux. The effective inductance of this circuit is due entirely to leakage-flux linkages and can be expressed as

$$2L = 2[M_0 - M(x, s)] \quad . \quad . \quad . \quad . \quad (11)$$

The validity of eqns. (10) and (11) was checked by measuring the effective inductances of such pairs of discs for various values of x and s. Some elaboration of the method was necessary in order to determine the equivalent single-limb values of L for the multi-limb transformers. The test circuits for tandem, single-opposition and double-opposition conditions are shown in Figs. 6 and 7. Corresponding discs on two, or three, limbs

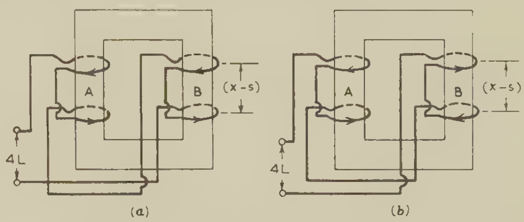


Fig. 6.—Circuits for determining the effective inductance, $2L$, of a pair of discs on an equivalent single-limb winding for a 2-limb transformer.

(a) Tandem condition.
(b) Single-opposition condition.
One disc is represented as a single turn.

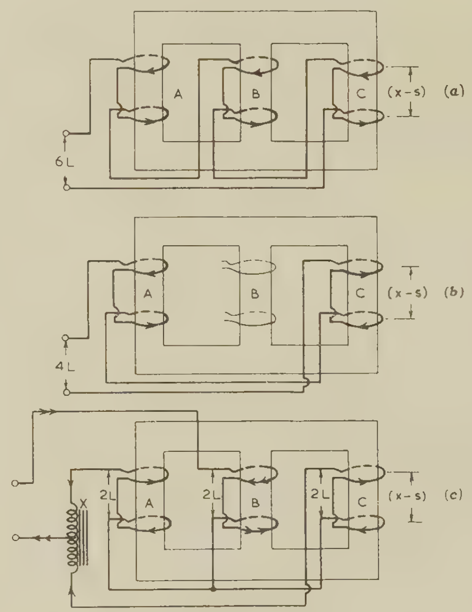


Fig. 7.—Circuits for determining the effective inductance $2L$ of a pair of discs on an equivalent single-limb winding for a 3-limb transformer.

(a) Tandem condition.
(b) Single-opposition condition.
(c) Double-opposition condition.
One disc is represented as a single turn.

were interconnected in such a way that the magnitudes and directions of the currents in them bore the same relation as the appropriate terminal voltages, as defined in the glossary. For the double-opposition condition, in order to ensure equality of the currents in limb windings A and C, the two branches of the circuit were supplied from a centre-tapped reactor, X, and an average was taken of the apparent inductances of the three pairs of discs. In the case of the 2-limb transformer, measurements were confined to discs having the normal number of turns (see Section 14).

The inductances of these circuits were measured at 50 c/s by comparison with a calibrated mutual inductor, selecting pairs of discs which were arranged:

(*a*) Asymmetrically with respect to the mid-point, with x variable and $s = \frac{1}{2}l$ (2-limb) or $s = l$ (3-limb).

(*b*) Symmetrically with respect to the mid-point, with x and s variable, $x + s = l$.

The differences between the results from (*a*) and those from (*b*) were found to be negligible, thus confirming the assumption that any effect of proximity to the yokes could be neglected. The change in effective inductance due to short-circuiting the inner winding was also negligible, which verified that the same core flux is produced by any element carrying a given current.

The variation of effective inductance per disc with separation $|x - s|$ between discs is shown in Figs. 8 and 9. For tandem conditions on both transformers the variation can be seen to be roughly exponential, as postulated in eqn. (10), whereas this is not so evident for opposition conditions. Indeed, the double-opposition results in Fig. 9 approximate to a straight line through the origin, corresponding to one of the hypothetical approximations tried by Pirenne, who concluded that the resulting induction equation agreed with that obtained by Bewley. According to Pirenne's interpretation of Bewley's equations the distribution of mutual inductance takes the form

$$M(x, s) = A - B(|x - s|)$$

where A and B are constants. When $x = s$ this becomes

$$M_0 = A$$

The effective inductance of two discs in series and in magnetic opposition is then

$$2L = 2[A - M(x, s)] = 2B(|x - s|)$$

and Pirenne shows that, using the present notation,

$$\omega_m^2 = \frac{m^4\pi^4\alpha^2}{2l^3BC_G(\alpha^2 + m^2\pi^2)} \quad . \quad . \quad . \quad . \quad (12)$$

Using this expression, B was evaluated from measured values of α, C_{G2} and ω_m for double-opposition condition. The corresponding linear distributions of L are shown in Fig. 9 for $m = 1$ and for $m = 9$. The distribution for $m = 9$ shows fair agreement with measured values.

In Section 5, values of ψ were found which, applied to Coleman's rigorous or approximate expressions, gave a reasonable fit between measured and computed frequency ratios. Using these same values of ψ and the measured frequencies, the corresponding values of M_0 were found from the equation

$$M_0 = \frac{\mu_m^2\nu_m^2}{2\omega_m^2\psi C_G l^2} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Then using eqns. (10) and (11) a computed curve for L was drawn for each assumed value of ψ, as in Figs. 8 and 9. These curves offer another critical test of the theory, since eqn. (13) embodies all the recognized winding parameters.

Comparing the computed and experimental distributions of inductance it can be seen that the values measured at 50 c/s are consistently the larger. This discrepancy could arise from errors in measuring any of the winding parameters involved in the right-hand side of eqn. (13) or it could indicate a shortcoming of the theory. The parameters concerned are C_G, ω_m, α and ψ, of which the first two are quantities which are measurable with reasonable certainty, apart from small experimental errors. As explained in Section 4, the determination of appropriate values of α is in some degree a matter of judgment, but it is most unlikely that these values were over-estimated to the extent needed to explain the discrepancies. There is more uncertainty about the values of ψ, but these were selected in an attempt to give a

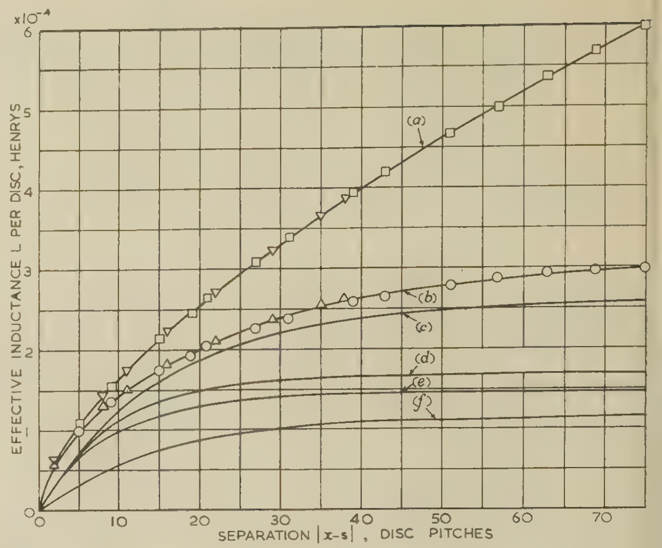


Fig. 8.—Effective inductance L per disc for 2-limb transformer.

(*a*) Single-opposition condition: □ $x + s = l$; ▽ $s = l$ (measured values).
(*b*) Tandem condition: ○ $x + s = l$; △ $s = l$ (measured values).
(*c*) Tandem condition. Computed inductance from eqn. (13) for $\psi = 6$ and $m = 10$ (Coleman).
(*d*) Tandem condition. Computed inductance from eqn. (13) for $\psi = 10$ and $m = 10$ (Coleman).
(*e*) Tandem condition. Computed inductance from eqn. (13) for $\psi = 10$ and $m = 1$ (Coleman).
(*f*) Tandem condition. Computed inductance from eqn. (13) for $\psi = 6$ and $m = 1$ (Coleman).

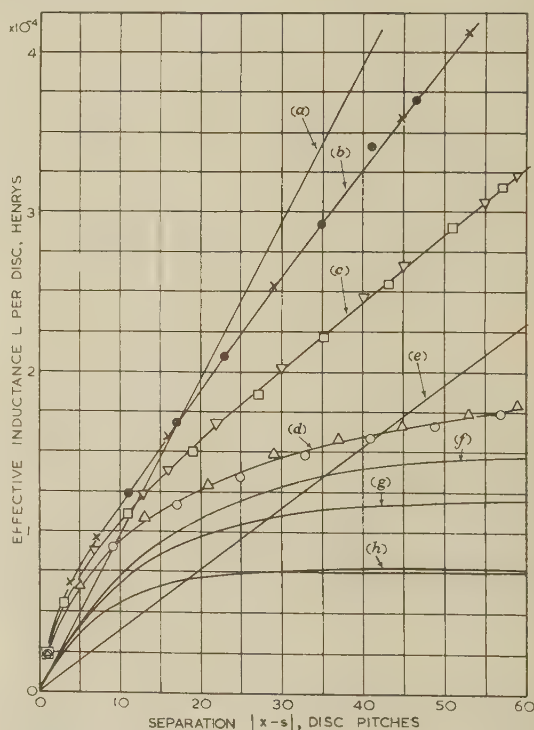


Fig. 9.—Effective inductance L per disc for 3-limb transformer.

(*a*) Double-opposition condition, computed from eqn. (12) for $m = 9$ [Pirenne (Bewley)].
(*b*) Double-opposition condition: × $x + s = l$; ● $s = l$ (measured values).
(*c*) Single-opposition condition: □ $x + s = l$; ▽ $s = l$ (measured values).
(*d*) Tandem condition: ○ $x + s = l$; △ $s = l$ (measured values).
(*e*) Double-opposition condition, computed from eqn. (12) for $m = 1$ [Pirenne (Bewley)].
(*f*) Double-opposition condition, computed from eqn. (13) for $\psi = 4$ and $m = 1$ or $m = 9$ (Coleman).
(*g*) Single-opposition condition, computed from eqn. (13) for $\psi = 5$, and $m = 1$ or $m = 9$ (Coleman).
(*h*) Tandem condition, computed from eqn. (13) for $\psi = 7{\cdot}7$ and $m = 1$ or $m = 9$ (Coleman).

reasonable fit between measured and computed frequency ratios. The position, then, is similar to that which confronted Blume and Boyajian, and Bewley, who adopted the expedient of arbitrarily varying an inductance term which had been assumed constant in the theory in order to obtain agreement with experiment. To make similar adjustments in relation to Coleman's theory would not further the aims of this investigation, nor would it be likely to be acceptable in a mathematical sense.

Pirenne[9] found that the mutual inductance between turns varied with frequency, though in his case both core-flux and leakage-flux linkages contributed to the measured quantities. Comparable measurements were made on the 3-limb transformer, using the circuit of Fig. 10 to eliminate core flux. The circuit

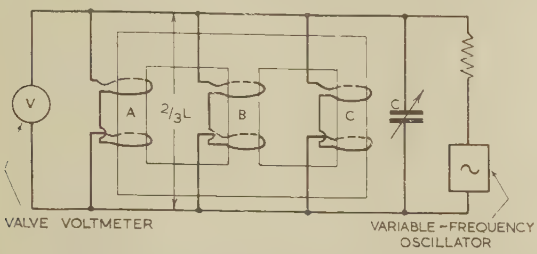


Fig. 10.—Circuit for determining effective inductance $2L$ of a pair of discs on an equivalent-single-limb winding corresponding to tandem condition for a 3-limb transformer, at winding-oscillation frequencies.

One disc is represented as a single turn.

was tuned to resonance with an applied high-frequency current by means of a variable capacitor C, and the effective inductance was deduced from the total capacitance after including an estimated value for the stray capacitance. Results for the first, third and fifth tandem frequencies are plotted against separation between elements in Fig. 11. All discs not in use were left

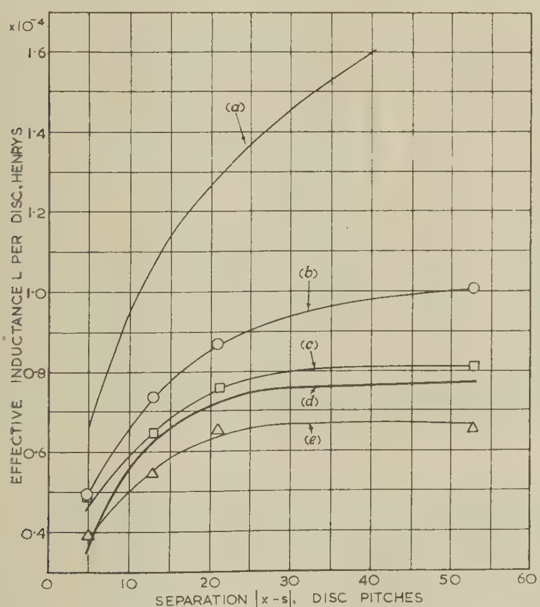


Fig. 11.—Effective inductance L per disc for 3-limb transformer, tandem condition.

(*a*) Measured at 50c/s [see Fig. 9 curve (*d*)].
(*b*) Measured at 43·8 kc/s ($m = 1$).
(*c*) Measured at 209 kc/s ($m = 3$).
(*d*) Computed from eqn. (13) for $\psi = 7 \cdot 7$ and $m = 1$ or $m = 9$ (see Fig. 9 curve (*h*) (Coleman).
(*e*) Measured at 418 kc/s ($m = 5$).

unconnected, but it is realized that the mutually-induced e.m.f.'s in them would undoubtedly have set up stray capacitive currents which in turn would have induced e.m.f.'s in the test circuit. The accuracy of these high-frequency measurements was therefore probably low; nevertheless, the results confirm that the apparent values of M_0 and ψ are frequency-dependent, or expressing the fact in other terms, that the equivalent circuit in its present form is incomplete. It appears probable that imperfect behaviour of the iron core is responsible, and that inductively-coupled circuits to represent eddy-current paths and resistances to represent I^2R and core losses need to be included if better correlation between theory and experiment is desired.

However, Fig. 11 shows that the computed curve of effective inductance per disc for the same transformer and the same terminal condition agrees reasonably with the high-frequency measured values as a whole. No high-frequency measurements of effective inductance were made for opposition conditions on the 3-limb transformer or for any terminal condition on the 2-limb transformer, owing to practical difficulties.

The results of this Section can be summarized as follows:

(*a*) An exponential distribution of mutual leakage inductance appears to be a satisfactory approximation for tandem condition.
(*b*) There are apparent variations of self and mutual leakage inductance with frequency.
(*c*) For computing oscillation frequencies from Coleman's equations, values of M_0 and ψ should be obtained from measurements of inductance distribution at a frequency within the range of frequencies to be considered.
(*d*) A linear distribution of mutual inductance, corresponding to Bewley's theory, is a possible approximation for double-opposition condition, but is less satisfactory than an exponential law when considered on the basis of frequency ratios.

(7) WAVE VELOCITIES

In some theories of oscillation in windings, e.g. those of Rüdenberg and Bewley, the solution can be expressed in standing-wave form as $a \cos \omega t \sin m\pi x/l$, which can be written alternatively in travelling-wave form as

$$\tfrac{1}{2}a \sin \omega(x/v + t) + \tfrac{1}{2}a \sin \omega(x/v - t)$$

where $v = l\omega/m\pi$ = axial velocity of propagation of travelling waves at angular frequency ω. Assuming that the waves actually travel along the path of the conductor, i.e. in a helical path of total length l_w, the phase velocity of the mth oscillation mode is

$$v_m = l_w \omega_m / m\pi \quad . \quad . \quad . \quad . \quad . \quad (14)$$

By assuming, as an approximation, that the spatial distribution of the mth oscillation mode is sinusoidal, the phase velocities have been deduced from the measured frequencies by applying eqn. (14), and the results are plotted in Figs. 2, 3, 4 and 5. These show that, for the larger values of m, the phase velocity is fairly constant in the region of 45% to 55% of the velocity of light in vacuo. This is comparable with the group velocity of travelling waves in a winding in air, estimated by Norris[14] at 65% and by Frid[16] at 67% to 73% of the velocity of light.

The predicted variation of velocity with m is given by the following expressions:

(Rüdenberg) $$(m^2\pi^2 + \alpha^2)^{-1/2} \quad . \quad . \quad . \quad . \quad (15)$$

(Bewley) $$m\pi(m^2\pi^2 + \alpha^2)^{-1/2} \quad . \quad . \quad . \quad . \quad (16)$$

(Heller and Veverka) $$(m^2\pi^2 + \psi^2)^{1/2}(m\pi^2 + \alpha^2)^{1/2} \quad . \quad (17)$$

Expression (17) can also be obtained from Coleman's theory by putting $v_m = m\pi$.

Considerable success has been achieved in predicting intersection voltages in windings from consideration of travelling waves. Norris describes a method based on Rüdenberg's

theory in which, as can be seen from expression (15), the phase velocity decreases with increase of m. The success of the method may well be due in part to the fact that the phase velocity is actually much less dependent on m than is suggested by expression (15), so that distortion of the travelling waves is less serious than would be expected.

It is interesting to note that phase velocities deduced for the air-cooled 3-limb transformer are slightly lower than those for the oil-immersed 2-limb transformer, contrary to the supposed effect on velocity of immersing a winding in oil.

This can be explained by the geometrical differences between the windings. It has been shown,[8] theoretically and experimentally, that the geometry of shielded coils has an influence on phase velocity independently of the permittivity of the insulating medium.

Except for the first two oscillation modes ($m = 1$ and $m = 2$), in either transformer it can be shown that, for a given mode or frequency, the differences in phase velocity between tandem and opposition conditions are small. Hence, when computing the surge performance of a winding from travelling-wave considerations, it appears that little would be gained by resolving the surge into its 'symmetrical' components.

(8) AMPLITUDES AND DECREMENTS OF OSCILLATIONS

Amplitudes and decrement factors of oscillations due to an applied wave approximating to a unit function were estimated from oscillograms of voltages in the 2-limb transformer under tandem condition. The applied wave had a wavefront duration of about 1 microsec and a wavetail which was substantially constant for the first 100 microsec. The first few symmetrical modes of oscillation, having voltage antinodes at the mid-point of the winding ($x/l = \frac{1}{2}$), were measured from the oscillogram of voltage to core at this point. The voltage between points at $x/l = \frac{1}{4}$ and $x/l = \frac{3}{4}$, containing alternate antisymmetrical modes only ($m = 2$, 6, etc.), yielded two modes of oscillation, and the fourth harmonic ($m = 4$) was extracted from the voltage between points at $x/l = \frac{1}{8}$ and $x/l = \frac{3}{8}$. The amplitudes plotted in Fig. 12 are the initial values ($t = 0$) estimated by extrapolation and are expressed as percentages of the applied peak voltage. Only for the first two frequencies was it possible to find decrement factors from the oscillograms. However, during the determination of frequencies (Section 5) using a variable-frequency oscillator, bandwidth measurements were made from which the decrement factors were determined for both transformers. These results and the two obtained from oscillograms are plotted in Fig. 12. Owing to a large degree of asymmetry in the resonance curves, measurements of bandwidths for antisymmetrical modes in the 3-limb transformer were mostly unsatisfactory.

The amplitudes of oscillation are predicted similarly by Bewley, Blume and Boyajian, and Rüdenberg (also by Coleman if the approximation $v_m \simeq m\pi$ is made) by the expression

$$2\alpha^2/m\pi(\alpha^2 + m^2\pi^2) \quad . \quad . \quad . \quad . \quad . \quad (18)$$

for unit applied voltage. This function is plotted in Fig. 12 for comparison with the experimental results, from which it appears that the prediction is satisfactory if allowance is made for experimental error and the difficulty of analysing the oscillograms.

From expressions given by Lacey[5] and Bewley it can be deduced that, in a winding having uniformly distributed resistance, the decrement factor varies

(a) As m^2 if the resistance is in series with the inductive elements.
(b) Inversely as $(m^2\pi^2 + \alpha^2)$ if the resistance is in parallel with the capacitance to core.
(c) As $m^2\pi^2/(m^2\pi^2 + \alpha^2)$ if the resistance is in parallel with the inductive elements.

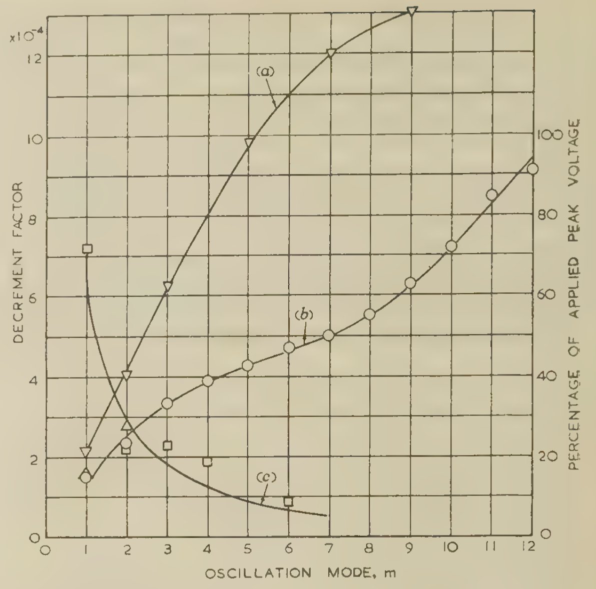


Fig. 12.—Amplitudes and decrement factors of oscillations in 2-limb transformer for tandem condition, and decrement factors of oscillations in 3-limb transformer for tandem condition.

(a) Decrement factors for 3-limb transformer; ▽ values from bandwidth measurements.
(b) Decrement factors for 2-limb transformer; △ values from transient oscillograms; ○ values from bandwidth measurements.
(c) Amplitudes for 2-limb transformer; curve computed from eqn. (18); □ values measured from transient oscillograms.

Referring to Fig. 12, although all three forms of resistance may be contributory to the observed decrements, the latter cannot be fully accounted for by these forms alone. This could be taken as another indication of the inadequacy of the equivalent circuit already suggested in Section 6.

(9) SPATIAL DISTRIBUTIONS OF OSCILLATIONS

A sinusoidal current was injected at the mid-point of one limb winding of the 2-limb transformer from a high-impedance source and the frequency was adjusted to resonance with the fundamental mode ($m = 1$). The voltage distribution along the winding was explored with a high-impedance valve voltmeter and the results are shown in Fig. 13. These measurements were made prior to the standardizing of terminal conditions for multi-limb transformers, and following the usual practice at that time the other limb winding was left floating. It is considered, however, that under these conditions the limb winding under test could be regarded as a single-limb winding; the latter is not identifiable with either the tandem or the opposition equivalent single-limb winding, as can be observed by comparing the resonant frequencies. The test was repeated for the fifth oscillation mode ($m = 5$), the results of which are shown in Fig. 14.

In each case there is some asymmetry between the distributions in the two halves of the winding which has not been accounted for, and in the case of the fifth oscillation mode this results in a variation of 20% in the anti-nodal voltages. Both distributions are approximately sinusoidal, but it is interesting to note that the fundamental oscillation is closer to the distribution predicted by Coleman than to a sine wave.

Generally similar results were obtained for other symmetrical modes and also for antisymmetrical modes. The latter were plotted by the same method except that the current was injected not at the mid-point, but at or near to a point in the winding where a voltage antinode would be expected.

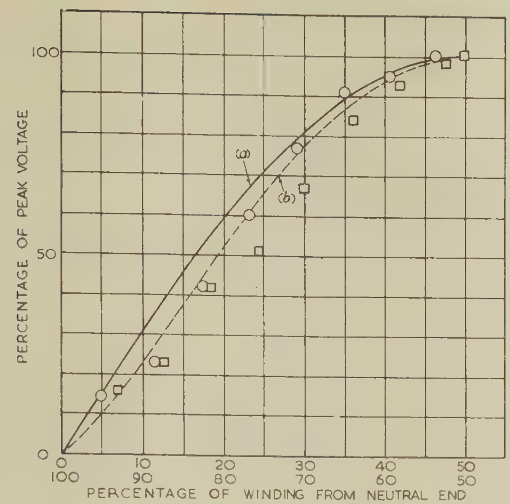


Fig. 13.—Spatial distribution of voltage in one limb of 2-limb transformer for oscillation mode $m = 1$, frequency $25 \cdot 3$ kc/s.

(*a*) Sine curve (Blume and Boyajian, Bewley, Wagner, Rüdenberg).
(*b*) Curve computed from eqn. (69) of Reference 1 (Coleman).
○ Values measured between neutral end and mid-point.
□ Values measured between line end and mid-point.

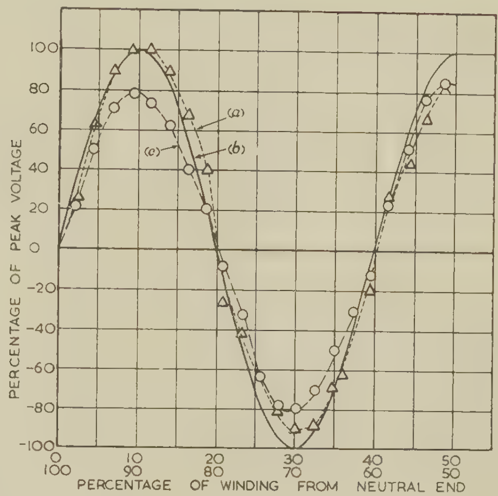


Fig. 14.—Spatial distribution of voltage in one limb of 2-limb transformer for oscillation mode $m = 5$, frequency 237 kc/s.

(*a*) Distribution measured between line end and mid-point.
(*b*) Sine curve.
(*c*) Distribution measured between neutral end and mid-point.

(10) ASYMMETRICALLY-APPLIED IMPULSES

Surges applied asymmetrically to the terminals of multi-limb transformers are common occurrences under service conditions and during impulse tests. Typical examples are

(*a*) Impulse voltage V_T applied to line terminal A of a 2-limb transformer with line terminal B earthed or connected to a low surge impedance. Limb windings A and B connected in series.

(*b*) Impulse voltage V_T applied to line terminal A(B) of a 3-limb transformer with line terminals B(A) and C earthed or connected to low surge impedances. Limb windings star-connected with neutral earthed.

(*c*) As for (*b*) but with neutral isolated.

According to Coleman the responses of the windings under these conditions can be synthesized by superposing the responses to tandem and opposition component impulses. The line terminal voltages can be synthesized from components as demonstrated in Table 3, noting that the tandem components in examples (*a*) and (*c*) are of the isolated-neutral type whereas all other components are of the earthed-neutral type.

The validity of this system of synthesis was checked experimentally using a recurrent-surge oscillograph to record voltages at selected points in windings under the asymmetrical terminal conditions and also under tandem and opposition terminal conditions. Summations of the component voltages were then compared with the asymmetrical response at numerous instants after the start of the wave.

Example (*b*) is illustrated in Fig. 15, which shows voltages at the mid-point of limb winding A of the 3-limb transformer. The curves are traced from oscillograms and the results of summing

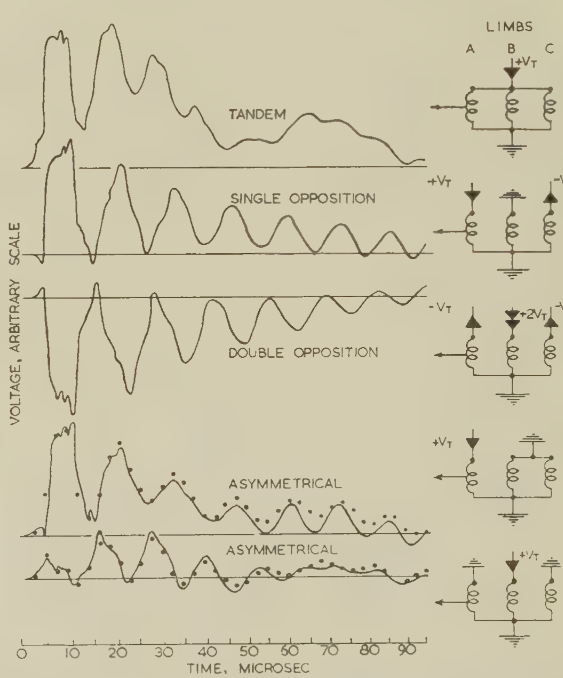


Fig 15.—Synthesis of voltages due to asymmetrical terminal voltages from components due to symmetrical terminal voltages and measurements of voltage to earth at mid-point of inner winding on limb A of 3-limb transformer.

Table 3

VOLTAGE COMPONENTS IN LIMBS A, B, AND C UNDER ASYMMETRICAL CONDITIONS

Component	Condition (*a*)		Conditions (*b*) and (*c*)			Alternative conditions (*b*) and (*c*)		
	A	B	A	B	C	A	B	C
Tandem	$\frac{1}{2}V_T$	$\frac{1}{2}V_T$	$\frac{1}{3}V_T$	$\frac{1}{3}V_T$	$\frac{1}{3}V_T$	$\frac{1}{3}V_T$	$\frac{1}{3}V_T$	$\frac{1}{3}V_T$
Single-opposition	$\frac{1}{2}V_T$	$-\frac{1}{2}V_T$	$\frac{1}{2}V_T$	0	$-\frac{1}{2}V_T$	0	0	0
Double-opposition	—	—	$\frac{1}{6}V_T$	$-\frac{1}{3}V_T$	$\frac{1}{6}V_T$	$-\frac{1}{3}V_T$	$\frac{2}{3}V_T$	$-\frac{1}{3}V_T$
Asymmetrical condition ..	V_T	0	V_T	0	0	0	V_T	0

the components in accordance with Table 3 are shown by the points. This, and other syntheses of a similar kind, prove that the principles of Coleman's analysis are sound.

It is found that the waveforms of component voltages are noticeably purer than those for asymmetrical conditions, owing to the absence of beats between oscillations nearly equal in frequency, exactly as would be expected from the theory. Tandem-condition waveforms differ markedly from either of the opposition-condition waveforms, but the differences between the single-opposition and the double-opposition waveforms are only slight.

Further verification of the multi-limb analysis was obtained in a wide range of observations of voltages and currents in the experimental transformers.

The process illustrated in Fig. 15 can be extended to the synthesis of the response of a parallel-connected (2-limb) or a delta-connected (3-limb) winding. With these connections, impulses are imposed simultaneously on line ends and neutral ends of limb windings, and to obtain the overall response it is necessary to superpose the responses of two series or star-connected windings formed by interchanging the functions of the line and neutral ends.

(11) CONCLUSIONS

Transient oscillations in a multi-limb winding can be synthesized by superposition of component oscillations in fictitious series or star-connected banks of single-limb windings. These components are mutually independent and are associated with winding parameters which can be determined from measurements on the multi-limb winding under certain specified symmetrical conditions (designated tandem, single-opposition and double-opposition). This verifies Coleman's theoretical treatment of multi-limb windings.

For uniform disc windings it is reasonable to assume an exponential law of variation of mutual leakage inductance between discs with axial separation between them, as assumed by Coleman.

There is an apparent variation with frequency of the self and mutual leakage inductances. This could be due to eddy currents in the core which are not accounted for in the equivalent circuit of any of the theories considered.

No critical frequency corresponding to zero phase velocity is reached or even approached in the range of frequencies measured. This result differs significantly from the predictions of Rüdenberg but it is in support of Coleman's theory. The results confirm that, as a rough approximation in computing inter-section voltages, the phase velocity may be assumed to be independent of frequency.

Measured and computed amplitudes of voltage due to an applied impulse agree within the limits of experimental error.

Decrement factors determined from transient oscillograms agree with those deduced from bandwidth measurements, but cannot be fully accounted for by the classical theories involving uniformly-distributed resistance.

The spatial distribution of the fundamental oscillation is non-sinusoidal, confirming prediction by Coleman.

It is justifiable to simplify the theoretical treatment of a 3-limb winding by assuming triangular symmetry, making single-opposition and double-opposition parameters identical.

(12) ACKNOWLEDGMENTS

The author is grateful to all his colleagues who have helped in the experimental and computing work described. He is particularly indebted to Mr. B. L. Coleman for his valuable advice and criticism, and wishes to thank the Director of the Electrical Research Association for permission to publish this paper.

(13) REFERENCES

(1) Coleman, B. L.: 'Oscillations in a Transformer Winding with particular reference to the Response to an Applied Surge' (see p. 432).

(2) Wagner, K. W.: 'Das Eindringen einer elektromagnetischen Welle in einer Spule mit Windungskapazität', *Elektrotechnik und Maschinenbau*, February, 1915, pp. 89 and 105.

(3) Rüdenberg, R.: 'Performance of Traveling Waves in Coils and Windings', *Transactions of the American I.E.E.*, 1940, **59**, p. 1031.

(4) Blume, L. F., and Boyajian, A.: 'Abnormal Voltages within Transformers', *ibid.*, 1919, **38**, p. 577.

(5) Lacey, H. M.: 'Surge Phenomena', E.R.A. Report Ref. S/T35.

(6) Bewley, L. V.: 'Traveling Waves on Transmission Systems' (John Wiley, 1951).

(7) Heller, B., Hlavka, J., and Veverka, A.: 'Surge Phenomena in Transformers', *Elektrotechniky Obzor*, 1948, **37**, No. 6, p. 100. E.R.A. Translation Ref. Trans./IB989.

(8) Poritsky, H., Abetti, P. A., and Jerrard, R. P.: 'Field Theory of Wave Propagation along Coils', *Transactions of the American I.E.E.*, 1953, **72**, Part III, p. 930.

(9) Pirenne, J.: 'Théorie générale des phenoménes oscillatoires dans les enroulements des transformateurs', *Revue Générale de l'Électricité*, 1940, **47**, pp. 19 and 53. E.R.A. Translation Ref. Trans./IB1024.

(10) Abetti, P. A.: 'The Surge Performance of Transformers and Rotating Machines—Survey and Classification of Published Data', *Transactions of the American I.E.E.*, 1958, **77**, Part III, p. 1403.

(11) Selig, K. L.: 'Surge Voltage Distribution in a Continuous-Disk Transformer Winding', E.R.A. Report Ref. S/T48.

(12) Waters, M.: 'The Measurement and Calculation of Axial Electromagnetic Forces in Concentric Transformer Windings', *Proceedings I.E.E.*, Paper No. 1587 S, February 1954 (**101**, Part II, p. 35).

(13) Allibone, T. E., McKenzie, D. B., and Perry, F. R.: 'The Effects of Impulse Voltages on Transformer Windings', *Journal I.E.E.*, 1937, **80**, p. 117.

(14) Norris, E. T.: 'The Lightning Strength of Power Transformers', *ibid.*, 1948, **95**, Part II, p. 389.

(15) Abetti, P. A., and Maginnis, F. J.: 'Natural Frequencies of Coils and Windings Determined by Equivalent Circuit', *Transactions of the American I.E.E.*, 1953, **72**, Part III, p. 495.

(16) Frid, E. S.: 'Basic Relationships of the Impulse Gradient in Transformer Windings', *Elektrichestvo*, 1947, No. 3, p. 28. E.R.A. Translation Ref. Trans./IB1032.

(17) Heller, B., and Veverka, A.: 'Electric Stresses in a Coil during Unit Surge', *Elektrotechniky Obzor*, 1951, **40**, Nos. 7–10, p. 143. E.R.A. Translation Ref. Trans./IB1182.

(14) APPENDIX: DETAILS OF EXPERIMENTAL TRANSFORMERS

2-Limb Transformer: Core-type oil-immersed.

Nominal rating ..	500 kVA
Voltage ratio	12 500/400
Impedance	4·6%

Core diameter ..	9·5 in
Core length	51 in
Inner winding ..	Two-layer helical coil, 41 turns per limb: i.d. 10·5 in, o.d. 12·14 in
Outer winding ..	1 282 turns per limb i.d. 14·02 in, o.d. 17·38 in Limb A: Discs 1 and 87 . . . 12 turns Discs 2, 3, 8, 14, 20, 26, 32, 38, 44, 50, 56, 62, 68, 74, 80, 85 and 86 . . . 14 turns Remaining discs . . . 15 turns Limb B: Discs 1 to 86 inclusive . . . 15 turns

Limb A has reinforced end-turn insulation. Limb B has substantially uniform insulation throughout except that every third radial duct is 0·312 5 in wide compared with 0·25 in for the remainder and for all ducts between normally insulated discs in limb A. Both limbs have insulation as for 33 kV.

3-*Limb Transformer*:	Core-type, air-cooled.
Rating	420 kVA
Voltage ratio	5 500/5 500
Impedance	4·24%
Core diameter ..	7·69 in
Core length	35 in
Inner winding ..	720 turns per limb i.d. 9·125 in, o.d. 11·375 in 60 single discs, 12 turns per disc Ducts between discs, 0·187 5 in wide
Outer winding ..	720 turns per limb i.d. 12·815 in, o.d. 15·125 in 60 single discs, 12 turns per disc Ducts between discs 0·187 5 in wide.

621.314.2.045

The Institution of Electrical Engineers
Paper No. 3287 S
Oct. 1960

OSCILLATIONS IN A TRANSFORMER WINDING

With Particular Reference to the Response to an Applied Surge

By B. L. COLEMAN, B.Sc., Associate Member.

(*The paper was first received 29th April, 1959, and in revised form 13th April, 1960.*)

SUMMARY

Existing single-limb theory is revised by differentiating between magnetizing and leakage inductance effects. Analysis of m.m.f. and flux in multi-limb transformers enables the theory to be extended to a winding on two or three limbs. Solution of these cases is obtained by resolution into equivalent single-limb windings. Transient response to surges is discussed and expressions are evolved for current and potential distributions.

LIST OF SYMBOLS

x, s = Distance from origin (neutral terminal).
t = Time.
$V(x, t)$ = Potential to earth at x at time t.
$I(x, t)$ = Wire current at x, positive in direction of x increasing.
$I_s(x, t)$ = Series capacitance current at x.
l = Length of winding.
$C_G = lC_g$ = Capacitance to earth of complete winding on one limb.
$C_S = C_s/l$ = Series capacitance of complete winding on one limb.
$\alpha = (C_G/C_S)^{1/2} = l(C_g/C_s)^{1/2}$
$\Phi(t)$ = Resultant magnetizing flux through winding.
$M(x, s)$ = Coefficient of mutual leakage inductance.
C_h, C_k = Inter-limb capacitance parameters.
$L(x, s), N(x, s)$ = Inter-limb mutual leakage inductance parameters.
N = Number of turns per unit length in axial direction.
$\lambda = \psi/l$ = Spatial decrement factor in expression for $M(x, s)$.
μ, ν = Constants defining spatial distribution of characteristic functions.

(1) INTRODUCTION

Transient oscillations in a single-limb winding have been analysed in great detail by many authors, the complexity of solution increasing with the refinement of measuring technique. Whereas, however, in power transformer analysis a distinction is made between core flux and leakage flux, which are dealt with by totally differing methods, earlier attempts to retain the distinction have been disregarded in recent transient analysis. Apart from restoring the accepted power-frequency analysis of flux the mathematical treatment of the single-limb winding follows the lines laid down by Pirenne,[1] and Heller, Hlavka and Veverka,[2] and is of the standing-wave variety.

The aim of the multi-limb analysis is to reduce the complex oscillations to the superposition of simpler single-limb oscillations in a manner comparable with the use of symmetrical components at power frequencies.

This paper is purely theoretical in nature and a companion paper[5] provides the experimental evidence needed for completeness.

Non-linear effects, such as hysteresis, eddy currents, etc., are ignored. Only the h.v. winding is considered, the presence of other windings being assumed to have no effect on the phenomena treated. Winding resistance is neglected, since its effects on the natural frequencies of oscillation are likely to be small. The winding is represented by an equivalent distributed circuit, neglecting frequency-dependence of circuit parameters.

Following a remark by Rüdenberg,[4] the results are applicable to normal disc windings provided that the parameters are interpreted accordingly.

(2) THE MAGNETIC CIRCUIT

(2.1) The Current-Carrying Turn on an Iron Core

The current-carrying turn gives rise to flux which links in part with every other turn on the core. The flux may be divided into two components; that which lies entirely within the iron circuit is called core flux; that whose path is only partly in iron, or entirely outside the iron circuit, is called leakage flux.

(2.2) Leakage Inductance

The mutual leakage inductance between two turns of the transformer is defined by some form of equation such as

$$V = -M\frac{di}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

self-leakage inductance being similarly defined. The concept holds between turns on different limbs.

(2.3) Core Flux in Windings on One Limb

The core flux induced by a turn is proportional to the current in the turn, the constant of proportionality being independent of the size of the turn or its position relative to the yokes. Practical analysis of single-limb transformers is then carried out according to the following rules:

(*a*) The resultant core flux in a limb is zero.

It follows from the constant of proportionality that the resultant m.m.f. on the limb is zero, and in this form the law is known as that of m.m.f. balance.

(*b*) Superimposed on this balanced system is a magnetizing flux concerned solely with generation of necessary back-e.m.f.'s. This in turn is ascribed to magnetizing current in the primary winding negligible in comparison with the load current therein.

(2.4) Core Flux in Multi-Limb Windings

In order to extend the foregoing ideas to the more general case, two assumptions are necessary. The first generalizes the constant of proportionality to embrace turns on different limbs. The second is

(*c*) The core flux induced in a limb by a turn returns in equal proportions through the remaining limbs.

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.
Mr. Coleman is in the Ministry of Defence, Government of Israel.

Rule (*a*) still applies as stated. The law of m.m.f. balance, however, undergoes significant change. Taking, for example, a 3-limbed core, let subscripts *A*, *B*, *C* refer to individual legs and let Φ_{AA} be the core flux induced in limb A by the current turns on limb A only, with similar definitions for Φ_{BB}, Φ_{CC}. By rule (*a*), the total core flux in limb A vanishes and application of rule (*c*) yields

$$\Phi_{AA} - \tfrac{1}{2}\Phi_{BB} - \tfrac{1}{2}\Phi_{CC} = 0 \quad . \quad . \quad . \quad (2)$$

Forming the corresponding equation for limb B and subtracting,

$$\Phi_{AA} = \Phi_{BB} = \Phi_{CC} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now the constant of proportionality shows that Φ_{AA} is proportional to the m.m.f. of the windings on limb A, etc., whence

(*d*) The resultant m.m.f. is the same on each leg.

Consideration of the zero phase sequence shows that the equality does not imply vanishing.

Suitable magnetizing fluxes must be introduced to generate the requisite back-e.m.f.'s, as in rule (*b*). They are restricted to conform to flux continuity by the equation

$$\Phi_A + \Phi_B + \Phi_C = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Φ_A being the magnetizing flux generated in limb A, etc.

Some applications of the methods to power-frequency problems may be found in a previous publication by the author.[3]

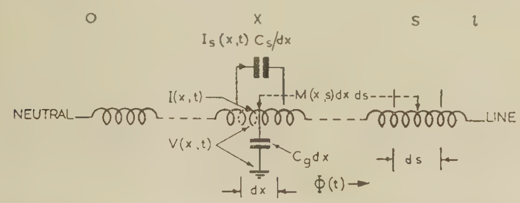


Fig. 1.—Equivalent network for single-limb winding.

(3) EQUATIONS OF OSCILLATION FOR WINDING ON ONE LIMB

(3.1) Equations of Oscillation

Representing the winding as in Fig. 1, application of Kirchhoff's laws to the differential element at *x* gives

$$\frac{\partial I(x, t)}{\partial x} = -C_g\frac{\partial V(x, t)}{\partial t} + C_s\frac{\partial^3 V(x, t)}{\partial x^2 \partial t} \quad . \quad . \quad (5)$$

$$\frac{\partial V(x, t)}{\partial x} = -N\frac{d\Phi(t)}{dt} - \int_0^l M(x, s)\frac{\partial I(s, t)}{dt}ds \quad . \quad (6)$$

$$I_s(x, t) = -C_s\frac{\partial^2 V(x, t)}{\partial x \partial t} \quad . \quad . \quad . \quad . \quad (7)$$

Applying rule (*a*), the law of m.m.f. balance,

$$\int_0^l I(x, t)dx = 0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

In these equations, $\Phi(t)$ is the magnetizing flux whose effect is shown explicitly in eqn. (6), the corresponding magnetizing current being neglected in comparison with $I(x, t)$. $M(x, s)$ refers only to leakage inductance. Pirenne's equations, based on the concepts of total current and inductance, do not include $\Phi(t)$, nor is there anything corresponding to eqn. (8). Otherwise they are formally identical with those given above. Writing $V(x, t) = V(x)T_1(t)$, $I(x, t) = I(x)T_2(t)$ leads to the reduced equations[1] for sinusoidal time-like oscillation with angular frequency ω:

$$I'(x) = \omega C_g V(x) - \omega C_s V''(x) \quad . \quad . \quad . \quad (9)$$

$$V'(x) = -N\omega\Phi - \omega\int_0^l M(x, s)I(s)ds \quad . \quad . \quad (10)$$

$$I_s(x) = \omega C_s V'(x) \quad . \quad . \quad . \quad . \quad (11)$$

$$\int_0^l I(x)dx = 0 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The boundary conditions of practical importance are those in which the line terminal ($x = l$) is earthed and the neutral terminal is earthed or isolated. The corresponding formulations for the reduced equations are

$$V(0) = V(l) = 0 \quad . \quad . \quad . \quad . \quad (13)$$

$$I(0) + \omega C_s V'(0) = V(l) = 0 \quad . \quad . \quad . \quad (14)$$

The problem has now been reduced to a typical boundary-value problem of mathematics. In general, solutions exist only for particular values of ω, called characteristic values, whilst to each such value corresponds a specific potential, current or flux distribution known as a characteristic function.

If subscripts *p*, *q* refer to characteristic functions of the different characteristic values ω_p, ω_q, it is shown in Section 10.1 that

$$\int_0^l V_p'(x)I_q(x)dx = 0 \quad (p \neq q) \quad . \quad . \quad . \quad (15)$$

a typical orthogonality relation of a boundary value problem which will be used to combine characteristic solutions to give the resultant oscillation following an applied surge.

(3.2) The Mutual Leakage Inductance Function

Pirenne[1] applied the theory of integral equations to deduce a general solution of his equations for an arbitrary mutual inductance function $M(x, s)$ dependent only on $|x - s|$. Even neglecting series capacitance, this general solution is too unwieldy for practical use. By making specific approximations for $M(x, s)$ he was able to classify all previous theories. The classification extends to the later work of Heller and his co-workers,[2] who choose an exponential approximation

$$M(x, s) = M_0\varepsilon^{-\lambda|x-s|} \quad . \quad . \quad . \quad . \quad (16)$$

where M_0 and λ are winding parameters assumed to be independent of frequency.

This is the approximation adopted in the present paper with the proviso that it refers to leakage instead of total inductance as in the original.

(4) CHARACTERISTIC OSCILLATIONS IN A WINDING ON ONE LIMB

(4.1) Transformer Windings

(4.1.1) General Solution.

Substitution of eqn. (16) in eqn. (10) and repeated differentiation using eqn. (9) gives the characteristic equation (see Section 11.2)

$$V''''(x) + (2\omega^2\lambda M_0 C_s - \lambda^2)V''(x) - 2\omega^2\lambda M_0 C_g V(x) = 0 \quad . \quad . \quad . \quad . \quad (17)$$

a fourth-order linear differential equation with constant coefficients. Its general solution is

$$V(x) = A_1 \cosh\frac{\mu x}{l} + A_2 \sinh\frac{\mu x}{l} + A_3 \cos\frac{\nu x}{l} + A_4 \sin\frac{\nu x}{l} \quad . \quad . \quad . \quad . \quad (18)$$

or a suitable equivalent, all the constants involved being real. Writing $\psi = \lambda l$, $\alpha = (C_G/C_S)^{1/2}$, substitution of eqn. (18) in eqn. (17) gives

$$\omega^2 = \frac{\mu^2\nu^2}{2\psi M_0 C_G l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$\mu^2 = \frac{\nu^2 + \psi^2}{1 + \nu^2/\alpha^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

There are, in all, six unknowns, namely the three ratios $A_1 : A_2 : A_3 : A_4$ together with μ, ν and Φ. The boundary conditions (13) or (14) provide two equations between them. Three more are a natural consequence of the derivation of eqn. (17) from lower-order equations, as shown in Section 11.2. These five form a set of five linear equations in four of the unknowns, namely the three ratios and Φ, the coefficients being functions of μ and ν only. For consistency of such a system its determinant must vanish identically, this yielding a second relation between μ and ν, which, taken with eqn. (20), allows their numerical evaluation and the complete solution follows immediately.

One consequence of eqn. (19) is that

$$\frac{f_p}{f_q} = \frac{\omega_p}{\omega_q} = \frac{\mu_p\nu_p}{\mu_q\nu_q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

holds between characteristic frequencies of any two distinct characteristic oscillations, irrespective of boundary conditions.

(4.1.2) Neutral Earthed.

The boundary conditions are eqns. (13). Following the procedure given above, it is found that two independent subsets of equations are obtained. Physically the phenomenon reflects division of the modes of oscillation into what Pirenne calls the symmetrical modes for which $V(x) = V(l - x)$ and the antisymmetrical modes when $V(x) = -V(l - x)$. These may be thought of as containing odd and even numbers respectively of half-wave spatial distributions.

The condition of consistency gives, for symmetrical modes,

$$\nu \tan \tfrac{1}{2}\nu + \mu \tanh \tfrac{1}{2}\mu + \frac{\mu^2 + \nu^2}{\psi} = 0 \quad . \quad . \quad (22)$$

and for antisymmetrical modes,

$$\mu \coth \tfrac{1}{2}\mu - \nu \cot \tfrac{1}{2}\nu + \frac{(\mu^2 + \nu^2)(2 + \psi)}{\psi^2} = 0 \quad . \quad (23)$$

Potential, current and flux distributions are given in Section 11.4, the most interesting distinction between the two sets of solutions being that in the symmetrical case the magnetizing flux Φ vanishes, but does not in the antisymmetrical modes.

Theoretically there are an infinity of solutions, but in practice, as the order of the harmonic increases, its wavelength approaches values comparable with the distance between adjacent turns, so that the theory can only be expected to give reasonable results when this limitation is borne in mind.

Numerical solutions may be obtained by the method given in Section 11.3. It is found that if m represents the order of the harmonic, or number of complete half-wave spatial distributions in the mode considered, $\nu_m \simeq m\pi$ and

$$\frac{f_m}{f_1} \simeq m\left(\frac{\psi^2 + m^2\pi^2}{\psi^2 + \pi^2}\right)^{1/2}\left(\frac{1 + \pi^2/\alpha^2}{1 + m^2\pi^2/\alpha^2}\right)^{1/2} \quad . \quad (24)$$

Rüdenberg[4] gives the corresponding results for earlier theories:

$$\frac{f_m}{f_1} = m\left(\frac{1 + \pi^2/\alpha^2}{1 + m^2\pi^2/\alpha^2}\right)^{1/2} \text{ (Rüdenberg)} \quad . \quad . \quad . \quad (25)$$

$$\frac{f_m}{f_1} = m^2\left(\frac{1 + \pi^2/\alpha^2}{1 + m^2\pi^2/\alpha^2}\right)^{1/2} \text{ (Blume–Boyajian)} \quad . \quad (26)$$

For large values of ψ, i.e. rapid decrease of $M(x, s)$ with distance, corresponding to an air-cored winding, eqn. (24) behaves much like eqn. (25) for the harmonics of practical importance, whilst for small values of ψ, approximating to the iron-cored winding, it is nearer eqn. (26).

A similar analysis can be made in the isolated-neutral case but no simplifying symmetry exists and the calculations are unwieldy. Since this case is of no practical importance it will not be pursued.

(4.2) Pseudo-Air-Cored Windings

The results of Section 4.1 complete the discussion of the single winding on one limb, as met with in practice in single-phase transformers. They do not suffice, however, for the discussion of multi-limb windings, and to this end the pseudo-air-cored winding is introduced. This may be regarded as an air-cored winding but with a ψ appropriate to an iron core.

The division of flux into leakage, core and magnetizing components no longer has any meaning and the equations are those of Heller and his co-workers. The reduced equations are (9), (10) and (11) but with $\Phi = 0$, and eqn. (12) vanishes. $M(x, s)$ now refers to total inductance.

The solution proceeds exactly as before, the differences in results appearing below and in Section 11.4.

(4.2.1) Neutral Earthed.

Since Φ vanishes, the solution for symmetrical modes is identical with that of Section 4.1.2. In the antisymmetrical modes, however, the current distribution is affected, reflecting m.m.f. unbalance. The new form of eqn. (23) may be obtained by writing ψ^{-1} for $(2 + \psi)/\psi^2$. Eqn. (24) may be recovered and may indeed be reached in Reference 2 if eqn. (33) in that paper is corrected.

(4.2.2) Neutral Isolated.

The consistency equation gives

$$\begin{aligned}&\psi^2(\mu^2 + \nu^2)(\mu \sinh \mu \cos \nu + \nu \cosh \mu \sin \nu) \\ &\quad + \psi[\mu\nu(\mu^2 - \nu^2) \sinh \mu \sin \nu + \\ &\qquad\qquad 2(\mu^4 + \nu^4 + \mu^2\nu^2) \cosh \mu \cos \nu + 2\mu^2\nu^2] \\ &\quad + (\mu^2 + \nu^2)(\mu^3 \sinh \mu \cos \nu - \nu^3 \cosh \mu \sin \nu) = 0 \quad . \quad (27)\end{aligned}$$

An approximate solution is given by $\nu = (2n \pm \tfrac{1}{2})\pi$. The potential distribution is given in Section 11.4.2.

(5) WINDING ON TWO LIMBS OF SINGLE-PHASE CORE-TYPE TRANSFORMER

(5.1) General Equations

Referring to Fig. 2, two additional parameters are introduced, namely capacitance and leakage inductance between limbs, appearing in the equations as C_k and $L(x, s)$. Using subscripts

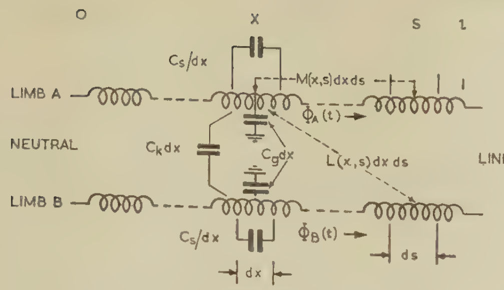


Fig. 2.—Equivalent network for 2-limb winding.

A and B to refer to the limbs, the following reduced equations may be written down:

$$I'_A(x) = \omega C_g V_A(x) - \omega C_s V''_A(x) + \omega C_k[V_A(x) - V_B(x)] \quad (28)$$

$$V'_A(x) = -N\omega\Phi_A - \omega\int_0^l M(x,s)I_A(s)ds - \omega\int_0^l L(x,s)I_B(s)ds \quad . \quad . \quad . \quad . \quad (29)$$

$$I_{sA}(x) = \omega C_s V'_A(x) \quad . \quad . \quad . \quad . \quad . \quad (30)$$

A similar triad may be obtained by interchanging A and B. The generalized rule for m.m.f. balance and the continuity condition for magnetizing flux supply two more equations:

$$\int_0^l I_A(x)dx = \int_0^l I_B(x)dx \quad . \quad . \quad . \quad . \quad (31)$$

$$\Phi_A + \Phi_B = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

(5.2) Resolution into Equivalent Single-Limb Equations

Write $V_1(x) = V_A(x) - V_B(x) \quad . \quad . \quad . \quad . \quad (33)$

and form similar combinations for the other variables. Simple operations on the equations of Section 5.1 yield

$$I'_1(x) = \omega(C_g + 2C_k)V_1(x) - \omega C_s V''_1(x) \quad . \quad . \quad (34)$$

$$V'_1(x) = -N\omega\Phi_1 - \omega\int_0^l [M(x,s) - L(x,s)]I_1(s)ds \quad (35)$$

$$I_{s1}(x) = \omega C_s V_1(x) \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$\int_0^l I_1(x)dx = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Comparing these with eqns. (9), (10), (11) and (12) it is seen that they are the equations of oscillation of an equivalent single-limb iron-cored winding of parameters $C_g + 2C_k$, C_s and $M(x, s) - L(x, s)$.

Similarly, forming the sum functions of the variables, using subscript 0, it will be found that the resolved equations are those of a pseudo-air-cored coil of parameters C_g, C_s and $M(x, s) + L(x, s)$.

It should be noted that the convention is such that

$$|M(x,s) - L(x,s)| > |M(x,s)| > |M(x,s) + L(x,s)|.$$

(5.3) Physical Interpretation

The significance of the resolution is well illustrated by considering possible types of oscillation when all four winding terminals are earthed. If the winding is supposed oscillating in tandem, i.e. potentials and currents are identical at corresponding points on the limbs, it follows that there is no stress on the inter-limb capacitance and hence no capacitance current between limbs. By continuity, magnetizing flux must vanish but it is not necessary for the resultant m.m.f. on each limb to be zero.

Oscillation of this type can now be identified with the 0-type deduced above and may be termed oscillation in tandem condition for a winding on two limbs of a core-type transformer.

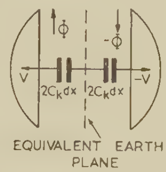


Fig. 3.—Single-opposition oscillation.

Fig. 3 represents a resonance in which potentials and currents are equal but opposite at corresponding points. By symmetry, the median plane between the limbs is effectively an earth plane, and thus the effective capacitance to earth for each winding is increased to $C_g + 2C_k$. Since the m.m.f.'s on the limbs must be equal, and yet are obviously opposite in sign, each must vanish identically, though in this case a magnetizing flux may exist. Finally, observing the sign convention for leakage flux, eqns. (34)–(37) may be written down by inspection. Thus oscillation of the 1-type may be termed oscillation in single-opposition condition.

The resultant oscillation in the winding may now be found by compounding the resolved types by

$$V_A(x) = \tfrac{1}{2}[V_0(x) - V_1(x)] \quad . \quad . \quad . \quad . \quad (38)$$

$$V_B(x) = \tfrac{1}{2}[V_0(x) + V_1(x)] \quad . \quad . \quad . \quad . \quad (39)$$

To complete the analogy with Section 4, the following approximations are made for the leakage-inductance parameters:

$$M_0(x,s) = M(x,s) + L(x,s) = L_0\varepsilon^{-\lambda_0|x-s|} \quad . \quad (40)$$

$$M_1(x,s) = M(x,s) - L(x,s) = L_1\varepsilon^{-\lambda_1|x-s|} \quad . \quad (41)$$

Boundary conditions are resolved in Section 10.5. Corresponding to eqn. (15), the biorthogonal relation

$$\int_0^l [V'_{Ap}(x)I_{Aq}(x) + V'_{Bp}I_{Bq}(x)]dx = 0$$

applies between oscillations in any two distinct modes for a given boundary condition, whether of tandem or opposition type, or one of each. Physically it represents the well-known phenomenon of energy transfer between limbs as observed in oscillographic records.

(6) WINDING ON THREE LIMBS OF CORE-TYPE TRANSFORMER

(6.1) General Equations

To cover the widest possible field of application it becomes necessary to introduce, in addition to $M(x, s)$, C_g and C_s, parameters C_k, C_h referring to distributed capacitance between adjacent and outer limbs respectively, and $L(x, s)$, $N(x, s)$ referring to the corresponding mutual leakage inductance functions. Using subscripts A, B, C to denote the limbs, it is assumed that the assembly is symmetrical about limb B. The reduced equations for limb A are

$$I'_A(x) = \omega C_g V_A(x) - \omega C_s V''_A(x) + \omega C_k[V_A(x) - V_B(x)] + \omega C_h[V_A(x) - V_C(x)] \quad (42)$$

$$V_A'(x) = -N\omega\Phi_A - \omega\int_0^l M(x, s)I_A(s)ds$$
$$- \omega\int_0^l L(x, s)I_B(s)ds - \omega\int_0^l N(x, s)I_C(s)ds \quad (43)$$
$$I_{sA}(x) = \omega C_s V_A'(x) \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Interchange of A and C gives the equations for limb C. For limb B they take the form

$$I_B'(x) = \omega C_g V_B(x) - \omega C_s V_B''(x)$$
$$+ \omega C_k[2V_B(x) - V_A(x) - V_C(x)] \quad (45)$$
$$V_B'(x) = -N\omega\Phi_B - \omega\int_0^l M(x, s)I_B(s)ds$$
$$- \omega\int_0^l L(x, s)[I_A(s) + I_C(s)]ds \quad (46)$$

M.M.F. balance and magnetizing-flux continuity give

$$\int_0^l I_A(x)dx = \int_0^l I_B(x)dx = \int_0^l I_C(x)dx \quad . \quad . \quad (47)$$
$$\Phi_A + \Phi_B + \Phi_C = 0 \quad . \quad . \quad . \quad . \quad . \quad (48)$$

(6.2) Partial Resolution into Simpler Constituents

It is unfortunate that in the general case the oscillations cannot be completely resolved into single-limb types. However, the symmetry of the structure does allow identification of one elementary type, reducing accordingly the complexity. Thus, writing

$$V_1(x) = V_A(x) - V_C(x) \quad . \quad . \quad . \quad . \quad (49)$$

etc., it is found that the variables with subscript 1 form an independent subset of equations of the single-limb iron-cored type, which will be referred to as oscillation in single-opposition condition, with equivalent parameters $C_g + C_k + 2C_h$, C_s, $M(x, s) - N(x, s)$. In this type of oscillation all variables are zero on limb B and assume equal but opposite values on limbs A and C, exactly as in the 2-limb case.

The complementary subset of equations, given in Section 11.6, may be solved as they stand, but the characteristic equation is now of eighth order and massive calculations ensue. Further resolution is only possible if simplifying assumptions are made, as illustrated in the next Section.

(6.3) The Triangular Core

This is the classic standby of the transformer analyst when difficulties arise in 'in-line' calculations. First, assume $N(x, s) = L(x, s)$. Complete resolution is now possible, as shown in Section 11.6, into a pseudo-air-cored type in which the three limbs oscillate in tandem, and an iron-cored type where the outer limbs are in tandem and variables on the middle limb are opposite to those at corresponding points on the outer limbs and double their magnitude. The first, with parameters C_g, C_s, $M(x, s) + 2L(x, s)$, will be referred to as the tandem condition and denoted by subscript 0, and the second, with parameters $C_g + 3C_k$, C_s, $M(x, s) - L(x, s)$, as the double-opposition condition, subscript 2.

Now set $C_h = C_k$, completing the triangular symmetry. It will be observed that the single- and double-opposition conditions now have the same parameters and that both are of iron-cored type. Consideration of possible delta or star connections shows that both equivalent resolved oscillations have the earthed-neutral boundary condition. It follows that the conditions coalesce in the sense that their defining equations become identical. By definition, the characteristic functions of either type satisfy equations of the form

$$V_A(x) + V_B(x) + V_C(x) = 0 \quad . \quad . \quad . \quad (50)$$

The distinction between single- and double-opposition conditions has now vanished and there is one degree of indeterminancy in the choice of the two conditions which are to partner the tandem condition in the definition of the resolved types. The situation is highly reminiscent of the choice to be made in the theory of symmetrical components, where the arbitrary resolution is made into positive and negative phase sequences, each of the components satisfying eqn. (50).

It seems less artificial to retain the three conditions deduced above, adjusting the amplitudes of the opposition conditions to fit excitation requirements according to the following scheme:

$$\left.\begin{aligned} V_A(x) &= \tfrac{1}{3}V_0(x) + \tfrac{1}{2}V_1(x) + \tfrac{1}{6}V_2(x) \\ V_B(x) &= \tfrac{1}{3}V_0(x) - \tfrac{1}{3}V_2(x) \\ V_C(x) &= \tfrac{1}{3}V_0(x) - \tfrac{1}{2}V_1(x) + \tfrac{1}{6}V_2(x) \end{aligned}\right\} \quad . \quad . \quad (51)$$

For a delta winding, or for a star winding with neutral earthed, all three are of neutral-earthed type. For a star winding with neutral isolated the tandem condition has neutral isolated and the opposition condition has neutral earthed. Only two sequences of characteristic values exist and the orthogonality condition takes the form

$$\sum_A \int_0^l V_{Ap}'(x)I_{Aq}(x)dx = 0$$

between any two simultaneous mono-frequency oscillations p and q.

(7) SURGE RESPONSE OF WINDINGS

(7.1) The Single-Limb Winding

Section 3 was devoted to setting up the general equations of oscillation and Section 4 to determination of the characteristic functions, which will now be combined to give the response to an arbitrary applied wave at the line terminal, assuming it to be a function of time only and unaffected by the current drawn by the winding. It is sufficient to calculate the response to the Heaviside unit function, other conditions being obtained by the Laplace transform.

The separation of the variables was carried out for sinusoidal time variation. There exist, in addition, singular solutions which are discussed in detail by Pirenne,[1] the results being quoted below without proof.

(7.1.1) Initial Distribution.

This corresponds to zero current and flux, the potential distribution being derived from the distributed capacitances. For neutral earthed it is

$$V(x, 0) = \operatorname{cosech} \alpha \sinh \alpha x/l \quad . \quad . \quad . \quad (52)$$

and for neutral isolated,

$$V(x, 0) = \operatorname{sech} \alpha \cosh \alpha x/l \quad . \quad . \quad . \quad . \quad (53)$$

These distributions exist only at $t = 0$, vanishing subsequently, being connected with the discontinuity at the origin of the Heaviside unit function.

(7.1.2) Axis of Oscillation.

This corresponds to the characteristic oscillation of zero frequency. The different forms are

Neutral earthed, iron-cored winding:

$$V(x, t) = x/l \quad \Phi(t) = -t/Nl \quad I(x, t) = 0 \quad . \quad (54)$$

Neutral earthed, pseudo-air-cored winding:

$$\left.\begin{aligned} V(x, t) &= \int_0^x \int_0^l M(y, z)dydz \Big/ \int_0^l \int_0^l M(y, z)dydz \\ I(x, t) &= -t \Big/ \int_0^l \int_0^l M(y, z)dydz \end{aligned}\right\} \quad . \quad (55)$$

Neutral isolated:

$$V(x, t) = 1, \quad I(x, t) = \Phi(t) = 0 \quad . \quad . \quad . \quad (56)$$

(7.1.3) **Solution as Series Expansion.**

The problem may be restated as follows: to find a solution of the general equations of oscillation, for a particular set of boundary conditions, in the form of an axis of oscillation together with an expansion in terms of characteristic functions such that the initial distribution is satisfied to $t = 0$. For an iron-cored winding with neutral earthed the required expansion is

$$V(x, t) = \frac{x}{l} + \sum_{n=1}^{\infty} a_n V_n(x) \cos \omega_n t \quad . \quad . \quad . \quad (57)$$

in which the ω_n terms are found from eqns. (19), (20), (22) and (23) and the $V_n(x)$ terms are given in Section 11.4.1, eqns. (69) and (70). The a_n terms are constants to be determined so that eqn. (52) is satisfied. This is done by means of eqn. (15) and the result is

$$a_n = \frac{I_n(l) + \omega_n C_s V'_n(l)}{\int_0^l V'_n(x) I_n(x) dx} \quad . \quad . \quad . \quad . \quad (58)$$

Eqns. (57) and (58), but with the appropriate ω_n, $V_n(x)$, apply for neutral earthed and for pseudo-air-cored windings save that the axis of oscillation must be amended accordingly.

(7.2) Winding on Two or Three Limbs

The first stage in the solution is to resolve the applied surge into tandem and opposition components using eqns. (38) and (39) or (51). Separate solutions are then calculated for the individual conditions and the results combined using these same equations.

(8) CONCLUSIONS

The underlying theme throughout has been unification of flux analysis with the accepted treatment at power frequencies, extending the latter to the multi-limb case as necessary. It has been argued by others that at the frequencies of oscillation encountered in practice there is negligible flux in the iron circuit, a viewpoint which overlooks that it is rate of change of flux, and not quantity of flux, which counts.

The 2-limb winding has been completely reduced to equivalent single-limb windings, the 3-limb case being only partly resolvable without simplifying assumptions.

The mathematics of the exponential approximation has been developed to a point where practical results are readily calculable and the frequency spectrum derived in a form admitting comparison with earlier theories.

(9) ACKNOWLEDGMENTS

The author wishes to thank the Director of the Electrical Research Association for permission to publish the paper and for assistance in preparing it for submission. Thanks are also due to the various staff members, and in particular to Mr. E. L. White, for valuable criticism and discussion, both as colleagues and subsequently.

(10) REFERENCES

(1) Pirenne, J.: 'Théorie générale des phénomènes oscillatoires dans les enroulements des transformateurs', *Revue Générale d'Électricité*, 1940, **47**, pp. 19 and 53. E.R.A. Translation Ref. Trans./IB1024.

(2) Heller, B., Hlavka, J., and Veverka, A.: 'Surge Phenomena in Transformers', *Elektrotechniky Obzor*, 1948, **37**, No. 6, p. 100.

(3) Coleman, B. L.: 'The Calculation of Currents due to Faults between Turns in Transformer Windings', E.R.A. Report Ref. Q/T121.

(4) Rüdenberg, R.: 'Performance of Travelling Waves in Coils and Windings', *Transactions of the American I.E.E.*, 1940, **59**, pp. 1031 and 1257.

(5) White, E. L.: 'An Experimental Study of Surges and Oscillations in Windings of Core-Type Transformers' (see p. 421).

(11) APPENDICES

(11.1) The Orthogonality Relationship

By eqns. (10) and (12), if p and q denote different characteristic solutions,

$$\int_0^l V'_p(x) I_q(x) dx = -\omega_p \int_0^l \int_0^l M(x, s) I_p(s) I_q(x) ds dx \quad (59)$$

Integrating by parts, substituting for $I'_q(x)$ in the resultant expression from eqn. (9) and integrating again by parts,

$$\int_0^l V'_p(x) I_q(x) dx = \Big[V_p(x)[I_q(x) + \omega_q C_s V'_q(x)] \Big]_0^l$$
$$- \omega_q \Big[C_g \int_0^l V_p(x) V_q(x) dx + C_s \int_0^l V'_p(x) V'_q(x) dx \Big] \quad (60)$$

Comparing with the boundary conditions, eqns. (13) and (14), the first bracket on the right vanishes. Equating the right-hand sides of eqns. (59) and (60) it is seen that all the integrals involved are symmetrical in p and q. It follows that, if $\omega_p \neq \omega_q$, the expressions vanish identically, giving eqn. (15). Further, these expressions represent the energy 'crosstalk' between modes, whence Pirenne's energy theorem[1] may be deduced.

(11.2) The Characteristic Equation

Substituting eqn. (16) in eqn. (10), successive differentiation gives

$$V'(x) = -N\omega\Phi - \omega M_0 \Big[\int_0^x \varepsilon^{-\lambda(x-s)} I(s) ds + \int_x^l \varepsilon^{-\lambda(s-x)} I(s) ds \Big] \quad (61)$$

$$V''(x) = -\omega M_0 \Big[-\lambda \int_0^x \varepsilon^{-\lambda(x-s)} I(s) ds + \lambda \int_x^l \varepsilon^{-\lambda(s-x)} I(s) ds \Big] \quad (62)$$

$$V'''(x) = 2\omega\lambda M_0 I(x) + \lambda^2 [V'(x) + N\omega\Phi] \quad . \quad . \quad . \quad . \quad (63)$$

and finally the characteristic equation (17). Comparison of eqns. (61) and (62) for $x = 0$ or l, and integration of eqn. (67) gives

$$V''(0) = \lambda[V'(0) + N\omega\Phi] \quad . \quad . \quad . \quad . \quad (64)$$

$$V''(l) = \lambda[V'(l) + N\omega\Phi] \quad . \quad . \quad . \quad . \quad (65)$$

$$V''(l) - V''(0) = \lambda^2[V(l) - V(0) + N\omega\Phi l] \quad . \quad (66)$$

which are the conditions referred to just after eqn. (20). Substituting from eqn. (18) in eqn. (17),

$$\omega^2 = \frac{\nu^2(\nu^2 + \psi^2)}{2\psi M_0 C_G l^2(1 + \nu^2/\alpha^2)} = \frac{\mu^2(\mu^2 - \psi^2)}{2\psi M_0 C_G l^2(1 - \mu^2/\alpha^2)} \quad (67)$$

whence eqns. (19) and (20) are derived.

(11.3) Numerical Calculations

Rewriting eqn. (22) in the form

$$\nu = 2 \text{ arc tan} \left[\frac{\mu \tanh \frac{1}{2}\mu + \psi^{-1}(\mu^2 + \nu^2)}{\nu}\right] \quad . \quad (68)$$

first approximations for ν are $\nu = \pi$, 3π, 5π, etc. Taking, for example, $\nu = \pi$, a corresponding approximation for μ is found from eqn. (20). Substituting these values in the right-hand side of eqn. (68) a new approximation for ν is obtained and the whole process is repeated until sufficient accuracy results. Similar devices obtain for the other equations of the paper.

(11.4) Potential and Current Distributions

(11.4.1) Neutral Earthed.

For the symmetrical modes the equations are

$$\left.\begin{aligned}
V(x) &= \cos\tfrac{1}{2}\nu\cosh\tfrac{1}{2}\mu\left(1 - \frac{2x}{l}\right) - \cosh\tfrac{1}{2}\mu\cos\tfrac{1}{2}\nu\left(1 - \frac{2x}{l}\right)\\
I(x) &= \frac{1}{2\omega\psi M_0 l^2}\left[\nu(\nu^2+\psi^2)\cosh\tfrac{1}{2}\mu\sin\tfrac{1}{2}\nu\left(1-\frac{2x}{l}\right)\right.\\
&\qquad \left. - \mu(\mu^2 - \psi^2)\cos\tfrac{1}{2}\nu\sinh\tfrac{1}{2}\mu\left(1 - \frac{2x}{l}\right)\right]\\
\Phi &= 0
\end{aligned}\right\} \quad (69)$$

and for antisymmetrical modes,

$$\left.\begin{aligned}
V(x) &= \sin\tfrac{1}{2}\nu\sinh\tfrac{1}{2}\mu\left(1 - \frac{2x}{l}\right)\\
&\qquad - \sinh\tfrac{1}{2}\mu\sin\tfrac{1}{2}\nu\left(1 - \frac{2x}{l}\right)\\
I(x) &= \frac{1}{2\omega\psi M_0 l^2}\left[-\nu(\nu^2+\psi^2)\sinh\tfrac{1}{2}\mu\cos\tfrac{1}{2}\nu\left(1 - \frac{2x}{l}\right)\right.\\
&\qquad - \mu(\mu^2 - \psi^2)\sin\tfrac{1}{2}\nu\cosh\tfrac{1}{2}\mu\left(1 - \frac{2x}{l}\right)\\
&\qquad \left. + 2(\mu^2 + \nu^2)\sin\tfrac{1}{2}\nu\sinh\tfrac{1}{2}\mu\right]\\
\Phi &= \frac{-1}{\omega\psi^2 Nl}2(\mu^2 + \nu^2)\sin\tfrac{1}{2}\nu\sinh\tfrac{1}{2}\mu
\end{aligned}\right\} \quad (70)$$

For the pseudo-air-cored coil, eqns. (69) are valid, but in eqns. (70) the flux equation vanishes and the corresponding term must be deleted from the current equation.

(11.4.2) Pseudo-Air-Cored Coil, Neutral Isolated.

The potential distribution is

$$\begin{aligned}
V(x) &= \psi(\nu^2\cosh\mu + \mu^2\cos\nu)\cosh\mu\left(1 - \frac{x}{l}\right)\\
&\quad + [\mu(\mu^2+\nu^2)\cos\nu - \psi\nu(\nu\sinh\mu - \mu\sin\nu)]\sinh\mu\left(1 - \frac{x}{l}\right)\\
&\quad - \psi(\nu^2\cosh\mu + \mu^2\cos\nu)\cos\nu\left(1 - \frac{x}{l}\right)\\
&\quad + [\nu(\mu^2+\nu^2)\cosh\mu + \psi\mu(\nu\sinh\mu - \mu\sin\nu)]\sin\nu\left(1 - \frac{x}{l}\right)
\end{aligned} \quad . \quad . \quad . \quad . \quad (71$$

(11.5) Resolved Boundary Conditions, 2-Limb Case

For earthed neutral, by eqn. (13) the resolved equations are

$$V_0(0) = V_0(l) = V_1(0) = V_1(l) = 0 \quad . \quad . \quad . \quad (72$$

For neutral isolated the actual conditions are

$$V_A(l) = V_B(l) = 0 \quad . \quad . \quad . \quad . \quad (73$$

$$V_A(0) = V_B(0) \quad . \quad . \quad . \quad . \quad . \quad (74$$

$$I_A(0) + \omega C_s V'_A(0) = -[I_B(0) + \omega C_s V'_B(0)] \quad . \quad (75$$

since the current flowing into the connecting wire is the sum o the wire and series-capacitance currents. Resolution gives

$$V_1(0) = V_1(l) = 0 \quad . \quad . \quad . \quad . \quad . \quad (76$$

$$I_0(0) + \omega C_s V'_0(0) = V_0(l) = 0 \quad . \quad . \quad . \quad (77$$

(11.6) Resolution of 3-Limb Case

After elimination of the single-opposition equations fron eqns. (42), using subscript T to denote the functions,

$$V_T(x) = V_A(x) + V_C(x) \quad . \quad . \quad . \quad . \quad (78$$

etc., and the remaining subsets are

$$\left.\begin{aligned}
I'_T(x) &= \omega C_g V_T(x) - \omega C_s V''_T(x)\\
&\qquad + \omega C_k[V_T(x) - 2V_B(x)]\\
I'_B(x) &= \omega C_g V_B(x) - \omega C_s V''_B(x)\\
&\qquad + \omega C_k[2V_B(x) - V_T(x)]
\end{aligned}\right\} \quad (79$$

$$\left.\begin{aligned}
V'_T(x) &= -N\omega\Phi_T - \omega\int_0^l [M(x,s) + N(x,s)]I_T(s)ds\\
&\qquad - \omega\int_0^l L(x,s)2I_B(s)ds\\
V'_B(x) &= -N\omega\Phi_B - \omega\int_0^l M(x,s)I_B(s)ds\\
&\qquad - \omega\int_0^l L(x,s)I_T(s)ds
\end{aligned}\right\} \quad (80$$

$$\left.\begin{aligned}
I_{sT}(x) &= \omega C_s V'_{sT}(x)\\
I_{sB}(x) &= \omega C_s V'_{sB}(x)
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81$$

$$\left.\begin{aligned}
\int_0^l I_T(x)dx &= 2\int_0^l I_B(x)dx\\
\Phi_T + \Phi_B &= 0
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83$$

With the assumption $N(x, s) = L(x, s)$ the subsets for tande and double-opposition conditions may be deduced by writing

$$V_0 = V_T + V_B = V_A + V_B + V_C, \text{ etc.,}$$

$$V_2 = V_T - 2V_B = V_A + V_C - 2V_B, \text{ etc.,}$$

and, in particular, it follows that

$$\Phi_0 = 0 \quad . \quad . \quad . \quad . \quad . \quad (8$$

$$\int_0^l I_2(x)dx = 0 \quad . \quad . \quad . \quad . \quad . \quad (8$$

showing that the tandem condition belongs to pseudo-air-co theory whilst the double-opposition condition is of the iron-co variety.

21.315.624 : 621.317.333.8

The Institution of Electrical Engineers
Paper No. 3317 S
Oct. 1960

THE SURGE FLASHOVER VOLTAGES OF AIR-GAPS ASSOCIATED WITH INSULATORS AND BUSHINGS

By G. W. BOWDLER, M.Sc., Associate Member, and R. C. HUGHES, B.Sc.

(*The paper was first received* 30*th March, and in revised form* 16*th June*, 1960.)

SUMMARY

The paper, which is based on E.R.A. Reports Ref. S/T92 and /T105, summarizes the results of tests made with surge voltages of /50 microsec waveshape on a wide range of insulators and air-gaps ver a period of 10 years. In determining 50% flashover voltages was found that the range of voltages over which transition from no ashover to flashover on each of 10 applications of the surge occurred vas somewhat variable but averaged about 15% for small and 7% for arge insulators and gaps. The extent of this range, in conjunction vith the finite number of surges (50–100) generally employed, could ccount for the variations obtained in the values, corrected to standard tmospheric conditions, of the 50% flashover voltage of a given nsulator.

In all the systems tested the electric field between the electrodes vas highly non-uniform, and a roughly common relationship was ound to exist between V, the average value of the positive and egative 50% flashover voltages, and the length of the corresponding lashover path.

The time-lag curves obtained in tests at higher voltages on systems vith small polarity differences approximated to exponential curves lecaying with a time-constant of 2·9 microsec to a voltage V and assing through $1\cdot 5V$ at 2 microsec.

(1) INTRODUCTION

The co-ordination of the insulation of a high-voltage overhead ine demands a knowledge of the flashover voltages under various test conditions of the insulators and protective gaps with which it is equipped, as well as of the voltages which vital equipment such as transformers will withstand without damage.[1]

One of the most important tests is the determination of the peak amplitude of a surge voltage of specified waveshape which has a 50% chance of causing flashover. Co-operative tests[2,3,4] have shown that the values obtained for the 50% flashover voltage of a given insulator or gap by various laboratories may differ by as much as 20%, even after allowance has been made for the effect of differing atmospheric densities and humidities. The variations obtained on different occasions in the same laboratory are not as great as this but may amount to 10%. Such large variations are not readily explained and may encourage the belief that small variations in the electrode shape and set-up of a gap could cause large variation in the flashover voltage.

During the course of surge flashover tests on insulators and associated air-gaps carried out over a period of 10 years the authors have accumulated much data relating to suspension strings of cap-and-pin insulators, stacks of pedestal-type post insulators, bushings and gaps between rod electrodes. These data are presented here and the extent to which they agree with or extend existing knowledge is discussed.

(2) TEST CONDITIONS

The surge voltages used throughout the tests had a waveshape which, in the absence of flashover of the test object, approximated closely to the 1/50 microsec standard. They were derived either from a generator consisting of any number of stages up to 20, the capacitance and voltage rating of each stage being 0·2 μF and 100 kV, or, when voltages above 1 500 kV were required, from a generator of eight stages, each of 0·12 μF and 400 kV rating. A stack of high-voltage capacitors (300–600 pF), connected in parallel with the test object, enabled the requisite wavefront to be obtained with a series damping resistance of 500–1 000 ohms.

All voltage measurements were derived either directly or indirectly (in the latter case using the generator charging voltmeter as a transfer instrument) from cathode-ray oscillograms of a fraction of the voltage across the test object, obtained from a resistive or capacitive voltage divider. The tests were carried out indoors under the prevailing atmospheric conditions and the measured voltages have been corrected to correspond to a temperature of 20° C, a pressure of 760 mm Hg and a humidity of 11 g/m³, in accordance with Appendix A of B.S. 137: 1941; the magnitude of this correction was never more than 7%.

The insulator strings, consisting generally of units 10 in in diameter and 5 in spacing, were suspended from a crane with the lower end of the string about 20 ft above floor level: a conductor clamp was attached to the bottom end of the string and a long horizontal metal tube was held at its mid-point in the clamp to simulate a line conductor (Fig. 1). The strings were tested with and without arcing fittings.

The post insulators were suspended from the crane via an

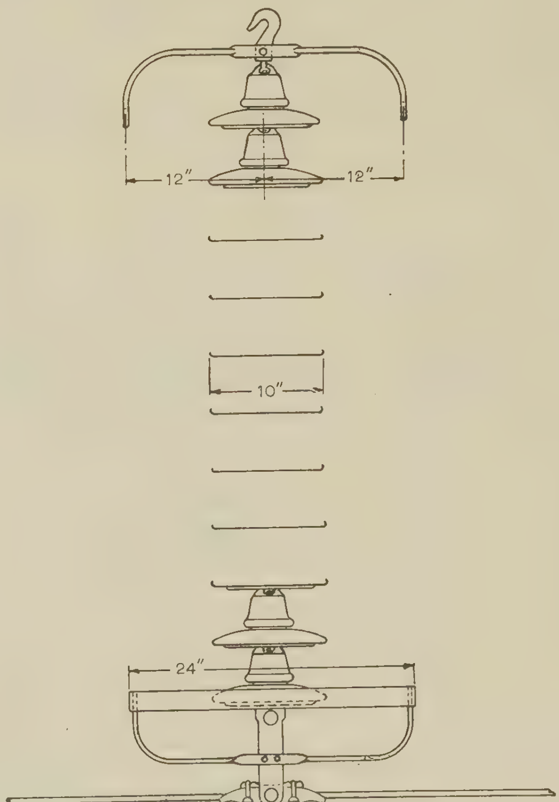


Fig. 1.—Suspension insulator string.

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.
The paper is an official communication from the National Physical Laboratory.

insulator string with the earthed lower end about 6 ft above floor level: an 8 ft length of 6 in × 2 in channel iron was bolted at its mid-point to each end of the post, the axes of the lengths of channel iron being at right-angles to that of the post and to each other (Fig. 2). Tests were also carried out with a ½ in diameter

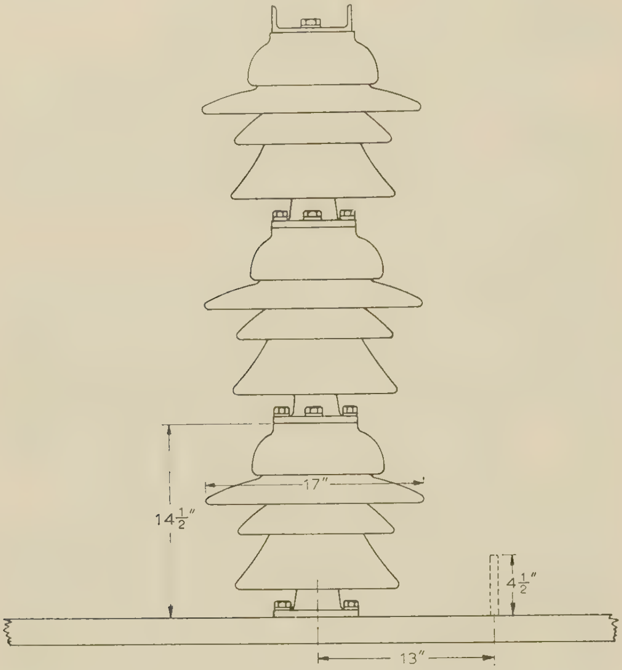


Fig. 2.—Post insulator stack.

bolt, shown dotted in Fig. 2, fixed to the bottom channel iron at a point 13 in from the centre line of the stack and projecting 4½ in above the surface of the channel, which eliminated a large polarity effect.

Some of the transformer bushings, which conformed to the T1 or T2 British Electricity Specifications, are illustrated in Fig. 3. They were mounted on an earthed plate with their lower ends immersed in transformer oil; the radial dimension of the earthed plate was approximately equal to the height of the bushing. Tests were also made on several small bushings (11–44 kV rating) without arcing fittings.

A range of standard rod gaps was tested: in these the electrodes consisted of ½ in square-section rods with square-cut ends mounted horizontally at a distance above an earthed plane of 4 in plus 1·3–2·0 times the gap length; the rods overhung their supports by at least half the gap length. In addition, tests were carried out on horizontal gaps between 5/8 in diameter rod electrodes, one of which was mounted on the cap of a bushing insulator and the other on a vertical earthed support to represent a typical co-ordinating gap structure mounted on a transformer. One of these gaps, 26 in in length, was used with the 132 kV bushing; the other, 46 in in length, was used with the 275 kV bushing [Figs. 3(*c*) and (*d*)].

(3) TEST RESULTS

(3.1) 50% Flashover Voltage

Over a small range of voltages, at each of which a group of 10 surges was applied, there was a transition from 0/10 to 10/10 flashover. The transition was not always regular or repeatable but the tests were continued* until a straight line could reasonably be drawn through the points in this range obtained by plotting on linear graph paper the percentage of surges causing flashover against the applied voltage: this procedure is discussed in Section 4.1. From the points at which this line intersected the abscissae corresponding to 0%, 50% and 100% flashover, the 50% flashover voltage and what will be termed the 0–100% transition range were determined. The extent of this transition range, expressed as a fraction of the 50% flashover voltage, is shown in Fig. 4 for most of the systems which were tested.

* A total of 50–100 surges was applied in the determination of the 50% flashover voltage.

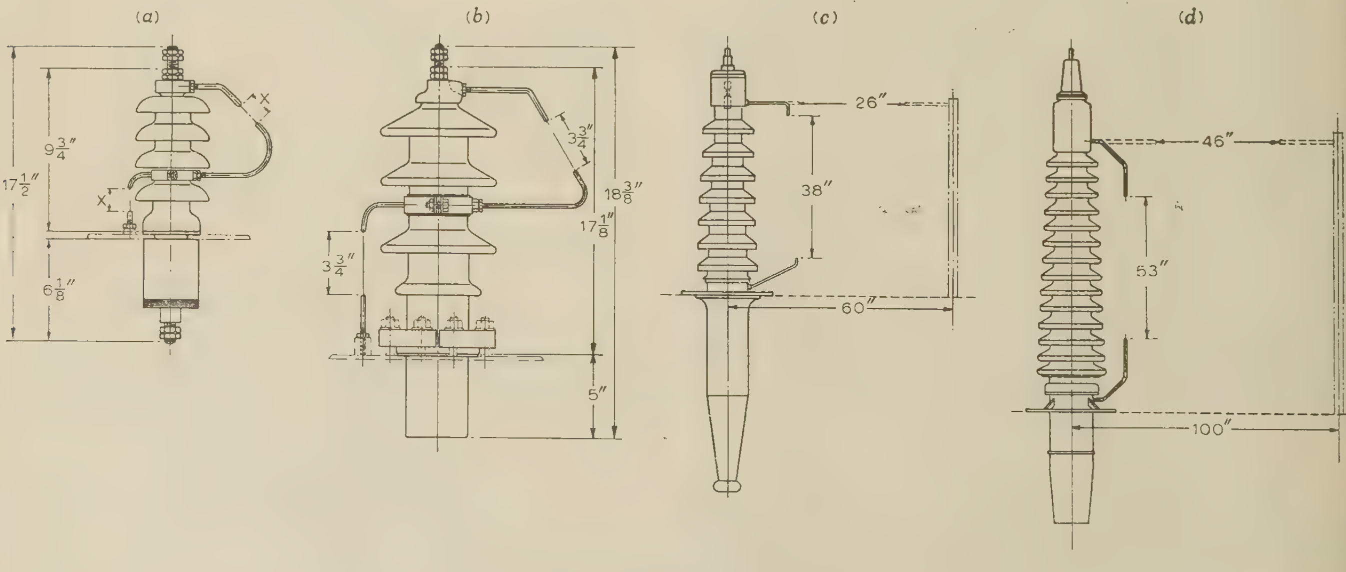


Fig. 3.—Transformer bushings.

(*a*) 6·6 kV and 11 kV bushing. Dimension *X* adjusted to 1 in for 6·6 kV and 1¼ in for 11 kV.
(*b*) 33 kV bushing.
(*c*) 132 kV bushing fitted with 38 in horn gap or, alternatively, 26 in co-ordinating gap (shown dashed).
(*d*) 275 kV bushing fitted with 53 in horn gap or, alternatively, 46 in co-ordinating gap (shown dashed).

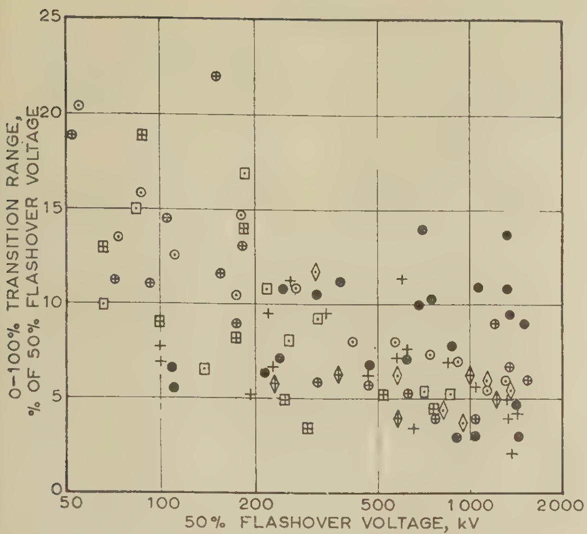


Fig. 4.—Relation between 50% flashover voltage and 0–100% flashover transition range.

Positive polarity	Negative polarity	
+	●	Insulator strings.
⊕	⊙	Rod gaps.
⊞	⊡	Bushings.
◊	◊	Posts.

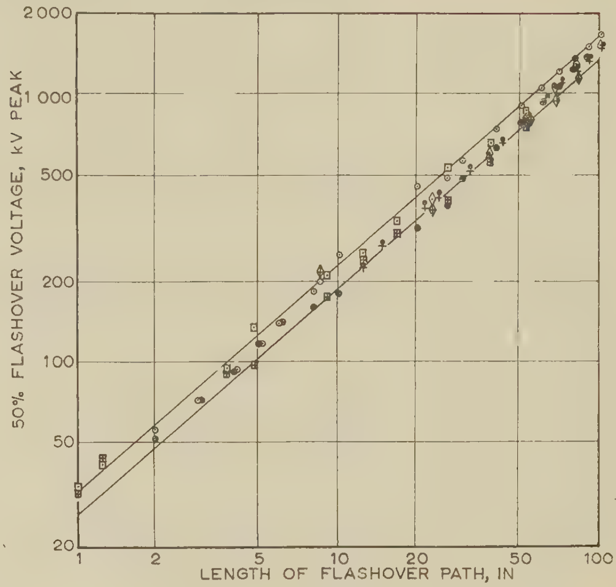


Fig. 5.—Relation between 50% flashover voltage (1/50 microsec wave) and length of flashover path.

Positive polarity	Negative polarity	
+	●	Insulator strings.
⊕	⊙	Rod gaps.
⊞	⊡	Bushings.
◊	◊	Posts.

Many of the insulators and gaps were tested more than once, in one case as many as 16 times. The resulting values of the 50% flashover voltage of a given assembly had a standard deviation of about 5% for small and rather less for large insulators and gaps. The mean 50% flashover voltages have been plotted in Fig. 5 on logarithmic scales against the corresponding length of the flashover path, i.e. the shortest path through air between the electrodes of the test object. This method of presenting the results not only shows the essential relation between flashover voltage and path length, but also avoids any undue impression of precision which might be conveyed by Tables of figures. The salient characteristics of the various types of insulator are briefly described below.

Insulator Strings.—There was about 5% difference between the positive and negative flashover voltages, the negative being the higher, for strings either with or without arcing fittings consisting of a 24 in diameter ring at the line end and horns of 24 in span at the earthed end. With these fittings the spark was clear of the insulators for strings having a flashover path length of 63 in (14 units) or less.

Post Insulators.—Without the projecting bolt fitted to the bottom channel iron (Fig. 2), the negative flashover voltage was about 15% greater than the positive for long stacks and 40% greater for short stacks of one or two units. The fitting of the bolt did not affect the positive flashover voltages, but the negative ones were reduced to such an extent that they were little greater than the positive; only the results with the bolt fitted have been plotted in Fig. 5.

Bushings.—The positive and negative flashover voltages of the small bushings fitted with series gaps [Figs. 3(*a*) and (*b*)] were approximately equal. A separate a.c. test established that the two gaps on these bushings shared the voltage about equally, and hence the flashover of the bushings is largely dependent on the flashover characteristics of the component gaps. The results on these bushings have therefore been plotted in Fig. 5 with abscissae corresponding to the lengths of the component gaps and ordinates corresponding to half the total applied voltages.

The negative flashover voltages of the 132 kV and 275 kV bushings [Figs. 3(*c*) and (*d*)] fitted with arcing horns and of the smaller bushings without arcing fittings exceeded the positive by amounts ranging from 10% to 30%.

Rod Gaps.—The negative flashover voltages of standard rod gaps exceeded the positive ones by 10–15% for long gaps (30 in or more) and by an inappreciable amount for short gaps of 6 in or less; for gaps of intermediate length they were as much as 40% greater than the positive. Many tests were made on rod gaps between 6 in and 20 in in length in an attempt to discover the cause of this local large polarity effect, but the results were inconclusive; the tests showed, however, that a precise specification of the shape of the electrode tip is not necessary, since conically-ended rods and square-cut tubes gave substantially the same results as the conventional square-cut square-section rods; ionization of the gap also had no effect on the flashover voltage.

The mean flashover voltages of the model co-ordinating gaps used in conjunction with the 132 kV and 275 kV bushings were not more than 5% different from those of standard rod gaps of the same length.

(3.2) Relation between Magnitude of Applied Voltage and Time to Flashover

Cathode-ray oscillograms of the voltage across the test object were recorded with surges ranging upwards in peak amplitude from the 50% flashover value until flashover occurred in approximately 1 microsec, or until the voltage limit of the generator was reached. From the oscillograms the time-lag curves shown in Figs. 6–8 were derived. These are the mean curves drawn through the experimental points; to include all the points obtained on a given assembly, a band extending 5% in the ordinate direction on either side of the mean curve would have to be drawn.

As noted in an earlier paper,[5] it was again observed that the discharge, which normally followed the shortest air path between

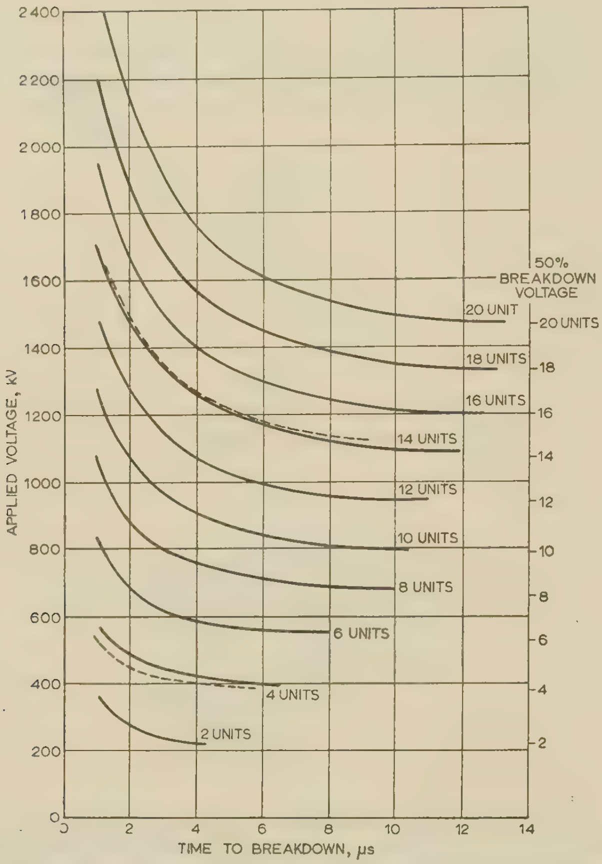


Fig. 6.—Time-lag curves of suspension strings of insulators, 10 in diameter × 5 in spacing, without arcing fittings (1/50 microsec wave).

Dashed curves are interpolated from American results to correspond to 4- and 14-unit strings.

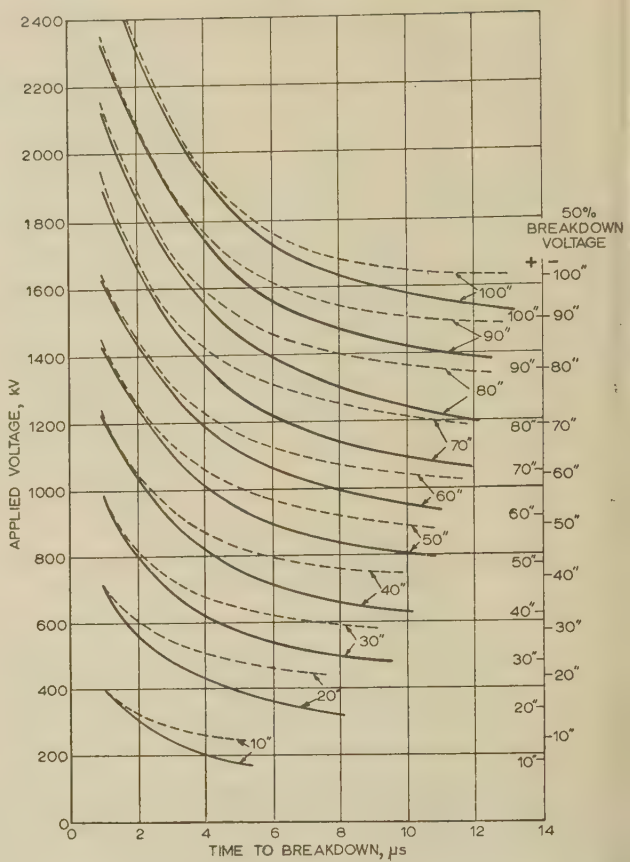


Fig. 7.—Time-lag curves of standard rod gaps (1/50 microsec wave).

—— H.V. electrode positive.
---- H.V. electrode negative.

the electrodes at voltages near the 50% value, tended increasingly to cling to the surface of the insulators as the voltage was raised.

(4) DISCUSSION OF RESULTS

(4.1) Consistency of Flashover Voltages

Statistical considerations assume that the actual transition from 0 to 100% flashover is sigmoid rather than linear, but that over the range 20–80%, to which particular attention was paid when drawing the line through the experimental points, it is substantially linear. Fig. 4 shows that the 0–100% transition range, derived from the slope of this line, varies widely; there is nevertheless a definite tendency for this range, expressed as a fraction of the 50% flashover voltage, to decrease as the length of the flashover path and the flashover voltage increase. There appears to be no appreciable difference in this respect between the various types of insulator and gap and little between the results obtained with positive and negative surges, i.e. the variously marked points in Fig. 4 are fairly evenly distributed.

Statistical considerations[6] also lead to a relation between the probable error in a determination of the 50% flashover voltage, the number of surges employed, and the extent of the 0–100% transition range; these indicate that, when 50 surges are applied for each determination, the standard deviation, σ, of a group of measured values of the 50% flashover voltage of a given insulator is approximately 0·3 times the 0–100% transition range, S, of that insulator. The values of σ and S derived from numerous tests on a range of standard rod gaps are given in

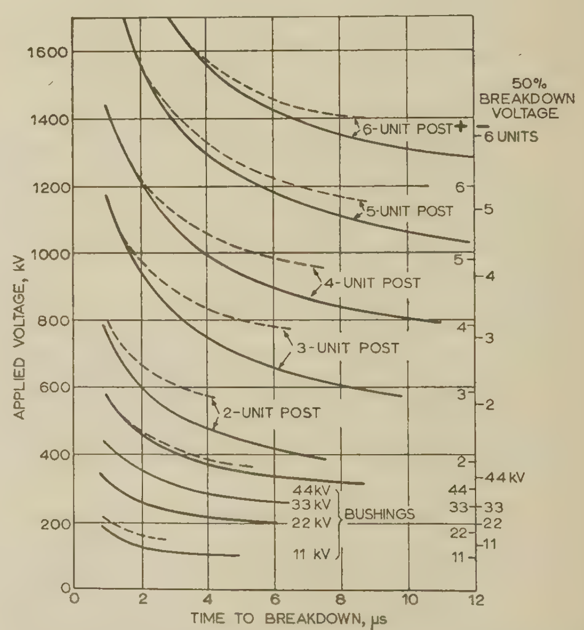


Fig. 8.—Time-lag curves of post insulators without horn at earthed end and of bushing insulators without arcing fittings (1/50 microsec wave).

—— H.V. electrode positive.
---- H.V. electrode negative.

Table 1

STANDARD DEVIATION AND MEAN EXTENT OF 0–100% TRANSITION RANGE OBTAINED IN REPEATED TESTS ON ROD GAPS

Gap length	Positive surges				Negative surges			
	Number of tests	S	$0\cdot3S$	σ	Number of tests	S	$0\cdot3S$	σ
in		%		%		%		%
6	11	17·4	5·2	6·5	10	16·5	5·0	2·9
8	16	19·3	5·8	4·9	14	19·2	5·8	4·9
10	13	15·1	4·5	5·9	15	12·8	3·8	6·5
20	4	10·5	3·1	1·8	5	15·2	4·5	3·3

Table 1. The rough agreement between corresponding values of σ and $0\cdot3\ S$ lends support to the view that some of the error in the determination of the 50% flashover voltage is due to the limited number of surges employed. Undue reliance should not be placed on this agreement, however, since the corrections applied to reduce the results to standard atmospheric conditions, and also the premisses on which the statistical considerations are based are subject to some uncertainty.

(4.2) Relation between 50% Flashover Voltage and Length of Flashover Path

The 50% flashover voltages obtained in the present tests on rod gaps and insulator strings agree, after allowing for slight differences in atmospheric conditions and voltage waveshape, to within 5% with the mean values obtained on equivalent gaps by five American laboratories,[2] except for standard rod gaps of 10 in and 20 in for which an abnormally large polarity effect, was observed in the present tests. It should be noted that for these gaps the Americans also reported a large dispersion of the results obtained with negative surges. The substantial agreement, shown in Fig. 5, between the flashover voltages of rod gaps and bushings having the same length of flashover path also agrees with the results obtained by Cole[7] on a wide range of bushings.

Most of the experimental points marked on Fig. 5 lie between two parallel lines which have been drawn to represent the upper and lower limits of the relation between flashover voltage, V, and gap length, L, in inches, given by the equation

$$V = 30L^{0\cdot85}\,\text{kV} \pm 10\% \quad . \quad . \quad . \quad . \quad (1)$$

The upper limit of tolerance indicated above is usually associated with negative polarity and the lower limit with positive polarity of the high-voltage electrode. All the results which lie outside these limits relate to systems with a large polarity effect, and even here the average of the positive and negative flashover voltages is well within the limits. If the results obtained by Hagenguth *et al.*[8] on gaps up to 250 in in length are plotted on Fig. 5, it will be found that they also lie between the parallel lines to which reference has already been made: eqn. (1) can thus be assumed to hold for gaps up to at least this length.

In any overhead line system it is undesirable to have a large difference between the flashover voltages with positive and negative surges. A gap with point-plane electrodes exhibits the greatest polarity difference, and insulator designs should avoid such extreme dissimilarity in the shape of the metal fittings. It is not always possible to predict by inspection of an insulator that it will possess a large polarity effect; experiment alone can decide this. Whenever the condition occurs, however, it can generally be alleviated by modifying the electrode which is positive, under the polarity giving the higher value of flashover voltage, so as to increase the stress at the point on this electrode to which flashover occurs. A device similar to that adopted here with the post insulators has been used by Allibone[3] to reduce the difference between the positive and negative flashover voltages of a pin insulator.

The voltage given by eqn. (1) is probably the minimum value at which a gap of length L, having no polarity effect, will flash over in 50% of the applications of a 1/50 microsec wave: it is the value obtained with pointed electrodes. In contrast, with uniform-field or large spherical electrodes, a maximum value, amounting to 3–4 times the above minimum, is obtained. On considering the way in which, for a given length of gap, the transition from the maximum to the minimum flashover voltage takes place as the curvature of the electrode is increased, it appears that there is a rapid fall as the ratio of maximum to mean inter-electrode stress departs from unity and that a value of this ratio is soon reached beyond which further increase causes no further decrease of flashover voltage. Thus, for a gap of one diameter between equal spheres, where the ratio of maximum to mean stress is 2·3, the sphere-gap Tables show that the flashover voltage has already fallen to about 65% of the uniform-field value, or halfway towards the value given by eqn. (1).

(4.3) Time to Flashover

Mean time-lag curves for standard rod gaps and insulator strings, using a 1·5/40 microsec positive wave, have been published in America.[9] Since the insulator unit used in the American tests differed from that of Fig. 1, a direct comparison of the time-lag curves for insulator strings is not possible. The two broken curves in Fig. 6 have been interpolated from the American curves, however, so as to correspond to the lengths of the 4-unit and 14-unit strings used in the present tests: the agreement of these with the full curves is good.

To avoid confusion, the American curves for rod gaps have not been added to Fig. 7; they lie, however, 0–5% below the positive-polarity curves for gaps of 60 in and upwards. For the smaller gaps the curvatures of the American curves are greater than those of the corresponding curves of Fig. 7, so that at approximately 2 microsec time-to-breakdown, where the difference is at its maximum, the flashover voltages derived from the American curves range from 10 to 20% below those obtained in the present tests.

From the time-lag curves the following general conclusions emerge:

(*a*) The mean time to flashover at the 50% flashover voltage increases with the length of the flashover path, but with a 50 microsec wave it tends towards a limit of about 15 microsec.

(*b*) Any difference in average breakdown time due to change of polarity becomes less, i.e. the positive and negative time-lag curves of a given system tend to coalesce, as the applied voltage increases.

(*c*) For insulators and gaps having little polarity effect the relation between breakdown voltage, V_1, and time to breakdown, t, is given approximately by the equation

$$V_1 = V(1 + \varepsilon^{-t/T}) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where V, the 50% flashover voltage, is calculated from eqn. (1) and T is a constant equal to 2·9 microsec. This equation gives values of V_1/V of 1·5, 1·25, 1·125 . . . for breakdown times of 2, 4, 6 . . . microsec.

In cases where there is an appreciable difference between the positive and negative flashover voltages, the values of V_1 and V in eqn. (2) are the means of the appropriate positive and negative values.

(5) CONCLUSIONS

The main conclusion to be drawn from an examination of the results presented here and in the references cited is embodied in Fig. 5 and eqns. (1) and (2), namely that for typical insulators and air-gaps associated with high-voltage overhead lines the

flashover voltage is dependent primarily on the minimum length of path through air between the electrodes and not on minor details of the electrode geometry. Scatter in the measured values of the 50% flashover voltage can be attributed largely to the extent of the 0–100% transition range and the limited number of surges employed for each determination; some may, however, be due to uncertainty in the values of the corrections applied to reduce the result to standard atmospheric conditions, which are known to be based on a very limited amount of experimental evidence.

(6) ACKNOWLEDGMENTS

The work described was carried out at the National Physical Laboratory on behalf of the British Electrical and Applied Industries Research Association. The paper is published by permission of the Directors of both these establishments.

(7) REFERENCES

(1) CLIFF, J. S.: 'The Co-ordination of Insulation of High-Voltage Electrical Installations', *Proceedings I.E.E.*, Paper No. 1485 S, March, 1953 (**101**, Part I, p. 39).
(2) BERGER, K.: Report of Study Committee No. 8, C.I.G.R.É., Paris, 1956, Paper No. 326.
(3) E.E.I.–N.E.M.A. Report: 'Flashover Characteristics of Rod Gaps and Insulators', *Transactions of the American I.E.E.*, 1937, **56**, p. 712.
(4) ALLIBONE, T. E.: 'International Comparison of Impulse-Voltage Tests', *Journal I.E.E.*, 1937, **81**, p. 741.
(5) BOWDLER, G. W., and STANDRING, W. G.: 'The Impulse Characteristics of Porcelain Insulators', *ibid.*, 1941, **88**, Part II, p. 443.
(6) BAUMANN, W.: 'Statistischer Fehler bei der Bestimmung der 50% Uberschlagstossspannung', *Elektrotechnische Zeitschift*, 1957, **78**, p. 369.
(7) COLE, H. L.: 'Flashover Characteristics of Transformer Condenser Bushings', *Transactions of the American I.E.E.*, 1939, **58**, p. 27.
(8) HAGENGUTH, J. H., ROHLFS, A. F., and DEGNAN, W. J.: 'Sixty-Cycle and Impulse Sparkover of Large Gap Spacings', *ibid.*, 1952, **71**, Part III, p. 455.
(9) DOWELL, J. C., and FOUST, C. M.: 'Impulse Strength of Insulators and Materials', *General Electric Review*, 1937, **40**, p. 141.

DISCUSSION ON 'THE DESIGN OF HOUSING-ESTATE DISTRIBUTION SYSTEMS USING A DIGITAL COMPUTER'*

SHEFFIELD SUB-CENTRE, 18TH MAY, 1960

Mr. G. R. Walker: If a real economic comparison between manual and automatic design is to be achieved it must be based on the overall cost of supplying the estate, which includes the cost of h.v. mains, substations and services as well as the m.v. mains. Unless the computer programme includes all the factors which are taken into account by the manual designer, I doubt very much whether any real saving would be shown on the overall costs.

To illustrate this point I would mention a few detailed factors which the designer considers when preparing his scheme. If eqn. (1) is applied indiscriminately for calculating the number of substations on estates of high load density, 1 000 kVA substations will be required. This, for well-known reasons, proves uneconomic.

The computer programme appears to design for single-sided mains in all cases. The layout of an estate may be such that, when taking into account both the m.v. mains and service costs, the double-sided mains system may in part prove to be the more economic. In Section 3.2 it is stated that the automatic design does not allow for m.v. cable between road sections 4 and 5; the manual designer has routed an m.v. cable along this road section as excavation is necessary for the h.v. cable between substations Nos. 1 and 2. Also, if the m.v. cable had been omitted a long service would have been required to the pavilion, thus increasing the service costs. A site suitable for an indoor substation may well, on the grounds of amenity, prove to be unsuitable for an outdoor type, and as the saving of some £400 can be achieved by the use of outdoor substations, these factors must be taken into account when selecting sites.

Dr. R. L. Grimsdale and **Mr. P. H. Sinclare** (*in reply*): We observed that there were no significant h.v. cost differences between automatic and manual designs for the four tested layouts. We therefore did not consider h.v. mains design to be vital to the automatic method, although we agree that it would be desirable; it would, in fact, be feasible provided that standards could be agreed upon.

If eqn. (1) results in an uneconomic number of substations for high load densities it could and should be amended. The automatic method was designed for single-sided mains, but provision is made for supplementing the single-feeder system where necessary (Section 2.3.2). When the designer selects feasible substation sites he could inform the machine whether indoor or outdoor stations are proposed, thus providing all relevant factors for an economic selection.

House services are associated with the nearest road section when the data are prepared, so that normal services need not enter into the design. For long services, such as the pavilion supply mentioned by Mr. Walker, the machine is supplied with all possible alternatives and then makes its choice of the most economic route.

* GRIMSDALE, R. L., and SINCLARE, P. H.: Paper No. 3131 S, December, 1959 (see 107 A, p. 295).

621.314.6

The Institution of Electrical Engineers
Paper No. 3215 U
Feb. 1960

SOME CONSIDERATIONS IN THE APPLICATION OF POWER RECTIFIERS AND CONVERTORS

By J. P. McBREEN, Associate Member.

(*The paper was first received* 29*th September, and in revised form* 17*th December*, 1959. *It was published in February*, 1960, *and was read before the* UTILIZATION SECTION 10*th March, the* NORTH-WESTERN CENTRE 5*th April, and the* SOUTH-WEST SCOTLAND SUB-CENTRE 20*th April*, 1960.)

SUMMARY

The paper reviews the more important considerations in the selection and application of power rectifiers and convertors. It begins with a discussion on the suitability and selection of semiconductor rectifiers and mercury-arc rectifiers and convertors for various applications, and proceeds to a discussion on the number of phases to employ and how this question is affected by supply and other considerations. The effect of rectifier-fed rolling-mill and winder drives on the supply system is discussed. A summary of the characteristics of the transformer connections in more general use is included, together with some notes on their application. Mention is made of parallel operation, and some examples are given of how phase multiplication is achieved. Then follows a discussion on possible faults and their prevention; protection requirements are outlined and the paper concludes with some notes on installation and ventilation. In the main, only British practice is discussed.

(1) INTRODUCTION

During the past 10–15 years there has been a tremendous increase in the use of rectifiers, not only for electrolytic and traction use, but also for many industrial applications. Mercury-arc rectifiers have mostly been used, but a few years ago, with the development of monocrystalline (germanium and silicon) semiconductor rectifiers suitable for high powers, a radical change began to take place. Many of the larger fields of application of the mercury-arc rectifier began to change over to germanium and silicon rectifiers. Simultaneously, a new and important field opened up for the application of mercury-arc rectifiers: they are now being used for many variable-speed and reversing drives, such as those encountered in winder and rolling-mill service, for which hitherto only Ward Leonard sets and a.c. drives had been considered.

A number of papers have been published describing the more novel applications[1–3] and particular types of rectifiers,[4–6] while methods of calculating rectifier performance[7] have been dealt with at length in some books.[8, 9] This paper is more general in scope, its object being to review the more important considerations which arise in the selection and application of such rectifiers and convertors in the hope that it will interest engineers generally and will lead to discussions beneficial to both manufacturers and users.

The term 'convertor' is often used when both rectification and inversion are involved in a single equipment, such as in a rectifier/inverter for a winder drive, but to save repetition the more familiar term 'rectifier' is used throughout the text when both rectifiers and convertors are referred to; similarly, the more familiar terms 'germanium' and 'silicon' are used in preference to 'monocrystalline'.

Mr. McBreen is in the Heavy Plant Division of Associated Electrical Industries, Ltd.

(2) CHOICE OF EQUIPMENT

The rectifier user in this country has a choice between

(*a*) Permanently sealed, glass or steel, multi-anode mercury-arc rectifiers.
(*b*) Permanently sealed, glass or steel, single-anode mercury-arc rectifiers.
(*c*) Germanium or silicon (monocrystalline) semiconductor rectifiers.
(*d*) Selenium-iron (polycrystalline) semiconductor rectifiers.
(*e*) A few special types of limited application.

Where their suitability overlaps, all are about equal in that they are able to give good reliable service, and for many applications they may differ little in first cost. The mercury-arc rectifiers are now mostly air cooled, and the tendency is towards air cooling for semiconductor rectifiers, although water cooling is used for some single-anode and semiconductor units.

Table 1 shows the present most suitable types for the more common applications. Often the considerations affecting choice are complicated, but the main guiding principles are as follow:

Capital costs.—Building, ventilation equipment and installation costs should be taken into account when making comparisons. They may be lower for germanium and silicon rectifiers.

Operating costs.—Efficiency may have an important bearing on these (see Table 2).

Duty and rating.—For very fast or sensitive voltage control and inversion duty, grid-controlled mercury-arc rectifiers are necessary for most ratings, although for small ratings without inversion, flux resetters[10] using germanium or silicon rectifiers are a possibility. For grid control of high powers and for high voltages, steel-tank rectifiers are preferable to glass-bulb units, since the latter are more prone to the deleterious effects of sputtered deposits on internal insulating surfaces.

When no voltage control is necessary or if the voltage control need not have a fast response, semiconductor rectifiers are an obvious choice.

Occasionally, for service on traction vehicles and for small drives, single-anode mercury-arc rectifiers may show an advantage over multi-anode types.

Reliability and servicing.—Semiconductor rectifiers require less auxiliaries and are inherently simpler. They are easier to service at long distances from the factory and are unaffected by low temperatures. The forward resistance of selenium-iron rectifiers increases with age, and this effectively limits their useful life to about ten years. Germanium and silicon rectifiers are not affected in this manner.

Space requirements.—For equivalent rating and operating conditions germanium and silicon rectifiers are more compact than mercury-arc rectifiers, and their associated transformers are usually smaller.

Table 2 shows the approximate efficiencies of the various types of rectifier (excluding the transformer) for the more usual voltages met with in practice.

(2.1) Voltage Control

Many types of grid control[1, 3] circuit have been developed for the voltage control of mercury-arc rectifiers; basically they all comprise a fixed negative bias on the control grids and a controlled firing pulse which overcomes the bias and permits

Table 1

APPLICATION RANGE OF SEMICONDUCTOR AND MERCURY-ARC RECTIFIERS

Application	Rectifier Type
Industrial uses of various kinds where voltage control (if any) can be of moderate speed, e.g. factory and dock supplies, cranes, lifts, motor starting and speed control, exciters for large alternators, etc.	*Medium and large sets.*—Germanium, silicon or multi-anode mercury-arc, depending on circumstances and voltage control. Increasing tendency to use germanium or silicon *Small sets.*—Selenium
Industrial uses requiring fast or sensitive voltage control or inversion, e.g. reversing-mill drives, winders, many auxiliary drives, paper machines, printing presses, field-forcing of large d.c. or a.c. machines, etc.	*Medium and large sets.*—Multi-anode steel-tank mercury-arc *Small and some special medium-size sets.*—Single-anode (hot-cathode, ignitron or excitron) mercury-arc
D.C. railway substations	Multi-anode mercury-arc, germanium or silicon
50 c/s locomotives and motor coaches, non-regenerative	Silicon, germanium, or single- or multi-anode mercury arc. Increasing tendency to use silicon or germanium
As above but with regenerative braking	Single- or multi-anode mercury-arc
High-power electrolysis, e.g. for aluminium, copper, zinc, chlorine, hydrogen, etc.	**Above about* 250 *volts.*—Germanium or silicon. *Up to about* 250 *volts.*—Germanium
Battery charging, electrolytic cleaning or tinning, electroplating	Selenium, germanium or silicon, with increasing tendency to use germanium or silicon
Mining service underground, d.c. electric furnaces, e.g. for titanium, and arc-welding	Germanium, silicon or selenium
Electrostatic precipitators, high-voltage d.c. testing equipment	*Usually selenium, but may be silicon in future
Anode supplies for radio transmitters	Usually special single-anode mercury-arc, but may be silicon in future
Nuclear research, usually involving high-power switching or inversion	Special mercury-arc, usually single-anode

* Although there are many existing mechanical rectifiers in these applications, it is unlikely that others will be provided in the future because of the outstanding advantages of semiconductor rectifiers for such duties.

the anodes to fire in sequence, the voltage control being achieved by varying the instant at which the main anodes are allowed to fire. Present-day applications usually require a very-fast-response static phase-shifting circuit. Sometimes on- or off-circuit tap-changers or regulating transformers are used in conjunction with the grid control, especially if prolonged operation at reduced voltage is necessary.

The method of voltage control used with semiconductor rectifiers varies according to the application and rating. For ratings up to a few hundred kilowatts it is possible to provide full-range voltage control by using a motor- or hand-operated regulating transformer of the moving-roller or moving-coil type. For larger ratings, on- or off-circuit tap-changers may be used,

Table 2

COMPARATIVE EFFICIENCES FOR THE VARIOUS TYPES OF RECTIFIER

Direct voltage	Efficiency*			
	Mercury-arc†	Selenium-iron‡	Germanium	Silicon
volts	%	%	%	%
25	—	92	97	94
50	—	92	98	97
100	82–83	92	98½	98
200	90–91	92	98½	98
300	93–94	92	98½	98
400	94½–95	92	98½	98½
500	95½–96	—	98½	98½
750	97–97½	—	98½	98½
1 000	97½–98	—	98½	98½
1 500	98½	—	98	98
3 000	98½	—	98	98

* Overall efficiencies, including transformer losses, are 1½–4% less than this depending on the rating of the equipment (transformer efficiencies increase with rating) and on the type of transformer connection used. Transformer efficiencies tend to be about ½% higher for semiconductor equipments with direct voltages above about 100 volts.

† Mercury-arc-rectifier efficiencies vary slightly with the different types and designs available.

‡ The efficiencies given for selenium-iron rectifiers are when new, and they fall with age. B.S. 2709: 1956 specifies that the fall in efficiency should not exceed 10% during the first 2 000 hours of service.

with or without transductor control to give smooth voltage control between the transformer taps. Flux resetters using germanium or silicon rectifiers and having the same speed of response and characteristics as grid-controlled mercury-arc rectifiers are possible up to about 100 kW, but they tend to be expensive and will probably be superseded by silicon controlled rectifiers when the latter become commercially available in suitable ratings.

(2.2) Rating

A certain amount of short-time overload capacity is often desirable or necessary for many applications, and providing this involves many complex considerations. The rectifier equipment consists of components having widely different heating time-constants; moreover the commutating ability of the rectifier rather than its thermal capacity is often a limiting feature. Care must thus be taken when specifying the magnitude and duration of overloads if unnecessary expenditure on equipment is to be avoided. The five rating classes of B.S. 1698: 1950 are a useful guide to accepted practice for such overloads, and it is obviously in the user's interests to specify the standard rating class most suited to his application.

The temperature limits specified in the relevant British Standard should be used whenever possible. Any increase in ambient or cooling air temperature increases the cost of the rectifier equipment, and any reduction of the low-temperature limit for mercury-arc rectifiers involves the provision of heating equipment.

(3) PULSE NUMBER*

(3.1) D.C. Considerations

For variable-speed machine drives supplied by grid-controlled mercury-arc rectifiers, or germanium or silicon rectifiers having similar characteristics in the d.c. output, the equipment and

* The pulse number is the number of main crests produced in the direct voltage during one period of the fundamental frequency of the supply voltage. It is the quantity mainly concerned in waveform considerations and is often loosely defined as the number of phases, but as the latter term may be confused with the number of transformer secondary phases, which is not always the same, the less ambiguous term pulse number is used in the following:

application of the whole drive must be taken into account. In general, 6-pulse or more is satisfactory so far as heating and commutation of the motor are concerned, but, unless the whole drive is taken into account, trouble may be encountered with the control, even with 6-pulse operation. As the voltage of a rectifier is reduced by grid control or its equivalent, the principal ripple in the direct current gradually increases until the current finally becomes discontinuous and, at this point, a change occurs in the regulation of the rectifier. Inductance in the d.c. circuit, which normally has no effect on the voltage, then acts as though it were in the anode circuit and the voltage/load regulation becomes very steep. Hunting may occur if the control system lacks the speed to cope with this steeper regulation, especially at the point where discontinuity begins. The higher the inductance in the d.c. circuit and the higher the pulse number, the lower will be the current at which this occurs.[1, 7]

The only other application of general interest needing careful consideration of the d.c. waveform is traction. For single-phase rectification with the rectifiers mounted on the motor-coaches or locomotives, the waveform is important for motor commutation, and an appropriate amount of inductance must be added to the d.c. circuit. To a limited extent, the more inductance added the greater will be the harmonics in the a.c. supply and—owing partly to this, but more to considerations of weight and cost—the value added must be a compromise. The present tendency is to use a traction motor which can accept in the direct current a ripple of ±30% of the mean value at full load.

For rectifiers in static substations supplying traction systems the waveshape is important because insufficient smoothing may lead to interference with telephone and signalling circuits. When smoothing is necessary there is an economic choice between 6-pulse operation with smoothing equipment or 12-pulse operation with little or none.

(3.1.1) D.C. Smoothing Equipment.

In its usual form for use with 6-pulse traction substation rectifiers, d.c. smoothing equipment consists of a series reactor with four resonant shunts, the shunts being arranged to form a by-pass for the sixth, twelfth, eighteenth and twenty-fourth harmonics, respectively, as shown in Fig. 1. For 12-pulse

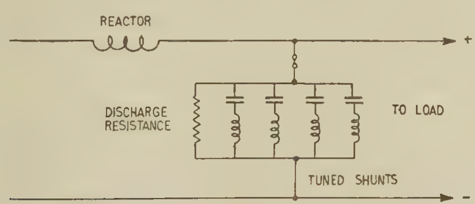


Fig. 1.—6-pulse smoothing equipment.

rectifiers with shunt characteristics the series reactor may with suitable design be omitted and only the resonant shunts for the twelfth and twenty-fourth harmonics included.

(3.2) Supply Considerations

In most cases rectifiers may be fed from the a.c. supply system with no more difficulty than other equipment of comparable rating, but for the larger installations it is necessary to consider the effects of waveform, since that of the current drawn from the a.c. supply by a rectifier has a harmonic content which becomes less as the pulse number is increased. This harmonic content of the supply current could itself, if excessive, cause a limited amount of interference with other users' equipment, but the harmonic voltages it sets up in the a.c. supply system are usually of more importance. The various harmonic currents passing through the supply system induce corresponding harmonic voltages across the impedances through which they pass, and these voltages are, in their turn, propagated throughout parts of the supply network. Under unfavourable conditions these voltages could cause interference with other consumer's equipment, such as certain types of ripple control apparatus, telephones and power-factor-correction capacitors. Whether such interference occurs depends on the magnitude of the harmonics, the arrangement and impedance of the a.c. supply system and the construction of the apparatus affected. Attention to any one of these can reduce the interference to an unobjectionable level, but considerations of cost usually prevent any of the three from reaching the ideal. In considering methods to guard against such interference, compromise is therefore necessary.

Until fairly recently it was the practice for the various manufacturers, users and local electricity undertakings to judge for themselves the magnitude of the rectifier harmonics to permit for each installation; but with the rapid growth in rectifier usage during the past 10–15 years this became no longer satisfactory, and an attempt has been made to standardize practice in this country. The position so far as England and Wales is concerned is now guided by recommendations[11] compiled after consultation with representatives of interested manufacturers, and issued by the Electricity Council Chief Engineers' Conference to all the Local Area Boards. These recommendations set out in graphical form (Fig. 2) a suggested limiting relationship between rectifier

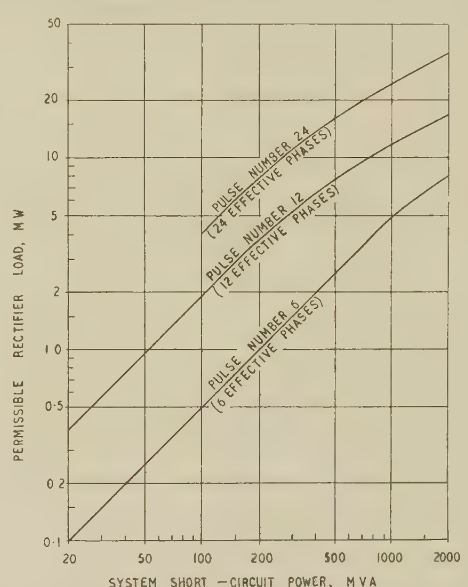


Fig. 2.—Curves relating rating and pulse number of rectifier installation to system short-circuit capacity.

load capacity, pulse number and system short-circuit capacity (supply impedance). It should be particularly noted that the recommendations are based on general conditions existing in this country and are not intended for application to railways and other large co-ordinated systems: moreover, the load capacity given in Fig. 2 is the maximum 30 min loading at the point of common coupling with other users, and the system short-circuit capacity referred to is the minimum operating value and not the maximum value, i.e. not the value normally stated for a.c. circuit-breaker rating.

These rules do not apply to rectifiers employing wide-range grid control other than for occasional starting. However, from the basis used in drawing up the rules and from the amount by which grid control increases the amplitude of the harmonics,

it readily follows that, where there is wide-range grid control, the permissible load should be 50–85% of the value given in Fig. 2 depending on the amount of grid firing delay and the number of phases used.

(4) POWER FACTOR AND A.C. SYSTEM LOADING

The manufacturer has practically no control over the power factor of a rectifier. For free-firing mercury-arc rectifiers and for semiconductor rectifiers without reactor control it is determined by the transformer reactance and magnetizing current and lies between 0·95 and 0·97 at full load. For grid-controlled mercury-arc rectifiers and semiconductor rectifiers using reactors and transductors for voltage control the power factor at maximum voltage is the same as for free-firing rectifiers and falls practically in direct proportion[12] as the voltage is reduced. Fig. 3 is typical of the manner in which the power factor of a

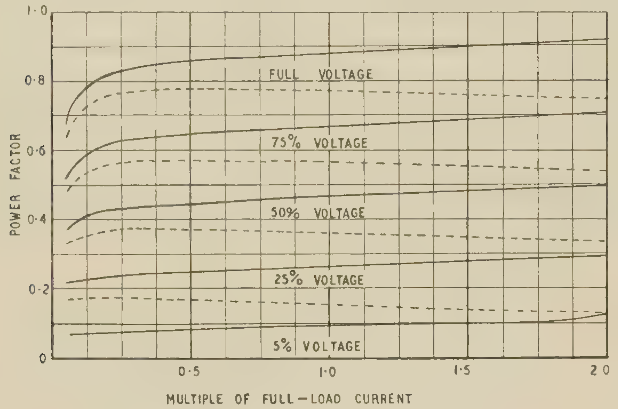


Fig. 3.—Typical variation of power factor with load when output voltage is varied by grid control.

Conditions: 12-pulse operation; $X = 8\%$; $I_m = 2\frac{1}{2}\%$.
Allowance of 15% for supply-voltage dip.
Voltage maintained constant at stated value.
—— Rectifying. ---- Inverting.

large winder drive supplied by a mercury-arc rectifier varies with voltage and load. This reduction of power factor with voltage has a very important effect. Reversing-mill and winder drives vary widely in loading and duty cycle, and it is common in such drives to have peak current demands of 2–3 times the normal rating at or near zero voltage and occurring at a frequency of the order of one or more per minute. When supplied from grid-controlled mercury-arc rectifiers, such drives must impose on the supply system large cyclic reactive-power demands, and these in turn produce fluctuations in the a.c. system voltage which are directly proportional to the demand and inversely proportional to the system short-circuit capacity.

The amount of such cyclic voltage fluctuation which can be tolerated must depend on the speed of the change and the frequency of the fluctuation. Little work has been done in this country on the subject of annoyance from such fluctuations, but a 1956 American paper[13] shows a curve relating their percentage and frequency to the borderline of irritation.

Voltage fluctuation could be the main consideration in determining the rating of the rectifier load producing such fluctuations which may be connected to a given supply system. With a limit to supply-voltage fluctuation of, say, 1%, the rectifier rating would always be within the limits for 12-pulse operation shown in Fig. 2, leaving harmonics the limiting factor for such loads only if the rectifier is 6-pulse connected.

For preliminary estimates the apparent power taken by any proposed rectifier equipment may be taken as 1·1 times the d.c. output for free-firing rectifiers and 1·3–1·45 times the d.c. output at maximum voltage for grid-controlled rectifiers for reversing-mill and winder service. When the voltage is lowered by grid control at constant current the apparent power drawn from the supply remains constant. Rectifiers do not add to the a.c. system short-circuit capacity.

(4.1) Power-Factor Improvement

A number of schemes, such as Bedford's rectifier sequence control[14, 15] and zero anode and asymmetrical grid control,[15] have been devised for the improvement of the power factor of

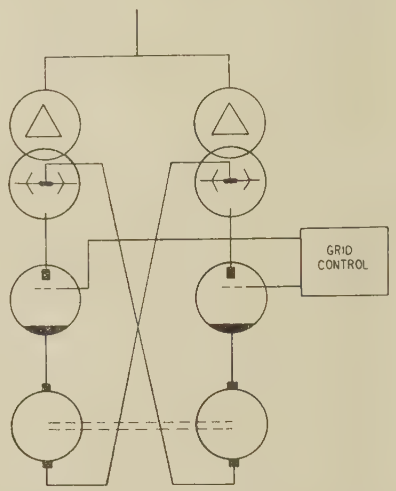


Fig. 4.—Bedford's connection with double armature drive.

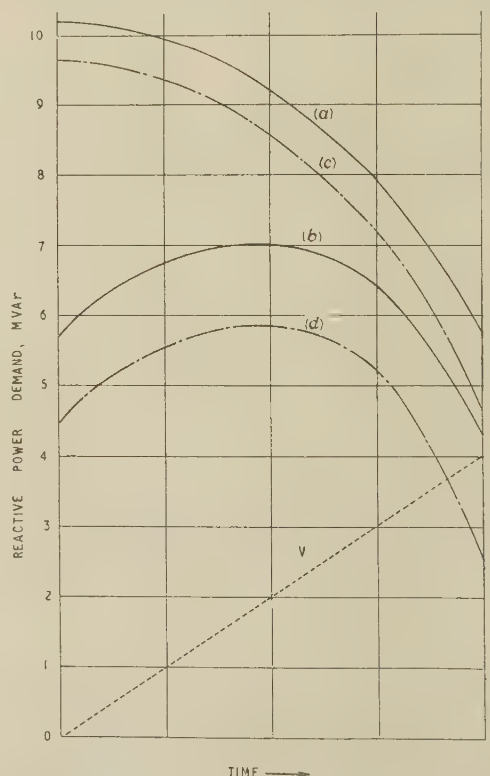


Fig. 5.—Variation in reactive-power demand during starting a 5000 h.p. winder drive controlled by a mercury-arc rectifier.

(a) Conventional grid control with allowance for 15% supply voltage dip.
(b) Bedford's connection with allowance for 15% supply voltage dip.
(c) Conventional grid control with allowance for 10% supply voltage dip.
(d) Bedford's connection with allowance for 10% supply voltage dip.

grid-controlled rectifiers when they are operating at reduced voltage, with the object of reducing reactive-power demand during motor starting. Bedford's scheme involves two half-voltage rectifiers in series (Fig. 4), controlled in sequence from full inversion to full rectification. It is the simplest and least expensive of these schemes, and Fig. 5 shows the order of reduction in reactive-power demand which can result from its use. Despite the apparent advantage of such schemes, they are all considerably more expensive and complicated than normal grid control and are economically justified only in borderline cases. Capacitors may be used for power-factor correction provided that suitable precautions are taken to avoid resonance, but they cannot prevent the supply-voltage fluctuations caused by swings in reactive-power demand.

(5) RECTIFIER AND TRANSFORMER CONNECTIONS

(5.1) General

The more commonly used rectifier and transformer connections together with their principal characteristics are shown in Fig. 6, and their applications are described in Sections 5.2–5.4.

(5.2) Semiconductor Rectifiers

Semiconductor rectifiers permit the use of double-way connections, such as the single-phase and 3-phase bridge connections, and thus enable transformers of standard design to be used. For 12-pulse operation the parallel star-delta 3-phase bridge has the advantage of resulting in a lower reactive regulation drop than most other connections. For very-low-voltage heavy-current applications the 6-phase double-star and diametric connections sometimes prove to be more economical overall.

(5.3) Single-Anode Mercury-Arc Rectifiers

Like the semiconductor rectifiers, single-anode mercury-arc rectifiers lend themselves to the bridge connections, albeit with some loss of efficiency; they may also be used in the same circuits as multi-anode rectifiers. Both of the single-phase circuits shown in Fig. 6 have been used for single-phase traction with the rectifier equipment on the coach or locomotive. In the overall assessment either connection may prove to be more economical in cost and weight, depending on the rating required and the rectifier ratings available.

(5.4) Multi-Anode Mercury-Arc Rectifiers

It is impossible to use the double-way connections with multi-anode rectifiers. For 3-pulse operation the 3-phase half-wave connection is used; for 6-pulse operation the double-star connection is the most frequently used, although the fork connection is often used for the lower ratings and for h.v. applications. Above about 300 kW the double-star connection permits a smaller and less expensive transformer than the fork connection and an r.m.s. current in the rectifier anodes which is lower in the ratio 1 : 1·41, thus permitting a higher rectifier d.c. rating. Against this is a 15% additional rise in voltage at loads below about 1%, but this can be suppressed if necessary (see Section 10).

For 12-pulse operation it is possible to use two 6-phase transformers with their primaries connected star and delta, respectively. However, this course is usually adopted only for the very high ratings or for some other special consideration; the normal choice lies between the quadruple star and the quadruple zig-zag connection, and, in general, the latter connection with a single primary tends to be more favoured because it results in a lower direct voltage regulation than the former for the same reactance. Both 12-pulse connections have a no-load voltage rise of 20% below about 5% load.

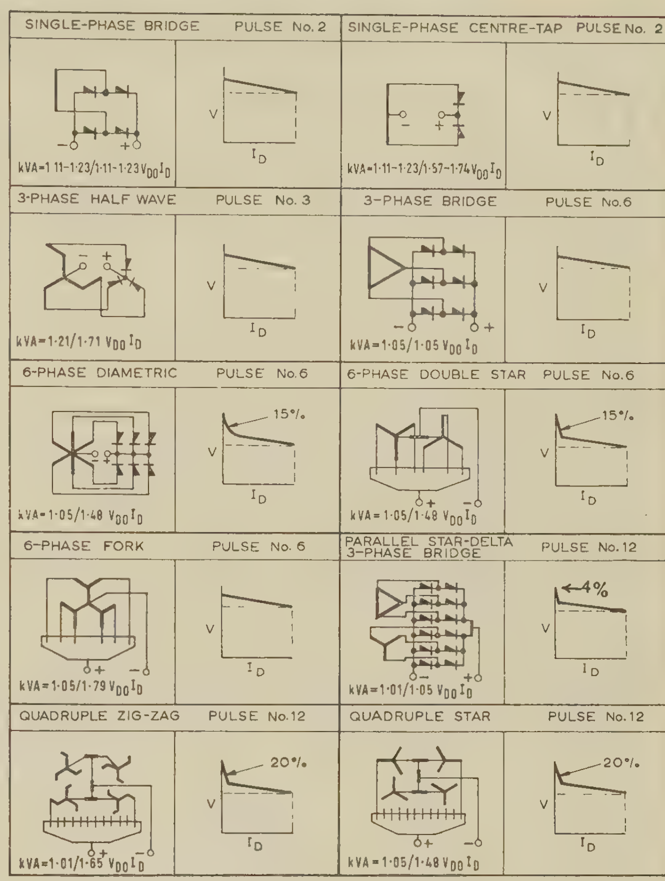


Fig. 6.—More commonly used connections with their regulation characteristics and transformer ratings.

V_{D0} = Theoretical no-load direct voltage at $\alpha = 0°$, i.e. value which would be obtained in the absence of rectifier drop and with inter-phase transformers (if any) fully magnetized.
I_D = Direct-current load.
Regulation percentages from light load to full load vary with reactance and transformer copper loss.

(5.5) Parallel Operation of Rectifiers and Rectifier Cells

When parallel operation of a number of multi-anode mercury-arc rectifiers is required it is necessary to provide a separate winding on the transformer for each rectifier, or a reactor in series with each anode, to ensure equal load sharing.

With semiconductor rectifiers the more general practice in this country is to derate or match the cells for parallel operation and to resort to split windings and reactors only for very large groupings. Some oversea manufacturers, however, use the equivalent of anode reactors for the individual cells with no derating or cell matching. Silicon controlled rectifiers, when they become

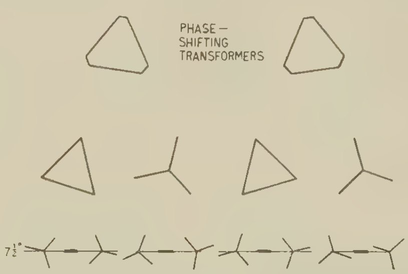


Fig. 7.—Increase of pulse number from six to 24 using phase-shifting auto-transformers.

commercially available for power circuits, may require balancing reactors for parallel operation and *RC* voltage dividers for series operation.

(5.6) Phase Multiplication

More than 12-pulse operation may be achieved in several ways. One method is to use 6-pulse units supplied through phase-shifting auto-transformers, as shown in Fig. 7, and another is to provide overshoot and zig-zag windings on the transformer primaries, as shown in Fig. 8.

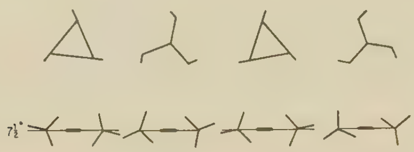


Fig. 8.—Increase of pulse number from six to 24 using extended primary windings on rectifier transformers.

(6) PARALLEL OPERATION OF RECTIFIER AND OTHER D.C. SUPPLY EQUIPMENTS

Rectifier equipments can readily be made to operate in parallel with other rectifier equipments and/or with shunt-wound rotating machines by matching their regulation characteristics. Operation in parallel with compound-wound generators and rotary convertors is also possible if an equalizing resistance is used, as shown in Fig. 9.

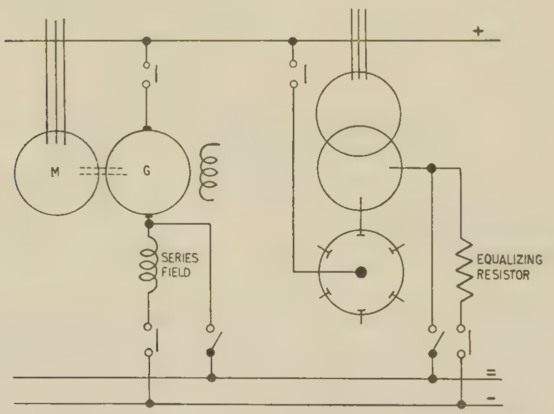


Fig. 9.—Parallel operation of rectifier with compound-wound generator.

Voltage drop across series field is matched with that across equalizer resistance and sharing is thus controlled by series field current.

(7) THREE-WIRE OPERATION

It is possible to provide mercury-arc or semiconductor 3-wire rectifier equipments for any degree of out-of-balance working, but, in the past, normal practice with the larger mercury-arc equipments has been to allow for only 15% of the full-load current to flow in the mid wire, in order to obtain the maximum efficiency. Since any future 3-wire rectifiers will probably be of the semiconductor type, this restriction on out-of-balance loading need not now apply.

(8) POSSIBLE FAULTS AND THEIR PREVENTION

(8.1) General Considerations

With any make or type of rectifier there is a current limit, a voltage limit and a temperature limit above which backfires will occur with great frequency, provided that some other fault does not intervene first. The nearer any of these limits is approached the greater is the probability of backfire, and British practice to operate well within them during normal service. Backfire may also be caused in mercury-arc rectifiers by incorrect temperature distribution; this is mostly a design matter, but occasionally it is affected by the application and the method of operation.

When a rectifier is supplying a back-e.m.f. load, a backfire results in a short-circuit on both the a.c. and d.c. systems (see Fig. 10). It is the severest fault a rectifier must withstand

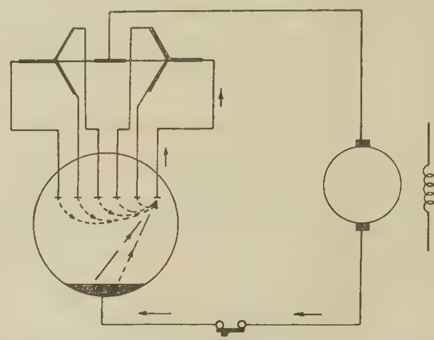


Fig. 10.—Backfire in a mercury-arc rectifier driving a motor.

although not the severest fault possible on either system— short-circuit produces larger fault currents. In a mercury-a rectifier a backfire may result in a fault current of up to 60 tim the normal current in the backfiring anode and its associate transformer winding. In a semiconductor equipment the fau current in the transformer windings is usually much less, but th backfiring rectifier cell or cells are destroyed.

So long as the protection is correct and backfires are not to frequent, they do not normally damage mercury-arc rectifier and such rectifiers may be returned into service immediate following a backfire without detriment, provided that the cau of the original fault, if any, is removed first.

(8.2) Semiconductor Rectifiers

Selenium-iron rectifiers have a fairly high forward resistan and do not require extra reactance in the circuit for fault limit tion. Both germanium and silicon rectifiers have a negligib forward resistance and a very low thermal capacity in the rectifi cells; depending on the speed of the protective device (unless th risk of destruction on short-circuit is accepted), they eith require a large amount of reactance in their supply, to limit th prospective short circuit current, or they must be derated. A semiconductor rectifiers are sensitive to surge voltages, and th is particularly so in the case of germanium and silicon rectifier

(8.3) Mercury-Arc Rectifiers

(8.3.1) Minimum Temperature Limits.

The current-carrying ability of a mercury-arc rectifier depend amongst other things, on the cross-sectional area of each a path and on the density of the mercury vapour in the rectifie the larger the cross-section and/or the higher the vapour densi the larger is the current which can be maintained. The area fixed for any particular design of rectifier, but the vapour densi varies rapidly with the temperature of the condensing surfac being halved for about each 8° C reduction in its temperature.

When the limiting current for any particular temperature reached an unstable condition results.[17] At this limit all t available atoms of the mercury are ionized, and under t influence of the voltage differences which exist in the rectifi

vessel they move towards the walls of the rectifier. This leaves the arc space momentarily almost free from unionized mercury vapour—the condition usually known as arc starvation—and thus the supply of fresh mercury ions and the flow of current are interrupted. This process takes only a few microseconds and is repeated until the rectifier attains sufficient temperature to stabilize the arc. When the current in the rectifier is interrupted in this manner, very high voltages are induced in all the inductances in the circuit, such as the transformer secondary winding, and these induced voltages can cause insulation failure if they are not restricted.

There are also other temperature limitations. In general, the minimum temperature of the rectifier vessel for ignition is about 5° C, that for the application of full load to free-firing rectifiers is about 10° C, and that for the safe operation of grid-controlled rectifiers is about 20° C. The latter are often expected to carry up to two or three times full-load current at about zero voltage on starting, and their limiting temperature for the elimination of arc-starvation surges must therefore be higher than for rectifiers more easily rated; moreover, maloperation of the control grids resulting in break-through (loss of voltage control) is liable to occur at vessel temperatures below about 15° C, and 20° C allows a reasonable margin of safety above this.

(8.3.2) General.

As mentioned in Section 8.1, mercury-arc rectifiers are sensitive to temperature distribution, and backfires are liable to occur if the temperature of the main condensing zone exceeds that in the vicinity of the anode seals and the anodes. This becomes increasingly important with increasing voltage and operating duty, and for some applications it is essential to provide anode heaters.

Special provision for low-temperature operation and starting may be provided. For most rectifiers in traction substation service and similar applications, fan control as a function of temperature or load is usually all that is necessary. For rectifiers subject to widely fluctuating and intermittent loads with no temperature control of the building or cooling air, both cathode and anode heaters are necessary in addition to fan control.

In equipment involving inversion it is essential that the transformer secondary voltage has sufficient margin to permit safe inverted operation at the lowest supply voltage which is liable to be encountered in normal service.[3] This must be carefully considered for each equipment and installation; too large an allowance will unnecessarily increase the size and cost of the transformer, lower the power factor and increase the reactive power demand (see Fig. 5); an allowance for 10–15% voltage dip should be adequate for most installations.

All mercury-arc rectifiers depend on a high degree of vacuum tightness for their operation. Partial failure of this vacuum by mechanical damage due to accidental causes or sustained overload or overheating could cause backfiring and ultimately the complete failure of the rectifier.

(9) PROTECTION

(9.1) General Considerations

The amount of reactance in the circuit and the speed of operation of the a.c. and d.c. protection must be such as to limit any possible peak fault currents and their duration to within the limits which the rectifier and its associated transformer can withstand without damage. Thus, faults on mercury-arc rectifiers must be so limited and cleared fast enough to prevent mechanical damage to, and excessive gassing in, the rectifier and overheating and mechanical damage to the transformer. External faults on semiconductor rectifiers must be limited and cleared fast enough to prevent destruction of any of the rectifying cells, and internal faults due to failure of a cell or string of cells must be cleared fast enough to prevent damage to the remaining sound cells.

The choice and arrangement of the protection may also be affected by the need for discrimination between a.c. and d.c. faults and between rectifier and feeder faults, by the fault capacity of the d.c. system and by the application. Each application and installation must be treated on its merits, but the following may be taken as a general guide.

(9.2) A.C. Protection

Ideally, the a.c. protection should guard against overloads as well as faults in the equipment, but it is usually too expensive, and unjustifiable, to provide complete inverse-time/over-current protection to match the operating duty and the characteristics of the equipment. Any protection against excessive overloads must therefore be a compromise and is frequently omitted. When included, overload protection usually takes the form of inverse-time/over-current relays, one or more relays being used as required, with settings as near as practicable to the operating duty. Fuses are also used for this duty in semiconductor rectifiers. In very large and important installations, winding- and/or oil-temperature indicators are sometimes used in the transformer.

Fault protection is always essential. For a small equipment it may consist of a simple switch-fuse unit or a small circuit-breaker with direct-acting overload trips. For larger equipments a circuit-breaker is always essential, and the required characteristics and fault tripping of this are dependent on the type of d.c. protection used and whether or not cell or anode fuses are used in the rectifier.

With glass-bulb, germanium and silicon rectifiers it is essential to clear the current flowing in the backfiring anode or faulty cell in the shortest time practicable, and h.r.c. fuses in series with the individual anodes or cells or groups of cells are indispensable. The fuses used for this duty must be fast enough to safeguard the equipment in case of backfire or cell failure, they must operate with certainty under conditions approximating to d.c. conditions at a voltage approaching the crest working voltage across the rectifier bulb or cells, and, in the case of semiconductor cell fuses, the arcing voltage must be within the limited over-voltage the cells can withstand. If this last condition is not met, the blowing of a fuse in service could easily start a chain reaction sufficient to destroy all the cells in the equipment. Fuse-failure alarm is usually included with semiconductor rectifiers but rarely with glass-bulb rectifiers.

Anode fuses are also occasionally used with steel-tank rectifiers for small equipments or when their use is justified by some special consideration appertaining to a particular application. For example, they may be justified as the only economical means of keeping the prospective backfire fault current within safe limits. They may also be justified in an application where the sudden interruption of the supply is undesirable, e.g. with a rectifier controlling a small rolling mill.

Where anode or cell fuses are used the a.c. circuit-breaker serves as back-up protection to the fuses and protects the transformer against internal faults. High-speed operation is not essential and almost any commercial circuit-breaker of the required rupturing capacity to suit the system may be used, although in low-voltage semiconductor equipments where no d.c. circuit-breaker is provided fast tripping may be essential to achieve correct discrimination with the cell fuses in the event of d.c.-circuit faults.

For steel-tank mercury-arc rectifiers, where no anode fuses are provided, the circuit-breaker must open fast enough in the event of a backfire to prevent damage to the rectifier and trans-

former. Instantaneous overload tripping of this circuit-breaker, possibly via a small fixed time delay to afford discrimination with the d.c. protection and to prevent tripping by transformer inrush current when closing the circuit-breaker, is usually essential. Inverse-time/over-current relays are seldom fast enough for this duty.

When using multiple-winding transformers or groups of transformers connected to the same circuit-breaker it is necessary to ensure that a branch-circuit fault will operate the protective circuits. Additional current transformers and relays are sometimes necessary for the branch circuits.

Earth-fault protection should be as for any other apparatus of equivalent rating.

(9.2.1) Arc Suppression.

With mercury-arc rectifiers an additional method of protection against backfire and d.c.-circuit faults is possible, although it is more often used as protection against backfires only. If a negative bias is applied to the control grids, the main anodes are blocked and cannot fire. If this negative bias is applied to the grids of a rectifier which is already carrying load, the anode or anodes which are carrying current continue to do so until the end of their firing period and all other anodes are prevented from picking up the load. By 'arc suppression' is meant the process of applying a negative bias to the control grids of a rectifier when a fault occurs, thus clearing the fault with the minimum disturbance to the circuit.

There are various methods of achieving this, and one[18] used for backfire protection is illustrated in Fig. 11. A signal from

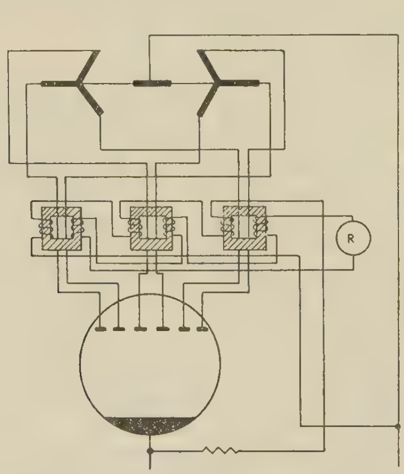


Fig. 11.—Arrangement of presaturated current transformers for application of arc suppression in the event of a backfire.

presaturated current transformers, acting on a special high-speed relay, results in the application of the negative bias in 1–2 millisec from the commencement of the fault, which is early enough to block the next anode and prevent it from adding to the backfire current.

Arc suppression is not a substitute for the more conventional forms of protection. It inherently cannot 'fail to safety' and therefore cannot be relied upon to be effective on every occasion the need arises. Nevertheless, its inclusion is justified in the larger industrial and traction installations, for it limits gas evolution in the rectifier and mechanical and electrical stresses in the equipment generally during faults. It also permits faster automatic reclosure. But even when arc suppression is included, the d.c. circuit-breaker must open to clear any feedback from the d.c. system during backfire.

(9.3) D.C. Protection

(9.3.1) Circuit-Breaker Classification.

Unlike their a.c. counterparts d.c. circuit-breakers are classified according to speed of operation rather than by rupturing capacity, although the latter must also be taken into account. Three speed grades are recognized, namely high speed, with a maximum fault clearance time of 0·025 sec, semi-high speed with a fault clearance time of 0·025–0·06 sec, and medium speed with a fault clearance time exceeding 0·06 sec.

(9.3.2) Application.

According to an informal but widely used code of practice medium-speed circuit-breakers should not be used when the total supply capacity connected to the d.c. system exceeds 4 kA or 500 volts, when frequent operation or automatic reclosure is required or when the circuit is highly inductive; moreover, their rating should not be less than one quarter of the connected supply capacity unless they are backed up by h.r.c. fuses. For such applications a semi-high-speed or high-speed circuit-breaker should be used.

High-speed circuit-breakers exert a definite current-limiting action and are essential in certain cases to protect the equipment against heavy fault currents or for discrimination. They are obtainable up to very high rupturing capacities and for service up to 3·5 kV.

Semi-high-speed circuit-breakers are also obtainable for high rupturing capacities but only for service up to 1·5 kV. Often they are fast enough to afford the correct measure of protection and to provide the necessary discrimination.

Within the above limits the type of d.c. circuit-breaker used depends on whether anode or cell fuses are used and on the application. When fuses are used the circuit-breaker gives back-up protection. It must also afford protection against feedback to faults inside the equipment and against faults in the d.c. circuit. Its speed of operation must be such that it will clear any d.c.-circuit fault before the cell or anode fuses blow.

For steel-tank mercury-arc rectifiers without anode fuses it is essential that the type and arrangement of the d.c. circuit-breakers be co-ordinated with the a.c. protection and the reactance of the equipment, to ensure that the magnitude and duration of any prospective backfire fault current is within the permissible fault rating of the rectifier(s) and transformer. For small equipments this condition may be satisfied with medium- or semi-high-speed circuit-breakers. For large equipments high-speed circuit-breakers are often essential, and sometimes it is necessary to sectionalize the rectifier output and to provide a circuit-breaker for each group of two or three rectifier tanks, in order to limit the currents which may be fed into the backfiring anode from the remaining healthy rectifier tanks in the equipment.

In many installations in which high-speed circuit-breakers are used, e.g. in traction substations, the rectifier circuit-breaker is connected to trip only on reverse current and the feeder circuit-breakers are arranged to trip only on forward faults; by this means only the affected unit or feeder is isolated and the maximum continuity of supply is ensured. In such installations automatic reclosure of the feeder circuit-breakers is often included. There are also many applications where bidirectional tripping of the rectifier d.c. circuit-breaker, or breakers, is desirable, such as in large motor drives, and it is possible to obtain high-speed circuit-breakers of substantially the same speed and rupturing capacity for either unidirectional or bidirectional tripping.

Direct-current circuit-breakers of all types employ some form of magnetic blow-out of the arc, but the resulting arcing voltage, which is not much different for the three types, is within the

withstand limits of mercury-arc rectifiers and d.c. motors. Semiconductor rectifiers can also usually withstand this arcing voltage without risk of cell breakdown, but this is not invariably the case and should always be verified.

When semiconductor rectifiers feed electrolytic cells whose ability to feed back current is limited and brief, the d.c. circuit-breakers are sometimes omitted—a matter of balancing the cost and complication of inclusion against the risk of omission. The risk involved, the chance of an accidental short-circuit in the rectifier cubicles and the failure of the cell fuses, is reasonably justifiable in equipments operating below about 100 volts, and under favourable conditions in equipments operating at higher voltages. Each case must be judged on its merits. When the d.c. circuit-breaker is omitted care has to be taken to achieve correct discrimination between the cell fuses and the a.c. circuit-breaker, as stated in Section 9.2.

In d.c. systems which are nominally insulated from earth but have 'leaky' insulation, such as most d.c. railway systems and electrolytic-cell installations, it is usual to provide circuit-breakers on the positive side only and an isolator on the negative side, since the resulting risk is small. The same practice is followed in many fully insulated industrial installations, with the addition of earth-leakage protection, although double-pole circuit-breakers are used in most of the smaller installations. When the mid-point of the system is solidly earthed, double-pole circuit-breakers or separate circuit-breakers in each line are necessary.

(9.4) Surge-Voltage Protection

Surge-voltage protection is normally not provided with selenium-iron rectifiers, but it is essential with mercury-arc, germanium and silicon rectifiers.

With mercury-arc rectifiers the best method of surge protection is prevention, i.e. by avoiding the conditions which give rise to high-voltage surges, but some form of supplementary protection is also necessary. This may be provided by non-linear resistors connected across all secondary windings, anode to neutral, and mounted inside the transformer tank under oil. Alternatively, non-linear resistors in series with a spark-gap may be used, connected in the same manner but mounted on the outside of the transformer tank or inside the rectifier cubicle(s). Capacitors instead of non-linear resistors are sometimes used with small glass-bulb equipments.

The momentary voltages which germanium and silicon rectifiers can withstand are usually limited to about 1·5–2·5 times the rated crest working voltage; such rectifiers must therefore be protected from surges produced by switching and other causes which might cause cell failure, and it is customary to provide them with a surge-suppression circuit or circuits. These normally comprise *RC* circuits in parallel across the transformer or cells.

When the a.c. and d.c. circuits are exposed to lightning, conventional lightning arresters may also be needed.

(9.5) Auxiliaries and Interlocks

It is general practice in large rectifier installations to provide electrical interlocks to ensure that all auxiliary circuits, i.e. fan and water-cooling circuits and—for mercury-arc rectifiers—excitation and temperature-control circuits, are healthy. Rectifiers with voltage control for individual drives also need some form of interlock to prevent full voltage being applied to a stationary machine. Grid-controlled mercury-arc rectifiers usually need an interlock or the controls so arranged to ensure that the grid circuits are energized before the d.c. circuit-breaker is closed. When thermostats are used with steel-tank mercury-arc rectifiers for fan control or low-temperature lock-out, it is necessary to take the signals away from them through insulating transformers, as shown in Fig. 12.

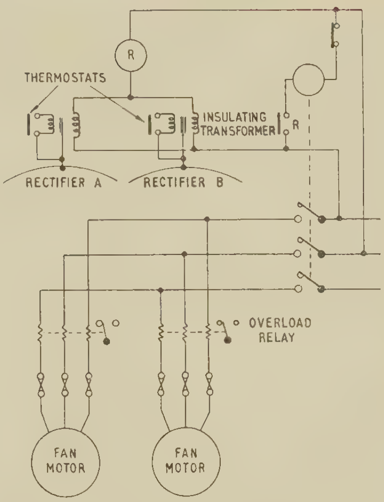


Fig. 12.—Typical fan control circuit for a mercury-arc rectifier showing use of insulating transformers.

The same rules[19] for fittings on rectifier transformers should be applied as for fittings on normal power transformers. The use of conservators, breathers and Buchholz relays should also be considered, even when not covered by these rules, when the operating duty is severe, such as when the transformer is supplying a grid-controlled mercury-arc rectifier in a reversing mill or winder drive. A Buchholz relay provides a sure indication of incipient insulation failure, and the conservator and breather combine to slow down oil deterioration and sludging.

(10) SUPPRESSION OF NO-LOAD VOLTAGE RISE

In the majority of cases the voltage rise, referred to in Section 5, which occurs with some connections at light load has no detrimental effects. Suppression is necessary only occasionally and may be accomplished in a number of different ways.

One method is to arrange for a small resistive load to be switched across the d.c. output when the load falls to a low value. Another method, more expensive in the first instance and slightly less effective in suppressing the voltage rise, is to use a static frequency-tripler to inject magnetizing current continuously into the interphase transformer.[16, 20]

(11) REGENERATIVE LOADING RESISTANCE

Unless designed as an inverter, which is possible only with mercury-arc rectifiers and is economically practicable only for certain applications, a rectifier cannot accept reverse power. This must be borne in mind when applying rectifiers to some forms of industrial load and to traction. Industrial loads often include cranes and conveyors which regenerate during part of their duty cycle, and facilities for regeneration are sometimes needed in traction systems. If the d.c. supply for such loads is taken from a rectifier and at the instant of regeneration there is no other connected load to absorb the regeneration, the direct voltage will rise and may cause motor flashover or insulation breakdown, particularly if the regeneration is caused by a rapid change in the field current of a motor.

This difficulty may be overcome by using an arrangement, such as that shown in Fig. 13, to switch a resistive load across the d.c. busbars. In this circuit any rise in the busbar voltage above a preset maximum causes the grid of the thyratron to become positive, thus permitting it to fire the main ignitron valve. After the ignitron has fired the relays take control. Such

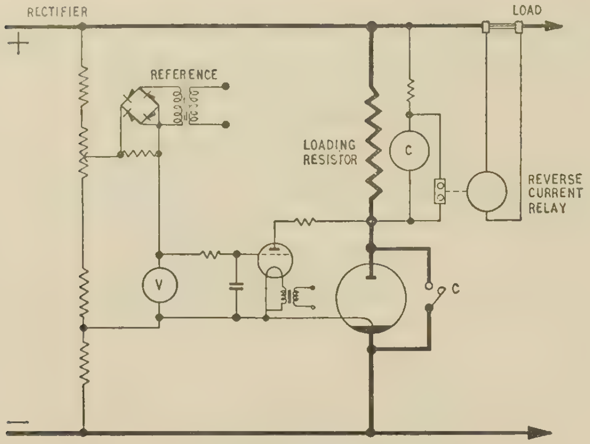


Fig. 13.—Basic circuit for ignitron loading equipment.

an arrangement may be made effective in limiting the rise in busbar voltage to a small momentary kick of about 20%.

An alternative using voltage-sensitive relays to detect any rise in busbar voltage is also possible, but because of the delays necessary for relays and contactors to operate, this arrangement is unsuitable when the voltage may rise very rapidly.

(12) INSTALLATION AND VENTILATION

Unlike machines, rectifiers require no special foundations, although air-cooled rectifiers may need attention to ventilation to ensure that the maximum cooling-air or ambient temperatures specified for them are not exceeded.

A small rectifier in a large room presents no difficulties if the air temperature of the room is below the specified maximum, provided that it is not too low for a mercury-arc rectifier. Whether it be blast or naturally cooled, the rectifier will draw in sufficient cooling air to suit its needs. Larger rectifiers, and in some cases small rectifiers in rooms with excessive air temperatures, may need their air intake to be positioned near ventilators to obtain the maximum benefit from cool air entering the room and, occasionally, special ducts may be needed to guide the air to the rectifiers. When use is made of incoming air in this manner, care must be taken to ensure that the temperature of mercury-arc rectifiers does not fall below the minima referred to in Section 8.3. Often it is necessary to use manually or thermostatically controlled louvres or controllable fans in the air intake to achieve this.

If a rectifier has to rely on air circulated in the substation for its cooling and the substation is relatively small, additional substation ventilation may be needed. When such a scheme is used the maximum substation air temperature must be taken into account when rating the rectifier. A preliminary estimate of rectifier losses may be calculated from the efficiencies in Table 2.

For special applications, such as for installation in a corrosive or very dirty atmosphere, it is advisable to provide a completely sealed rectifier with its own air circulator and an air-to-water heat exchanger. With germanium and silicon rectifiers such an arrangement is easier to provide. For very large installations under such conditions a still better arrangement—although more expensive in building costs—is to provide the complete room or substation with a closed air-cooling system, thereby enabling the control gear to operate in clean air without increasing the difficulty of access.

Rectifier transformers should be treated like any other transformer. Small units may be installed indoors near the rectifier. Larger units, i.e. those containing more than about 200 gal of oil, should preferably be installed over an oil soaking pit in a separate outoor cell.[21]

With the exception of the anode connections for mercury-arc rectifiers and connections exposed to lightning, standard cabling practice may be used throughout. The anode connections for mercury-arc rectifiers should be capable of withstanding the same test voltage as the transformer secondary,[22] namely thrice the direct voltage plus 5 kV, and they should have a reasonably high impulse-voltage strength. They should be armoured only when in 3-phase groups, and in installations with large current ratings and lead-covered cables the lead sheath should be earthed at one end only.

(13) ACKNOWLEDGMENTS

The author wishes to thank the management of the A.E.I. Heavy Plant Division for permission to publish the paper and the Chairman of the Electricity Council Chief Engineers' Conference for permission to reproduce Fig. 2.

(14) REFERENCES

(1) Bingley, P.: 'The Characteristics and Control of Rectifier-Motor Variable-Speed Drives', *Proceedings I.E.E.*, Paper No. 1157 U, June, 1951 (**99**, Part II, p. 189).
(2) Calverly, H. B., Jarvis, E. A. K., and Williams, E.: 'Electrical Equipment for Rectifier Locomotives', *ibid.*, Paper No. 2339 U, March, 1957 (**104** A, p. 341).
(3) Abram, L., McBreen, J. P., and Sherlock, J.: 'D.C. Winder Drives using Mercury-Arc Rectifier/Inverters', *ibid.*, Paper No. 2324 U, February, 1957 (**105** A, p. 77).
(4) Read, J. C.: 'Mercury-Arc Rectifiers for Medium-Voltage Applications', *ibid.*, Paper No. 1368, November, 1952 (**99**, Part I, p. 252).
(5) Read, J. C., and Gimson, C. F.: 'High-Power Mechanical Contact Rectifier', *ibid.*, Paper No. 1939 U, October, 1955 (**102** A, p. 645).
(6) Kinman, T. H., Carrick, G. A., Hibberd, R. G., and Blundell, A. J.: 'Germanium and Silicon Power Rectifiers', *ibid.*, Paper No. 1936 U, October, 1955 (**103** A, p. 89).
(7) Read, J. C.: 'The Calculation of Rectifier and Inverter Characteristics', *Journal I.E.E.*, 1945, **92**, Part II, p. 495.
(8) Marti, O. K., and Winograd, H.: 'Mercury-Arc Power Rectifiers' (McGraw-Hill, 1930).
(9) Rissik, H.: 'Mercury-Arc Current Converters' (Pitmans, 1947).
(10) Sherlock, J.: 'The Flux Resetting Magnetic Amplifier', *Electrical Energy*, 1959, **3**, p. 2.
(11) 'Harmonic Distortion caused by Mercury-Arc Rectifiers', Central Electricity Authority Engineering Recommendations G5/1.
(12) B.S. 1698: 1950, Appendix C.
(13) Kroneberg, A. A.: 'Voltage Dips and Flicker', *Transactions of the American I.E.E.*, 1956, **75**, Part II, p. 349.
(14) British Patent No. 431757 (now void).
(15) Korb, F.: 'Sequence Control for Reducing the Reactive Power of Rectifiers', *The Engineers Digest*, June, 1958, p. 233.
(16) Blake, L. R.: 'The Double 3-Phase Rectifier with Inter-Phase Reactor Excited from a Frequency Tripler', *Proceedings I.E.E.*, Paper No. 1509 S, June, 1953 (**100**, Part II, p. 310).
(17) Hull, A. W., and Elder, F. R.: 'The Cause of High-Voltage Surges in Rectifier Circuits', *Journal of Applied Physics*, 1942, **13**, p. 372.
(18) American Patent No. 2169866.
(19) B.S. 171: 1959, Table 19.
(20) Blake, L. R.: 'The Frequency Tripler', *Proceedings I.E.E.*, Paper No. 1508 S, June, 1953 (**100**, Part II, p. 296).
(21) British Standard Code of Practice. CP231.102 (1950).
(22) B.S. 1698: 1950.

DISCUSSION BEFORE THE UTILIZATION SECTION, 10TH MARCH, 1960

Dr. W. G. Thompson: In this country we made a notable step forward in developing a pumpless rectifier with air cooling, but with semiconductor rectifiers the possibility of water cooling with economic advantages has again come into the picture in certain cases. A recent example of water cooling applied to mercury-arc rectifiers, in which the emphasis is concentrated on extracting the maximum advantage from the cooling by producing a rectifier vessel with very much reduced dimensions, shows that in many cases, especially at higher working voltages, this type can occupy a smaller space than the corresponding semiconductor rectifier

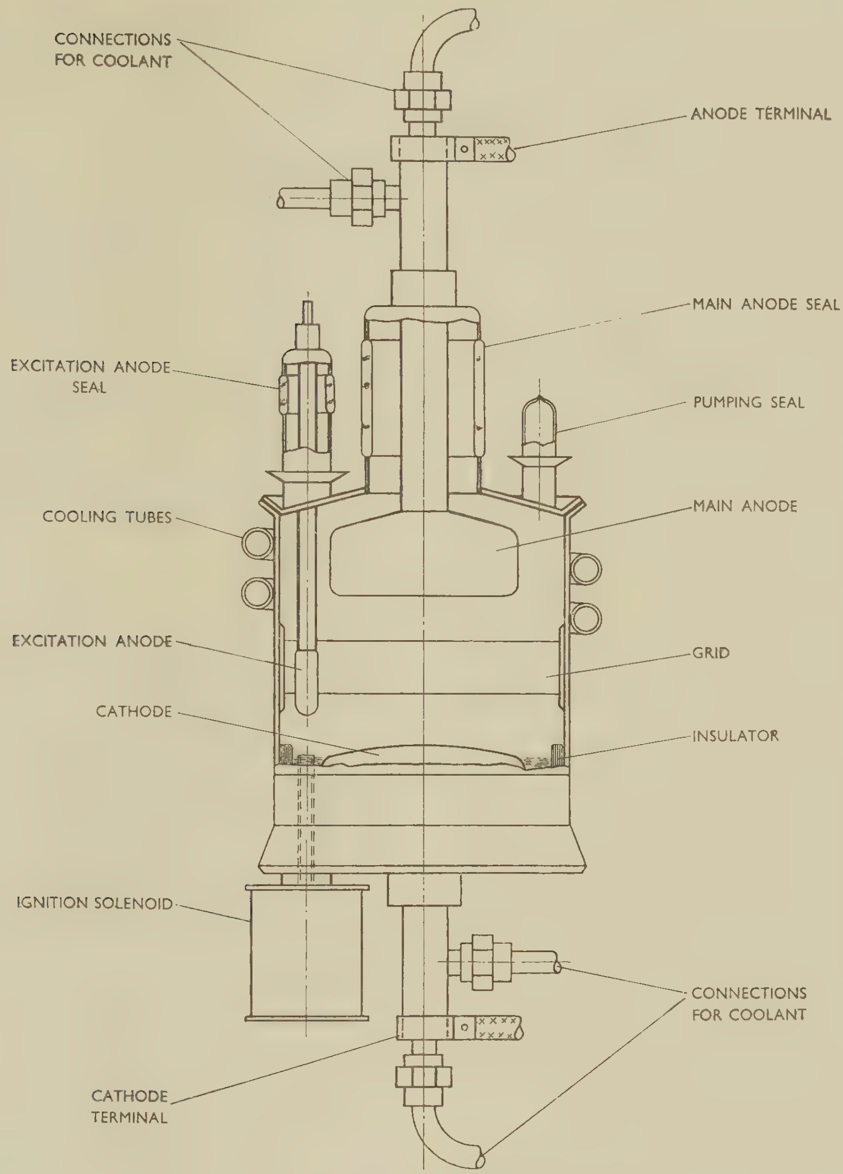


Fig. A.—General arrangement of the Compak rectifier.

equipment. Fig. A shows a practical form of the new rectifier, and some of these have now operated successfully in traction service for about six months. These rectifiers overcome a number of the points raised in the paper, particularly those arising from the length of arc path and temperature distribution in the old type of rectifier. With a Compak rectifier weighing only about 60 lb the mercury vapour has not far to travel and takes advantage of an anchored spot on a water-cooled cathode which produces, without excess, enough mercury vapour for the transport of the current. The water cooling establishes the the proper temperature conditions in the rectifier. All of this gives the rectifier both good starting and heavy overload capabilities, combined with high efficiency.

Table 2 of the paper gives the comparative efficiencies of the mercury-arc and semiconductor types. As a result of its much shorter arc path, the Compak rectifier is equal or better in efficiency to semiconductor rectifiers at voltages of 750 volts upwards. It is also capable of withstanding very high reverse voltages and has operated successfully at 1·5 kV d.c. The advantages of series operation or bridge connection with a working voltage of 3 kV will also be apparent. It is also effective as a grid-controlled rectifier, particularly in cases of impulse loading.

The author suggests germanium rectifiers for high-power electrolysis up to 250 volts, but with high ambient temperatures it may be preferable to take advantage of the silicon rectifier, and these have been used for electrolysis below 250 volts for that reason. Incidentally, mercury-arc rectifiers have been used for mining service underground for many years, and there are many multi-anode rectifiers providing supplies for radio transmission.

Reference is made in the paper to discontinuous burning of the arc with grid-controlled rectifiers phased back to give a low voltage output. This, however, presents no difficulty in operating variable-speed drives, because by fitting suitable high-speed response control circuits a completely smooth adjustment and operation is obtainable from crawling to full running speed.

Mr. T. B. Rolls: For many years rectifiers have been used for

2-wire d.c. supplies to cranes, rolling auxiliaries and similar drives. Usually such supplies are operated as 'insulated' systems, but there are several snags which have never been resolved satisfactorily. First, where applicable, No. 20 of the Electricity (Factories Act) Special Regulations must be observed, and either an earth screen in the transformer or a spark-gap must be used. Secondly, simultaneous earth faults can cause fortuitous operation of a contactor coil, thereby setting a machine or crane in uncontrollable motion, possibly with dire results. Prompt location and clearance of earth faults is therefore essential, but there is no easy way of achieving this.

Some electricity tariffs are so harsh on low power factors that anything under 0·98 is expensive. The lagging power factor on rectifiers then places them at a disadvantage with respect to synchronous rotating electrical machinery, since the latter gives scope for providing the extra corrective capacitance to compensate for the shortcomings of other, inductive loads.

Dr. J. C. Read: Much misconception exists regarding the relative merits of germanium and silicon power rectifiers, and I should therefore like to attempt a realistic comparison, relating to industrial (non-military) service.

Consequences of maximum working temperature (germanium, 75°; silicon, 150° C.)

Reliability: No difference.

Field of use: Both suitable up to 40–45° C air temperature; silicon preferable or necessary above 45°.

Bulk and weight: For industrial service the difference is negligible; on railway vehicles the difference decisively favours silicon in some, but not all, cases.

Air circulation: For small sets silicon permits larger ratings of naturally-cooled type.

Overvoltages.—Satisfactory protection is feasible for either, consequently they are equal.

Overloads.—Long-period overload capacities are inherently similar, because percentage safety margins in temperature rise are similar. Short-period overload capacities are inherently similar, because working current densities are similar.

Operating Experience.—The reliability of germanium is proved over several years and on the largest scale to be of very high order. Experience of silicon is brief but very promising; deterioration by thermal fatigue on fluctuating (traction) loads is believed to be now overcome, but time has not yet permitted full proof in service.

Efficiency and Cost.—The forward voltage drops per cell are approximately 0·5 volt for germanium and 1·2 volts for silicon; the maximum crest working voltages per cell are 300 and 600 volts respectively. Thus the theoretically obtainable efficiencies (i.e. where the direct voltage is high enough) are almost equal. When manufactured under similar conditions, costs also tend at present to be equal in this theoretical case. However, actual efficiencies and costs may be unequal in practice, owing to

(*a*) Direct voltage too low for full utilization of cells rated for 600-volt working.

(*b*) Direct voltage requiring comparatively unfavourable fractional number of 600-volt cells in series.

(*c*) 600-volt cells requiring 'single-way' transformer where germanium can use (cheaper and higher-efficiency) bridge-connected transformer.

Thus, for average requirements, the comparison normally favours germanium up to 200–250 volts d.c. Above this silicon proves superior in some cases and germanium in others, according to how the detailed requirements fit one or the other, and each case has to be judged on its merits.

I would emphasize that these remarks refer only to fully modern practice and to conditions as they exist throughout the world at this present date.

Mr. F. Moores: Section 4.1 states that 'Capacitors may be used for power-factor correction'. This is a very simple statement, but in practice the problem presents difficulties. Recently I have been concerned with the power-factor improvement of some 1 200 h.p. d.c. rectifier colliery winders. The rectifiers are grid-controlled, and it is found that, at the beginning of the wind, the reactive power loading peaks to a high value for about 1 sec and then reduces to much lower values for the remainder of the winding cycle. The consumer is concerned to keep the maximum demand to a minimum, and we have to consider the average loading over the winding cycle, including the stop. In one instance the winding cycle, including stops, takes only 45 sec, and it was found that the average power loading for a 1 200 h.p. motor was 300 kW and the average power factor approximately 0·40, lagging.

When considering average power factor we must remember that the rectifier is drawing reactive power from the system, whether it is rectifying or converting; furthermore, when retarding, the motor acts as a generator and power is supplied to the system. The kilowatt-hour meter therefore reverses and the number of units recorded is reduced, yet the sine units continue to be integrated.

Could the author say more about the possibility of improvement by capacitors, and to what power factor should improvement be considered; alternatively for what amount of reactive power loading should compensation be provided?

Mr. J. M. Waddell: One point which the author has mentioned but to which he has perhaps not given sufficient emphasis is the question of the silicon controlled rectifier. In Table 1 he lists various applications and says that in some cases germanium and silicon rectifiers can be used, and in others it is necessary to use mercury-arc rectifiers because grid control is required. In the body of the paper he occasionally mentions the silicon controlled rectifier, but only in the context of an existing silicon or germanium rectifier, i.e. where grid control is not essential. I think that a silicon controlled rectifier could be used in all cases where a mercury-arc rectifier with grid control could be used.

I agree with nearly all of Dr. Read's remarks, especially that the choice between germanium and silicon should be made on the merits of each case. However, I believe that the merit of small size will decide a large proportion of the cases in favour of silicon, because, other things being equal, size costs money.

The efficiency obtained from a monocrystalline rectifier depends a great deal on circumstances, but I cannot work out the circumstances the author has used in arriving at the last two columns in Table 2, and I shall be glad if he will explain this point.

The hardness of the solders used in making silicon rectifiers depends on the process employed. There are methods of making devices with quite soft materials, apart, of course, from the silicon, which is inherently hard.

Mr. P. Whitaker: The author mentions resonant-shunt smoothing circuits in traction substations. In small industrial equipments where good smoothing is sometimes required, a single-stage circuit with series reactive and parallel condenser would be adequate. It would not have to be tuned, it would be cheaper and resonance difficulties would be avoided.

The Germans tend to call convertors 'mutators', but in this country some supply authorities and users look upon them, not as mutators, but as mutilators of the a.c. system. The C.E.G.B. recommendations, however, ensure that mutilation of the system does not occur, and, in fact, provide a very safe code of practice. This code is not applied oversea, where much greater harmonic distortion is often tolerated, which suggests that our safety factor may be too high. In addition, measurements made at one steelworks in this country showed the harmonic distortion to be less

than originally calculated. For these reasons the recommendations should be kept under review in the light of experience and field measurements, and relaxed if found possible.

Rather stringent conditions are also imposed by the supply authorities to make certain that voltage fluctuations do not cause annoyance to other users on the same system. This principally affects convertor-fed mill reversing drives and mine winders. Where there are several drives off the same system, the coincidence of peaks is often taken into account in deciding what size of convertor the system can take. Here again field tests are necessary to decide what voltage fluctuations and what frequency of fluctuation can be tolerated, and whether coincidence of peaks needs to be considered seriously.

Mr. J. A. Broughall: From time to time reference has been made to the traction use of rectifiers and it was for traction that they first became important. I can call to mind a number of these occasions, and thereafter The Institution has heard nothing more of them. In 1950 reference was made to the installation of air-cooled multi-anode rectifiers for the electrification of the Liverpool Street–Shenfield line; nothing more has been heard of them since, for the excellent reason that there is nothing to say about them. They do not backfire, they do not have to be repaired; they just go on working. In 1952 another kind of rectifier was described to The Institution, for the Manchester–Sheffield line, and again the story is the same; and there are other stories of a similar nature. In 1931 the London Midland and Scottish Railway were starting to use glass-bulb rectifiers: of these installations, over half have never been back to the makers for any reason whatever and have worked for over 144 000 hours.

This is a point worth making, because once a paper has been read here about a project, that is normally the last that we hear about it, but it is useful to know how things go on. The railways are already probably the biggest users of rectifiers in the country, and with our a.c. electrification schemes we are at the moment engaged in buying and setting to work something like a million kilowatts of rectification plant of one sort or another. We have examples of most kinds of rectifier and in a few years' time we shall be in a position to compare them on a realistic basis. We only hope that the rectifiers which we are now buying, whether mercury-arc, silicon or germanium, will give us as good service as those which we have bought in the past.

There are two technical matters in the paper to which I wish to refer. Table 1 leaves a free choice of the kind of rectifier to use. The margin between them is narrow, and it may be that what will finally decide between the mercury-arc and semiconductor rectifiers is the absence of the need to warm up the latter.

There is one point which I do not think is brought out very clearly in the paper. Nearly all these rectifiers now comprise a series of relatively small components, 192, 56, 16, 8 or 4. We have some number to play with. What is important, in the absence of much in the way of overload capacity in any of them, is to have a clear understanding between purchaser and manufacturer, so that the manufacturer can install the right number. This being so, it is not so essential, as the paper implies, to keep to the British Standard; it is more important to have a clear understanding of what the duty is going to be.

Fig. 2 is a very cautious curve. It is fortunate that it does not apply to the railways, so that we can make a bargain. The parameters are obviously right, but I know of a case which has given no trouble for years where the power connected in 12-phase rectifiers is about three times that allowed by this curve.

The paper seems to imply that filters ought to be employed if 6-phase rectification is used. This again in my experience is untrue. There are many railways in different parts of the world on 6-phase working without filter circuits.

Mr. D. J. E. Evans: The author refers to a number of novel applications of power rectifiers. It is interesting to note that the gas circulators of one of the nuclear generating stations now under construction are to be driven by d.c. motors of some 2 200 h.p. and 1 000 r.p.m. supplied by steel-tank mercury-arc rectifiers. This does not mean that power rectifiers are the best and cheapest for all applications, because the type of drive for gas circulators is but one of the parameters in the overall station optimization, and it would be quite wrong to isolate a single parameter and give it undue significance. The author states that, in general, 6-pulse working or more is satisfactory so far as heating and commutation of the motor are concerned. It is generally assumed, I believe, that the use of rectifiers reduces the brush life by about 15–20%. It is appreciated that it is difficult to obtain accurate figures for this, but I should like to know the author's experience.

In the quadruple-star shown in Fig. 6, phases 1, 5 and 9 are associated and operate in parallel with 3, 7 and 11. Similarly, the even phases 2, 6 and 10 are associated and operate in parallel with 4, 8 and 12. Assuming a twin-tank arrangement for clarity, if all the odd phases were connected to the anodes in one tank and all the even phases to the second tank, the ideal currents in both tanks would be constant and equal. If phases 1–6 were connected to anodes in one tank and 7–12 in the other, the currents in both tanks would oscillate. What determines the sequence of connection of multi-anode rectifiers?

In a previous paper it was indicated that trouble had been experienced during system transients with a.c. induction motors. What is the experience of the operation of mercury-arc rectifiers in parallel with a.c. induction motors, particularly during system transients?

[The author's reply to the above discussion will be found on page 459.]

NORTH-WESTERN CENTRE AT MANCHESTER, 5TH APRIL, 1960

Mr. T. H. Hill: In the early days of electric traction, when the systems were d.c. supplied, with ground collection with either third rail or third and fourth rail, the distribution system consisted of 11 kV h.v. cables feeding into rotary-convertor substations; these were rather large buildings, constructed with generous proportions and resembling a large hall, to provide a craneway for maintaining the convertors; furthermore, the substations were manually controlled. Latterly the rotary convertors were dispensed with in favour of the mercury-arc rectifier and have shown a vast improvement, the traction substations being smaller and remotely controlled, so that further economies resulted.

The Manchester–Altrincham line completed in 1931 has two substations, both of which contain rotary convertors and mercury-arc rectifiers. One of the rectifiers is a very early type, a continuously evacuated steel-tank type, and has given more than 190 000 hours of service with little trouble. The Manchester–Bury line again has two substations, both of which now contain mercury-arc rectifiers of the glass-bulb type, and these rectifiers have given service exceeding 160 000 operational hours. These rectifiers were installed in 1933, displacing rotary convertors. On the Liverpool–Southport electrification some glass-bulb rectifiers have over 180 000 hours of recorded operation.

The more recent d.c. overhead electrification systems have utilized steel-tank mercury-arc rectifiers. The Liverpool Street–Shenfield line (1949) employs the pumpless type, while

the more recent Manchester–Sheffield–Wath system (1954) employs steel-tank rectifiers continuously evacuated; both water-cooled and air-cooled types are installed, and from experience with these to date it would appear that the water-cooled type is preferable, mainly because of its smaller size. Hence for the particular application and rating it has been found possible to use one tank unit instead of two, with a consequent reduction in the number of auxiliaries and hence maintenance.

The rectifiers on the Manchester–Sheffield line have been in service up to 50 000 hours, and again little trouble has been experienced. Two cases of cracked porcelain rings have occurred and a certain amount of tripping out on thermal overload has occurred during hot spells, owing to inadequate cooling of the building.

Little trouble has been experienced with glass-bulb mercury-arc rectifiers, although a recent backfire caused 17 anode fuses out of 18 on three bulbs of a 6-bulb unit to blow simultaneously.

With the recent decision to electrify the main lines at 25 kV and 50 c/s, the rectifiers are now installed on the locomotives. The earliest locomotive (1958) was the converted gas-turbine unit now renumbered 2001, which employs glass-bulb rectifiers; some trouble was experienced from unequal load-sharing among individual bulbs, but this was overcome when it was discovered that the cables to the motors were of unequal length. Later locomotives and multiple-unit sets at present undergoing trial running employ both single- and multi-steel-tank mercury-arc rectifiers, with germanium rectifiers on some multiple-unit sets. Troubles with germanium rectifiers have arisen mainly from overheating due to overload, causing breakdown of individual cells, which has been cumulative causing failure of the rectifier; two cases have been experienced on the experimental units operating on the Morecambe–Heysham line.

Has the author knowledge of any trouble experienced on mercury-arc rectifiers through loss of vacuum, and if so, what was the cause? Although the mercury-arc rectifier is electrically a fairly robust unit and can withstand for short periods up to 400% full load, is it possible that future developments of semiconductor rectifiers, e.g. silicon, may produce a comparable unit to meet the growing demands of electric traction, particularly now that the rectifier is fitted to the locomotive where space and weight are prime considerations?

Dr. R. Feinberg: From Table 2 it is not obvious why the efficiencies of mercury-arc valves, excluding the transformer losses, do not further increase above 98·5% for direct voltages above 1 kV, and why the efficiencies for semiconductor devices decrease at that voltage.

The displacement-factor/cos ϕ diagram may be advantageous to the design engineer of the particular plant to which it refers, but from the general point of view a cos ϕ/time diagram would have been interesting as an indication how cos ϕ changes when a large winder drive supplied by a mercury-arc convertor is in operation.

The statement in Section 5.2 on the economy conditions for very-low-voltage heavy-current applications of semiconductor rectifiers requires some elucidating remarks to explain the underlying reason.

The explanation for the functioning of arc starvation in Section 8.3.1 appears to be based on over-simplification of the mechanism of current conduction in the mercury-arc path.

Any attempt to use the term 'convertor' where appropriate instead of 'rectifier' is to be welcomed as a significant and helpful step forward in the effort to use a technical language based on a logically constructed terminology.

Mr. J. A. F. Cornick: The paper indicates that the momentary voltages which germanium and silicon rectifiers can withstand are usually limited to abont 1·5–2·5 times the rated peak working voltage. This statement appears to imply that users of such devices can thus allow them to operate 'momentarily' at voltages higher than the rated peak working voltage, but this may not in all cases be true. In general, the rated peak working voltage (frequently referred to as peak inverse voltage) is the absolute maximum allowable voltage on the device. The peak voltage to be expected in the circuit, including any short-period over-voltages, must be covered by the manufacturer's peak inverse voltage rating, and sufficient cells in series used to accommodate this voltage. Only where the manufacturer of the rectifier specifically states the over-voltages which can be tolerated should these be used in designing equipments.

The peak inverse voltage rating of a rectifier is specified by the manufacturer with a number of requirements in mind. One important feature of any semiconductor rectifier is the permissible amount of reverse power dissipation. The reverse leakage current is exponentially dependent on the temperature, and the system can be shown to be thermally unstable under certain circumstances. Since reverse power dissipation is a function of the inverse voltage, this voltage must have regard to the thermal-stability criterion. Some other important factors entering this problem are the maximum allowable junction operating temperature, the rate of increase of reverse current with temperature and the thermal characteristics of the device itself. Obviously the rating of the device must be such as to leave a minimum margin of safety when considering the operation of a device having the most unfavourable characteristics in the most unfavourable circuit and ambient conditions. The user does not in general know the margin of safety allowed, and is thus advised to adhere strictly to the recommended peak inverse rating unless some other limit is clearly indicated.

Mr. R. W. Whitehead: In previous literature I have noticed that three methods of defining the percentage ripple in the direct current have been used, namely

$$(a)\quad \frac{\text{Maximum d.c. ripple} - \text{Minimum d.c. ripple}}{\text{Twice d.c. mean value}}$$

$$(b)\quad \frac{\text{Peak value of second harmonic}}{\text{D.C. mean value}}$$

$$(c)\quad \frac{\text{R.M.S. value of ripple}}{\text{D.C. mean value}}$$

Would the author specify which of these he has used in obtaining the ±30% d.c. ripple given in Section 3.1? Was the figure quoted a result of the author's experience or was it an optimum value computed from results given in earlier papers?

Mr. D. H. Vertigen: Two of the types of a.c. locomotive at present being supplied to British Railways have almost identical multi-anode steel-tank rectifiers, three rectifiers being used per locomotive, fed in parallel through a pair of 3-limb inter-tank reactors from a centre-tapped single-phase transformer. On the one type of locomotive, backfire and overload protection of the rectifiers and traction motors is by means of high-speed d.c. circuit-breakers in the output leads from the transformer secondary. On the other type of locomotive, individual d.c. overload relays open the electro-pneumatic contactors in series with each traction motor in the event of motor faults, while rectifier backfire and over-current faults are detected by high-speed a.c. overload relays in the transformer-secondary output leads, tripping the air-blast circuit-breaker on the primary side of the transformer and subsequently opening the motor contactors. Both of these a.c. relays have three trip elements connected between the inter-tank reactors and the three rectifiers, in order to give close protection and yet afford the necessary discrimination with the d.c. motor overload relays. On both

types of locomotive, earth-leakage protection of the rectifier and motor circuit is provided, holding the rectifier cathodes at earth potential. I should like the author to comment on the relative merits of these two systems of protection.

Mr. J. Hindmarsh: What is the maximum likely rating of the silicon controlled rectifier, and is there any reason to doubt that it will ultimately replace the grid-controlled mercury-arc rectifier entirely?

SOUTH-WEST SCOTLAND SUB-CENTRE AT GLASGOW, 20TH APRIL, 1960

Mr. G. F. Kirkpatrick: In Section 2 the author states that for grid control of high powers and for high voltages, steel tanks are preferable to glass-bulb units. This has not been my experience with a 3·2 MW grid-controlled glass-bulk installation with a maximum direct voltage of 550 volts commissioned in about 1936. At the outbreak of the war the engineers responsible for the rectifier thought that pumpless steel-tank units would be preferable to glass bulbs in the event of bombing. Two cubicles were therefore converted to steel tanks, but since trouble was experienced on them it was decided not to convert any more until further experience had been gained. Since glass bulbs proved satisfactory, no more cubicles were converted and after the end of the war the two cubicles were reconverted to glass bulbs.

Semiconductor rectifiers have no starting or exciter circuits, but they have a fan-failure relay complete with timing unit and a fuse-failure alarm, so that they are no simpler than mercury-arc rectifiers. My experience of glass-bulb mercury-arc rectifiers is that they require practically no servicing. The cubicles are blown out and the fans are greased about once or twice a year. Dirt does not worry the bulbs, and I have seen equipments where the bulbs have never been cleaned, running trouble-free after many years. I understand that semiconductor cells must be kept clean, and I should like the author's comments.

In Table 1 I should like to have seen something mentioned about the fact that, with mercury-arc rectifiers, the equipment is not automatically shut down on fan failure, as in semiconductor rectifiers. This feature is important in steelworks and heavy foundries. With a fan failure, mercury-arc units can carry one-third of full load continuously; I should like to know the figure for air-cooled semiconductor rectifiers.

In Section 5.4 it is stated that, with 6-phase transformer connection, there is a 15% voltage rise at loads below 1%. This is correct for an interphase transformer with a hot-rolled-steel core. If cold-rolled steel is used the voltage rise does not occur until the load has dropped to ½%.

Similarly, with 12-phase connection, the 20% voltage rise would not occur until the load fell below 1%.

Fig. 6 gives regulation characteristics with no values. The regulation of a mercury-arc rectifier is generally in the region of 6–7%. What is the value for semiconductor rectifiers?

THE AUTHOR'S REPLY TO THE ABOVE DISCUSSIONS

Mr. J. P. McBreen (*in reply*): For convenience, replies have been grouped under subject headings.

Compak Rectifier.—Air-cooled glass-bulb rectifiers employing similar design principles are also available.

Germanium versus Silicon.—The relative merits of germanium and silicon rectifiers have been clearly stated by one of the contributors.

Efficiencies.—The efficiency of semiconductor rectifiers will vary with the connection used, the number of cells in series and the cooling-equipment loss (if any), and will vary slightly between manufacturers. The fall in efficiency at 1·5 and 3 kV is because of the conservative use of cells at these voltages.

Earth-Fault Protection.—Earth-fault protection may be provided by a sensitive relay connected between earth and the mid-point of a high-resistance potentiometer connected across the d.c. busbars.

Power Factor.—The amount of power-factor correction used with any reversing rectifier-fed drive can only be a compromise based on the operating duty and local electricity tariffs.

Silicon Controlled Rectifiers.—These will not become a practical alternative to the larger grid-controlled mercury-arc rectifiers until higher-rated cells are available. Prototype cells have been operated at a crest working voltage of 350 volts and a mean direct current of 100 amp, and there is no fundamental reason why practical cell ratings of up to 300 crest working volts and up to 250 amp mean direct current should not be achieved.

Smoothing.—Smoothing is not always necessary in 6-pulse traction substation installations. It is rarely necessary in industrial installations of 6-pulse and over.

Supply-Voltage Fluctuations.—The Electricity Council Chief Engineers' Conference have established a committee to study this problem.

Reliability and Service.—The newer semiconductor rectifiers are also achieving a reputation for reliability. Equipments installed or on order from British manufacturers now total nearly 500 MW and many have given reliable service for close on 5 years.

Rating and Overloads.—The use of British Standard ratings removes any ambiguities from the specification of the equipment. With all types of rectifier overloads are a question of rating.

Brush Life of Motors.—So many factors can influence brush life by up to 20% that data are difficult to obtain, but any effect on brushes by rectifiers should be negligible.

Grouping of Anodes.—Normal practice is to connect a 6-phase group to each rectifier vessel.

Transients.—Supply-voltage transients, if of sufficient magnitude and duration, could possibly affect grid-controlled-rectifier drives.

Grid Control.—Present-day applications require fast response control of high powers and voltages and glass-bulb rectifiers are inherently unsuitable for this duty.

Maintenance.—For all rectifiers the effort required for cleaning is negligible.

Fan Failure.—Fan failure in semiconductor rectifiers may be made to trip the equipment out of service after a short delay. The percentage load which mercury-arc rectifiers can carry without their cooling fans operating varies with the rectifier type and rating from 10 to 33⅓%; corresponding figures for semiconductor rectifiers are 20–30%.

No-Load Voltage Rise.—The values given in Section 5.4 for the commencement of the no-load voltage rise cover all cases; in most cases the values are much lower.

Regulation.—The voltage regulation of germanium and silicon rectifiers is usually 7–10%, depending on the application. Lower values can be provided.

Choice of Connections.—The 3-phase bridge connection basically requires three cells in parallel and one in series; the 6-phase double-star and diametric connections require six in parallel and one in series. When the loading is such as to require more than three cells in parallel and only one in series, the

6-phase connections permit a better utilization of cells and often this more than compensates for the more expensive transformer.

Surge Conditions.—The figures of 1·5–2·5 times the rated crest working voltage quoted in Section 9.4 refer to transients only.

Ripple.—For single-phase traction it is usual to express the ripple as plus or minus the percentage variation from the mean. For all other applications formula (*c*) in Mr. Whitehead's contribution is used.

Protection.—Provided that the requirements of Section 9.1 have been met, both methods of protection used on the British Railways locomotives should give satisfactory results.

DISCUSSION ON 'DISCRIMINATION BETWEEN H.R.C. FUSES'*

Before the North Staffordshire Sub-Centre *at* Stafford *11th December*, 1959, *the* South Midland Supply and Utilization Group *at* Birmingham *8th February*, *and the* North-Western Utilization Group *at* Manchester *15th March*, 1960.

Mr. A. R. Galanides (*at Stafford, communicated*): If a 'minor' fuse is subjected to a low over-current its element may exhibit one short break at the middle where the temperature rise is a maximum. This will result, in the first instance, in a stable arc which will cause the remnants of the element to burn back relatively slowly. The corresponding arcing time may be very protracted, particularly with fuses in d.c. circuits of large time-constant or with high-voltage a.c. fuses. Experience has shown that in a number of such cases the fuse cartridge may burst through the excessive heat produced (thermal failure) and the arc continue to burn in the open. Discrimination between this and the 'major' fuse under these conditions may be a matter of conjecture.

Mr. W. J. Elliott (*at Birmingham*): I agree that discrimination at high fault-current levels must be determined on the basis of I^2t, but disagree with the author's method of presentation. This is one of those 'useful tools' which requires more thought in its application than the paper suggests.

Questions on the comparative values of I^2t for an h.r.c. fuse, a semi-enclosed fuse and a cable of equal rating indicate a lack of appreciation of the significance of I^2t. The melting I^2t for a 60 amp fuse element, a 17 s.w.g. copper-wire element and a 19/·044 in cable are in the ratios 1 : 38 : 5 600; but these ratios are irrelevant, since we are not protecting the cable against melting. If we fix the cable temperature rise at 40° C (the terminal temperature limitation in B.S. 88) the ratio becomes 1 : 38 : 115, showing that either the h.r.c. fuse or the semi-enclosed fuse would give adequate thermal protection to the cable.

There is, of course, more in the problem than this. The low value of I^2t for h.r.c. fuses enables them to limit mechanical, as well as thermal, stresses when handling very high prospective currents. Semi-enclosed fuses are never used at these prospective short-circuit levels. The h.r.c. fuse loses heat much more rapidly than a semi-enclosed fuse of similar rating, and at the time of operation of a 60 amp semi-enclosed fuse for currents within its expected performance range the I^2t value for the h.r.c. fuse is much higher than that determined for higher prospective currents. This must be so or the h.r.c. and the semi-enclosed fuses with fundamental I^2t ratio of 1 : 38 could not be of similar current rating.

Because of these complications I doubt the value of I^2t as a 'simple tool'; it is a necessary device for the fuse designer, but the user is not a specialist and must be able to make a quick and accurate comparison of characteristics without the dangers which can arise from misinterpretations of I^2t. This is best achieved by the use of time/current curves as with the lower fault currents.

The virtual-time concept gives a way of presenting the information on the time/current characteristics and removing the confusion referred to above. The author opposes the idea of virtual time on the difficulty of interpreting its definition as given in a certain British Standard. Virtual time, however, is simply I^2t/I^2_p (where I_p is the r.m.s. symmetrical prospective current) and is the time in which the fuse would operate if carrying a steady current equal to I_p. The pre-arcing-time/current characteristics can be plotted on a separate sheet from the total-time/current characteristics and so prevent confusion due to curves crossing.

The author denies that, for correct discrimination, the total operating time of the minor fuse must be less than the pre-arcing time of the major fuse, but, in fact, the statement is quite true if the times referred to are virtual times.

Finally, the author claims accurate discrimination between fuses with I^2t values less than 8% apart. Since I^2t is proportional to the square of the area of the element, the very fine silver wires used must be accurate to within ±1% even if there are no other manufacturing variations at all. Is this possible?

Mr. J. B. Brockbank (*at Birmingham*): Can the author give some idea of the discrimination which can be obtained between fuses of the same general type but of different makes? It is vital in every installation to obtain discrimination between the consumer's fuses and the supply-undertaking service fuses, but the consumer has no control over the make of fuse which the undertaking installs.

Mr. I. G. Edwards (*at Birmingham*): Is I^2t appreciably different in the short-centre fuse (say 3⅝ in centre) as compared with the long-centre (6½ in centre) fuse of the same rating, since it would appear that the heat dissipation for the same current in the former would be less than in the latter? If the I^2t is appreciably less for short-centre fuses, it would appear that grading troubles would arise where equipment using them is operated in conjunction with plant using long-centre fuses, e.g. the Fig. 10 layout.

Difficulties are often experienced in obtaining discrimination

* Jacks, E.: Paper No. 2805 U, January, 1959 (see 106 A, p. 299).

between an h.r.c. fuse and a relay. While it is appreciated that the primary duty of the former is to protect against short-circuits, can anything be done to modify its characteristic such that when used in conjunction with a relay having a characteristic in accordance with B.S. 142 the fuse operates before the relay?

Mr. R. Rockliffe (*at Birmingham*): The medium-voltage network in Birmingham is supplied by distribution transformers having a capacity of up to 1 MVA, many of which are protected on the h.v. side by current-transformer-operated trip coils shunted by time-limit fuses. These fuses do not possess the non-deteriorating quality of medium-voltage h.r.c. fuses. A situation can therefore arise where, although satisfactory discrimination is obtained on the m.v. network, the time-limit fuses may operate through deterioration after passing the fault current which is normally cleared by the m.v. fuses. Is any investigation being made to overcome this problem by producing non-deteriorating time-limit fuses?

In Section 3.13 the author states that 'h.r.c. fuses can, with proper care, be made to conform to tolerance bands which are well within the requirements of normal service duty, but it is incumbent on the user to know the tolerance band when choosing fuses to give discrimination'. I do not think there is any tolerance value stated in B.S. 88, and I should like to know the manufacturer's tolerance for h.r.c. fuses and whether they are likely to have any appreciable effect on the discrimination.

Mr. H. M. Fricke (*at Birmingham*): I should like the author's comments on installation having 60 amp h.r.c. fuses at the point of intake, a wired type of fuse on a 30 amp distribution board for sub-circuits and the final load taken by 2 kW convector heaters, each of which has 13 amp fused plugs.

Mr. J. L. Bagley (*at Birmingham*): I should like more information on the use of h.r.c. fuses in d.c. circuits. I am particularly interested in d.c. distribution system in heavy industry and have recently been involved in the installation of such a system incorporating many thousands of horse-power of motors and 5 MW of rectifiers. I should like to know how to assess the potential fault currents, and I should like any information there may be on the subject of discrimination.

Mr. V. D. Long (*at Birmingham*): Is it correct to assume that there is no reason why quite different types of fuse-link, such as the silver-wire type and the dual-element type, should not discriminate satisfactorily, provided that the characteristics are suitably graded?

Mr. D. A. Picken (*at Manchester*): One cannot consider discrimination between fuses without considering associated parts of the circuit. There are no adequate data regarding the let-through capacity of electrical devices on a sufficiently wide scale to facilitate full utilization of the author's technique for fuses.

It is true that in many cases a cable is unlikely to suffer damage when protected by an h.r.c. fuse under short-circuit conditions, for the small common size of cable, 7/·029 in, should be safe, from a danger-to-persons aspect, with fuses rated as high as 100 amp; but under restricted fault conditions this value is much too high.

Problems also arise in the discrimination between one type of fuse and another. While adequate information is available in many cases for let-through capacity of the normal type of fuse, information is not available for the specialized type such as are used for heavy-current starting-motor conditions where the values might be expected to vary considerably between a cold fuse and a hot fuse of certain designs.

The author refers to very high fault currents, whereas in practice, despite the high fault levels obtaining on h.v. systems, it is rare to find low-voltage systems where the fault level exceeds 25 kA, and a high proportion of the fuses must operate in systems where the fault level is 15 kA or less. This will not seriously affect the discrimination between fuses, but it is a factor which has to be borne in mind in some circumstances.

The great need in protective equipments is for some common standard of evaluation to be adopted, and for manufacturers to rate their equipments in accordance with this technique; for if such a standard were adopted—and I^2t seems to fulfil most of the requirements—the problems of designing protective systems would be much simplified.

Mr. C. Ayers (*at Manchester*): The author gives several of the factors affecting the discrimination between h.r.c. fuses which rely to a large extent on the appreciation that the fuse is a precise piece of electrical apparatus. It is thus essential that the user appreciates that published characteristics of fuses relate to the results of tests carried out under specific conditions and that the characteristics may be modified if the fuses are used under different conditions.

The major factors quoted by the author are worthy of further mention and are

(*a*) Condition of fuse and its non-deterioration properties.
(*b*) Pre-loading of the fuse.
(*c*) Rating of major and minor fuses.
(*d*) Position and environment of the fuse.
(*e*) Size of the connections to and from the fuse.
(*f*) Manufacturer of the major and minor fuse.

The first three criteria have been adequately dealt with by this and other papers. Factors (*d*) and (*e*) may affect fuse performance, and I should like the author's view on their general effect. The last criteria is one which the user can control to a large degree and it is well to remember that fuses of the same nominal rating but of different manufacture can exhibit a spread of 10 : 1 in pre-arcing time for a given operating current.

The author refers to degrees of discrimination and distinguishes lower degrees, high degree, positive and economic. I suggest that two fuses either discriminate or not and that qualification of the 'degree of discrimination' is a matter of circuit design unconnected with fuses and their characteristics. The author appears to approach this view in Section 5·3, where he states that 'in some instances discrimination may be foregone with proper discretion'. In this connection his choice of words in the Conclusions is unfortunate when he states that 'the assessment need not be critical'. Surely a critical approach to fuse utilization is essential, although it may not be necessarily accurate?

Mr. F. Mather (*at Manchester*): When faults occur on public-supply distributors the current will seldom reach the cut-off value, and it would therefore appear from Section 3.9 that successive steps of current rating will give discrimination without harm to the major fuse. Is this correct?

Section 5 indicates that for vital circuits the ratio of ratings may have to exceed 2 : 1 for discrimination. It would be interesting to know why, since Figs. 7 and 8 indicate that even 1·5 : 1 is adequate in the higher current ratings.

Would it be possible for manufacturers to facilitate discrimination by adopting a common characteristic curve for induction relays to B.S. 142, semi-enclosed fuses to B.S. 3036 and cartridge fuses to B.S. 88?

Section 3.13 states that it is incumbent on the user to know the tolerance band when choosing fuses. Should this factor not be included in the relevant British Standard?

Mr. J. M. Fellows (*at Manchester*): In the steel industry and on large testing installations, high-power d.c. systems capable of developing prospective short-circuit currents of 40–100 kA are frequently encountered. Examination of the fuses used in a majority of these applications will show that they were primarily designed for use on a.c. systems only. Very few fuse manufacturers are prepared to guarantee satisfactory performance of

their fuses on these high-power d.c. systems. Where fuses are used on d.c. systems, care must be taken to ensure that suitable suppression is included to prevent high-voltage transients causing secondary stresses or damage. The characteristics of a d.c. system would suggest that the arcing times would generally be greater than those encountered on a.c. systems. Does the author agree with this, and what would he consider a reliable rule-of-thumb ratio to obtain discrimination between major and minor fuses on d.c. systems?

Mr. E. Jacks (*in reply*): Fuses should be designed to avoid the difficulties mentioned by Mr. Galanides. Fuse-links which burst under any conditions are obviously not suitable for the duty to which they have been subjected. Discrimination must obviously depend upon fuses being used within their declared ratings.

I do not disagree with Mr. Elliott's further advocacy of the virtual-time method or the additional points which he makes in support of it. I also appreciate that there is a good deal of technology implicit in Fig. 7 which is not obvious to the uninitiated. To use his own words: Fig. 7 is a 'useful tool'. It is unnecessary for the user to understand completely the technology upon which it is based any more than it is necessary for a calculating-machine operator to understand the details of the design of the machine from which she must produce arithmetical results.

I believe that the virtual-time method is less direct than the curves given in Fig. 7, but I respect the opinions of those who, like Mr. Elliott, prefer the more abstract concept.

The claims for accurate discrimination between fuses within I^2t values less than 8% apart are based on actual tests carried out over a comprehensive range of fuses. This narrow margin does not depend solely upon the dimensions of the fine silver wires used in the fuse construction, but rather on the thermal characteristics of the silver itself. It can be demonstrated that silver may be heated to a point of absorbing a considerable amount of latent heat before losing its ability to resist distortion. This phenomenon can be used to advantage by careful design, which accounts for the practical results actually obtained.

Several speakers refer to discrimination between fuses of differing characteristics. The degree of discrimination obtained under these circumstances will obviously depend upon the divergence between the characteristics, but provided that the necessary data are available, this degree can be assessed on the lines indicated in the paper. The different physical dimensions mentioned by Mr. Edwards are not so significant as may be imagined, because the I^2t values for fuse-links of the same current rating but of different lengths are not necessarily widely different. Mr. Fricke refers to a specific installation, and it would require specific data relating to the items he mentions to make a proper assessment. It may be expected that the 13 amp plug fuses will discriminate with the 30 amp rewirable fuses, but these cannot be expected to discriminate with the 60 amp h.r.c. fuses at prospective fault currents above 2 kA.

Time-limit fuses of the types mentioned by Mr. Rockliffe are somewhat outside the scope of the present paper, because the circumstances under which they are used are not comparable with distribution circuits in which h.r.c. fuses are used. The question raised by Mr. Rockliffe in this respect therefore requires special treatment.

Mr. Picken refers to the let-through capacity of the electrical equipment which is protected by fuses. Information on this point is being rapidly accumulated, and the I^2t data relating to fuses as shown in Fig. 7 are useful for direct comparison with the capacity of protected equipment.

The effect of pre-heating fuses and of varying their environment is dealt with in Section 4 of the paper. Both Mr. Picken and Mr. Ayers have emphasized the importance of these considerations.

Mr. Ayers seems to have misconstrued my intention in referring to degree of discrimination. The choice of fuses depends on many factors other than discrimination and some compromise may be expedient. Where such a compromise is made, it is essential to know the degree of risk. The paper attempts to outline the data required and the methods to be applied in order to assess this risk. Mr. Mather points out an apparent contradiction in the paper on this theme. A ratio of greater than 2 : 1 may be necessary where the factors external to the fuse are extreme. The likeliest factors are those involving the possibility of human error, leading to unintentional abuse.

Mr. Bagley and Mr. Fellows raise the question of discrimination on d.c. systems. The first point to be ascertained in this connection is that the fuses proposed are, in fact, rated for d.c. duty. If so, the principles laid down in the paper apply. The values of arcing I^2t for a given fuse on a d.c. circuit may be greater than on an a.c. one. This will tend to widen the ratio required between fuses for discrimination, but any difference will obviously depend upon the particular design of fuse which it is proposed to use.

1.357.1

The Institution of Electrical Engineers
Paper No. 3196 U
Feb. 1960
©

ELECTRICITY IN THE MANUFACTURE OF HYDROGEN PEROXIDE

By B. E. A. VIGERS, M.A., A.M.I.C.E., M.I.Chem.E., and R. O. FLETCHER, B.Sc., A.M.I.Mech.E., Associate Member.

(The paper was first received 7th September, and in revised form 9th November, 1959. It was published in February, 1960, and was read before the UTILIZATION SECTION *18th February, 1960.)*

SUMMARY

The paper gives a brief history of the development and uses of ydrogen peroxide. The electrochemical process is given in some etail, and types of electrolytic cell are mentioned with reference to e electrical characteristics affecting the design.

A modern factory with an 11 MW electrolytic load and the main eatures of the electrical system peculiar to this type of plant are escribed. Reference is made to the other services concerned and the se of back-pressure turbines and generating plant.

The paper concludes with a note on the future of the electrochemical rocess.

(1) INTRODUCTION

(1.1) Discovery and Development

The discovery of hydrogen peroxide in the early 19th century can be attributed to the French chemist Louis-Jacques Thenard.[1] He found that when barium peroxide was treated with an acid a residual liquid with new properties was produced. Later work proved that this liquid was in fact hydrogen peroxide, but production was not of any real commercial significance until 1879, when barium peroxide became more readily available.

Experiments by Berthelot[2] in 1878, which showed that persulphuric acid might be produced by electrolysis of sulphuric-acid solution and readily hydrolized in solution to produce hydrogen peroxide and sulphuric acid, led to the establishment in 1908 of the first commercial electrolytic process. Although the electrolytic process is still the principal method employed, an organic autoxidation process has now reached commercial application in Britain and the United States, and there is also a development using isopropyl alcohol in a partial combustion process.[3]

Hydrogen peroxide is a clear colourless liquid which, although it can be decomposed by a wide range of catalysts, in the pure state is stable and can be stored for years without appreciable loss of strength. It will decompose on heating or by the addition of a catalyst such as permanganate of potash to give oxygen and water, which may be in the form of steam if the hydrogen peroxide is of sufficiently high strength. Concentrations higher than 90% by weight having an exceptionally high degree of purity and stability are now manufactured on a large scale. Hydrogen peroxide, even of the highest strength, can be handled quite easily and safely provided that simple precautions are observed, cleanliness being of great importance. The metal most commonly used for storage is aluminium, but some of the modern plastics can also be used.

Concentrations of hydrogen peroxide are nowadays referred to as 'weight per cent of H_2O_2' in aqueous solution. Table 1 gives approximate values of the heat liberated by decomposition and of the additional heat that can be generated by combustion of a suitable fuel with the oxygen evolved.

Mr. Vigers and Mr. Fletcher are with Laporte Chemicals Ltd.

Table 1

PHYSICAL PROPERTIES OF HYDROGEN PEROXIDE

Weight per cent of H_2O_2	Density at 25° C	Heat of decomposition	Additional heat by combustion with carbon to CO_2
	g/cm³	C.H.U./lb of solution	C.H.U./lb of solution
0	0·997	0	0
20	1·069	113·2	277
40	1·149	267·4	553
60	1·236	404	830
80	1·334	554·3	1 107
100	1·443	688·9	1 383

(1.2) Uses of Hydrogen Peroxide

Although Thenard had considerable accomplishments in applied chemistry, he found little practical application for hydrogen peroxide, which remained a chemical curiosity for the next 60 years or so after its discovery. By 1885, however, hydrogen peroxide came into use as a bleaching agent for hair and wool and as an antiseptic in the treatment of wounds. It was later used for bleaching straw, and strengths of up to 6% became available. Improved techniques in the chemical process and also the introduction of the electrolytic process made possible the production of a much purer peroxide of high stability, and a greater degree of concentration (up to 35%) proved possible without incurring serious losses through decomposition. Bulk transport now became possible, which enabled factories to continue manufacture even when local requirements for hydrogen peroxide disappeared.

In non-military applications the principal consumption of hydrogen peroxide continues to be as a bleaching agent, either as the liquid or converted to sodium percarbonate or perborate for admixture with washing powders. It is also finding increased uses as a reagent in the manufacture of organic chemicals.

High-strength hydrogen peroxide can serve as a highly compact source of energy which can be released by decomposition using a suitable catalyst, yielding oxygen and steam at high temperatures. Large quantities were used by Germany in the last war as a power source in various weapon systems, and it is now employed in missile engineering and assisted-take-off units. Another use, also initiated by the Germans, is in the propulsion of submarines.

(2) COMMERCIAL ELECTROLYTIC PROCESS

In general industry the electricity supply is used for lighting and for motive power. The cost is only a small fraction of the total production costs, e.g. in the wool-textile industry[4] it is around 4%. Electrochemical and electrothermal processes used in the treatment and purification of raw materials use a much larger amount; for instance, in the production of certain ferrous

alloys by reduction and treatment of the ore in electric-arc furnaces, the cost of electricity is about 28% of the total production cost.

By comparison the electrolytic production of hydrogen peroxide has only one important raw material—electricity—which can account for up to 50% of the total production cost. For economic production, not only must a cheap supply be available, but the process must be designed to use this power supply to the very best advantage.

Direct formation of hydrogen peroxide in an electrolytic cell is not possible in usable quantities, but peroxy compounds can be formed at the anode, which by subsequent hydrolysis will yield hydrogen peroxide. The choice of electrolyte and electrode materials will materially affect the operating voltage and current efficiencies and therefore the overall energy efficiency of the process.

The electrolyte in common use consists of ammonium bisulphate and sulphuric acid in solution with water. The overall reactions of the process may be represented by the following formulae:

In the electrolytic cell,

$$2NH_4HSO_4 \rightarrow (NH_4)_2S_2O_8 + H_2\uparrow$$

In the hydrolysis column,

$$(NH_4)_2S_2O_8 + H_2O \rightarrow 2NH_4HSO_4 + H_2O_2$$

From these equations it can be seen that the bisulphate reformed at the hydrolysis stage is equal to and becomes available as feed to the electrolytic cells. Some adjustment to the strength of solution is, however, necessary for technical reasons and to make up losses.

it is therefore fractionally condensed in a column to give a final yield of peroxide which commonly has a strength of 35–50%. The combined system of hydrolysis and fractionating columns is maintained at a low pressure of about 1·5 in Hg absolute by means of condensers and vacuum pumps.

(3) TYPES OF PRESENT-DAY CELLS

It is the usual practice to incorporate a porous diaphragm between the anode and cathode of the electrolytic cell, thus separating the electrolyte into anolyte and catholyte liquors. Many efforts have been made to operate without a diaphragm, but none has yielded energy efficiencies approaching that obtained in the more conventional cell. In one type of cell the diaphragm consists of asbestos material wound closely round the cathode, and the catholyte is present only as a thin film of liquid between asbestos and cathode. An alternative is to use a ceramic diaphragm enclosing the cathode. The purpose of the diaphragm is, of course, to prevent anolyte liquor coming into contact with the cathode, where it would be decomposed.

Two general designs[5, 6] of cell have been employed, one based on the flow of anolyte liquor only and the other using liquor flow through both anolyte and catholyte compartments. In the single-flow cell some leakage of anolyte through the diaphragm must be permitted in order that acid conditions are maintained around the cathode. If this were not allowed there would be formation of ammonia, which would lead to breakdown and blocking of the diaphragm, causing a build-up of cell voltage. A battery of single-flow cells may consist of about 50 cells connected in series electrically and fed with liquor at one end. The liquor passes through all the anolyte compartments in series and emerges as a solution containing ammonium per-

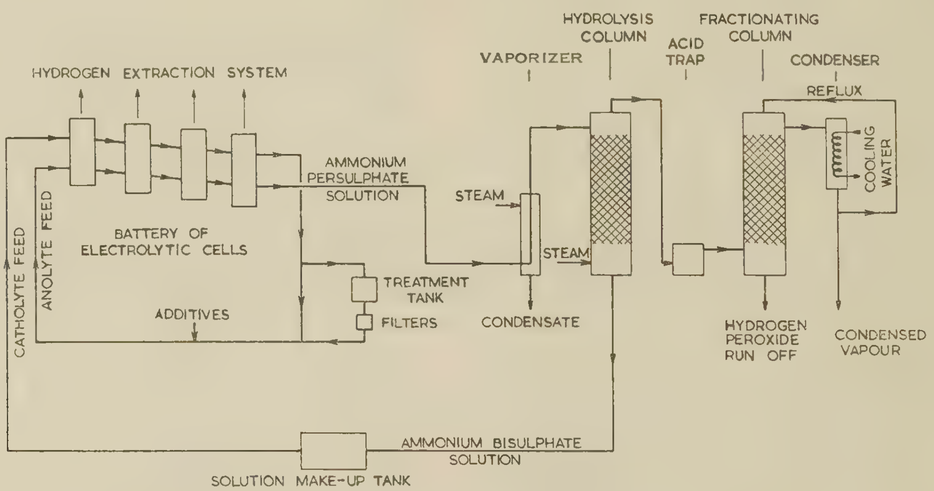


Fig. 1.—Liquor flow diagram for ammonium persulphate process.

Fig. 1 shows a simplified flow diagram for the process. Return liquor from the hydrolysis column, after adjustment for strength, passes to the electrolytic cells. These are described in more detail in the next Section, and it will suffice here to say that the liquor flows first through the catholyte chamber and is then pumped back as anolyte feed. After leaving the anolyte chambers, the electrolyte, now containing ammonium persulphate, is passed through a steam-jacketed vaporizer where hydrolysis begins. The hydrolysis is completed in a packed stoneware column where the hydrogen-peroxide vapours separate from the ammonium bisulphate liquor, which is returned as feed to the electrolytic cells. At this stage the hydrogen-peroxide vapour mixed with water vapour is only about 4% strength and

sulphate at the other end of the battery. Feed liquor is made up of an aqueous solution of sulphuric acid and ammonium sulphate or, more accurately, since these constituents react together, ammonium bisulphate with excess sulphuric acid. Constitution of feed liquor and product liquor may vary widely, but a typical example is given in Table 2, it being realized, of course, that the amount of persulphate formed will depend upon the liquor flow rate, the number of cells in series and the electric current passing through the cells.

There are advantages in the use of the double-flow cell in which the electrolyte is passed through the cathode compartments and returned as feed to the anolyte side. Excess acid in the catholyte feed maintains acid conditions in the cathode

Table 2

TYPICAL COMPOSITION OF ELECTROLYTE

Constituent	Feed	Return (product)
	g/l	g/l
H_2SO_4	260	160
$(NH_4)_2SO_4$	210	76
$(NH_4)_2S_2O_8$	0	232

chamber, so preventing build-up of alkalinity, and it is therefore no longer necessary to allow leakage of anolyte through the diaphragm to obtain this result. Loss of persulphate in the anolyte is therefore prevented and the acid conditions in both compartments assist in the conductivity of the solution.

At present, platinum is nearly always used as the anode material, and the design must obviously economize in the use of this expensive material as much as possible. Various forms of anode have been devised, the object being to present the required surface area of platinum suitably disposed with a minimum ohmic resistance in the distribution of the current. Examples are sheets of platinum foil and various arrangements of platinum wires. The cathodes are made of lead or graphite. A considerable amount of heat is generated in the cell, owing to resistive voltage drop, and it is necessary to incorporate a suitable cooling system in which water is circulated through either glass or lead coils which may form part of the cathode design. The electrolysis baths are made in various materials, stoneware and plastic-lined tanks having been used for the purpose. The size of bath depends to a large extent upon whether it contains only a single-cell unit, by which is inferred a single voltage drop, or a number of cells in series separated by means of partitions built into the bath.

Fig. 2 shows a simplified outline of a single-cell bath in stone-

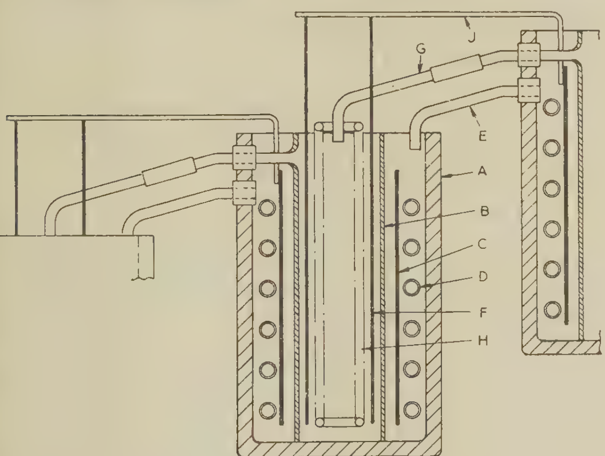


Fig. 2.—Type-A single-cell bath.

A. Stoneware bath.
B. Ceramic diaphragm.
C. Cathode.
D. Cooling coil.
E. Catholyte feed.
F. Anode.
G. Anolyte feed.
H. Anolyte cooling coil.
J. Series electrical connection.

ware. Although it is single cell so far as voltage drop is concerned, it consists of a number of anode assemblies in separate diaphragms constituting a parallel arrangement of cells.[7] A battery of these cells, for convenience called type A, is generally arranged in a cascade formation consisting of 25 baths in series,

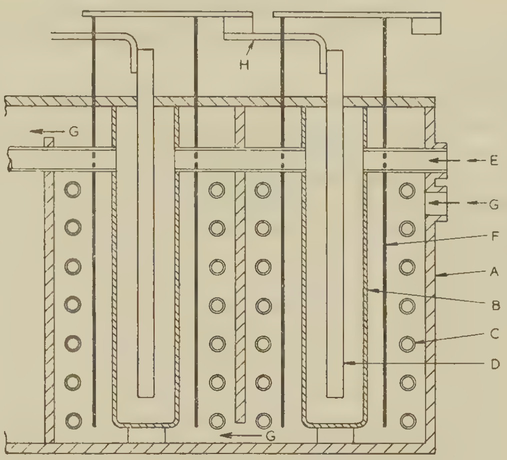


Fig. 3.—Type-B 10-cell bath.

A. Plastic-lined tank.
B. Ceramic diaphragm.
C. Glass cooling coil.
D. Cathode.
E. Catholyte flow.
F. Anode.
G. Anolyte flow.
H. Series electrical connection.

while four cascades are connected in electrical series to allow for operation at 500 volts.

Type B is a 10-cell bath manufactured in plastic material as shown in Fig. 3. This construction makes total enclosure of the top of the bath an easier proposition and therefore tends to give cleaner operating conditions than type A. It is more suitable for the smaller factory, although there is little to choose as regards overall cost.

The later Sections deal more particularly with the type A battery installed in a large factory, where the total electrolytic load approaches 11 MW.

(4) ELECTRICAL CHARACTERISTICS OF TYPE-A CELL

Broadly speaking, any voltage in excess of the decomposition potential of the products at the anode represents a resistive voltage drop. As the decomposition potential is essential to the formation of ammonium persulphate, any attempt at reducing the overall voltage and hence improving the efficiency of the process must be aimed at the resistive drop. A high current density is necessary at the anode in order to raise the potential to give preferential formation of persulphate rather than oxygen, but the remaining portion of the total voltage is within the control of the designer by choice of suitable cathode material and surface area, concentration of electrolyte and disposition of cell components.

The decomposition voltage can be evaluated by consideration of the heat balance of the electrochemical reaction. In the electrolytic cell, complex ionization of the liquor takes place, but the overall reactions can be condensed into the formula:

$$2NH_4HSO_4 \rightarrow (NH_4)_2S_2O_8 + H_2 - W$$
$$2(-240) \qquad (-373)\ (0)$$

The figures in brackets are the heats of formation of each compound in kilocalories per gramme-molecule. Consequently, W represents the theoretical energy required to produce the chemical change. From this it is possible to calculate the theoretical voltage necessary to produce and maintain the reaction at constant temperature. The result will be very nearly the same as the decomposition potential calculated from the change in free energy of the compounds.

From the above, $W = 107$ kcal/g-mol of product or, by converting the units, $W = 224$ kJ per gramme equivalent of ammonium persulphate.

In accordance with Faraday's law, the quantity of electricity required to liberate one gramme equivalent of a substance is 96 500 coulombs; hence, if V_1 is the theoretical potential, $W = V_1 \times 96\,500$ joules per gramme equivalent.

Equating these values of W gives the theoretical potential, $V_1 = 2{\cdot}32$ volts.

In practice the voltage applied to each cell is about 5 volts, resulting from the components shown in Table 3. The relation between current and voltage in a typical type-A cell is shown in Fig. 4.

Table 3

DISTRIBUTION OF CELL VOLTAGE

	Voltage volts
Electrode potentials	2·9
Polarization and over-voltages	1·3
Anolyte liquor	0·3
Catholyte liquor	0·2
Diaphragm	0·3
Total	5·0

Conductivity of the electrolyte is controlled by maintaining excess acid conditions in both anode and cathode chambers. Fig. 5 shows the conductivity of neutral solutions of ammonium

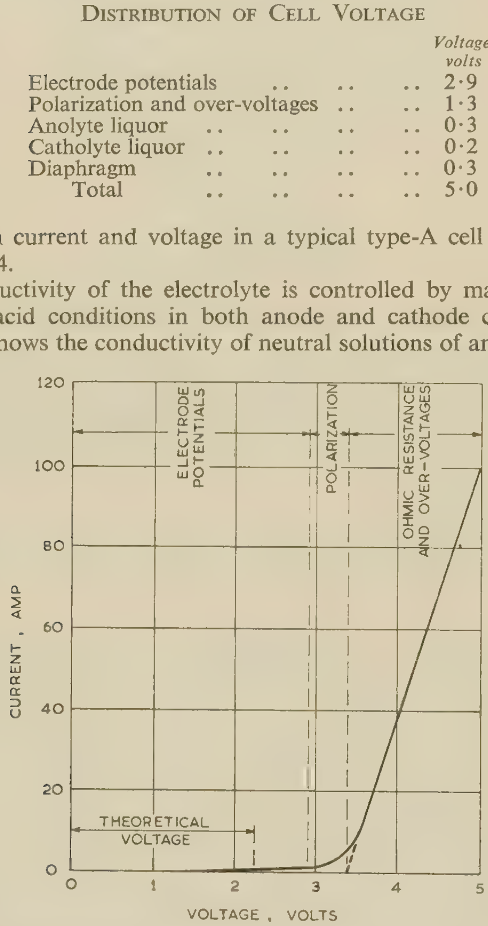


Fig. 4.—Relation between current and voltage of type-A 4-anode cell.

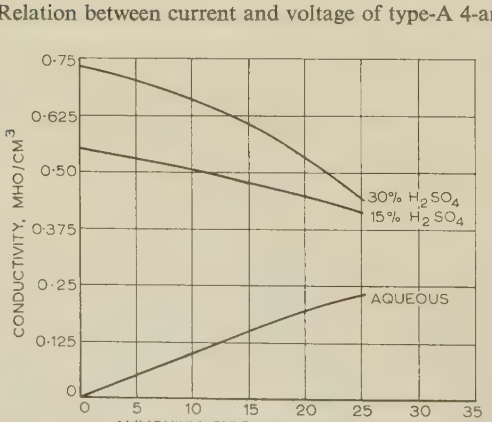


Fig. 5.—Conductivity of ammonium-sulphate/sulphuric-acid solution.

sulphate compared with a solution containing sulphuric acid and demonstrates the considerable difference in conductivity. Disposition of the electrodes will have some effect on voltage drop, and it is important that the anodes are equally spaced to prevent interference between current flow in the high-density region. There is an optimum distance between the cathode and the diaphragm since, if the space is too restricted, the hydrogen bubbles evolved at the cathode will raise the gas/liquor ratio to such an extent as to reduce the conductivity.

A great deal of work has been carried out on the development of suitable ceramic diaphragms to give adequate resistance to corrosion as well as minimum leakage between anolyte and catholyte. The ceramic must have a high degree of interconnected porosity to limit the voltage drop across the diaphragm.

When due regard has been paid to these factors and optimum values have been selected for current densities and spacing of components, there still remain the electrode potentials and over-voltages in excess of the theoretical voltage for production of persulphate. The result is that considerable heat is evolved in both anode and cathode compartments as well as in the body of diaphragm. Efficient cooling must be provided, since excessive temperatures, particularly in the anolyte, will cause decomposition or prevent formation of the ammonium persulphate.

Calculation of the amount of heat to be taken away by the cooling system entails a knowledge of the current efficiency at which the cell will operate. Current efficiency is defined as the ratio of the theoretical quantity of current required for a particular reaction to that actually consumed. The difference between these two quantities results from changes, in addition to the chemical change desired, that may take place or from some of the main product that may be lost by secondary chemical changes. In the case of the persulphate cell these two effects are evident as liberation of oxygen at the cathode. For a battery of cells in good condition the current efficiency will be around 80%, i.e. 20% of the current goes to making oxygen.

Based on the foregoing it is now possible to prepare the formula for the total heat balance in the cell, namely

Total energy supplied

= energy in formation of persulphate
+ energy in formation of oxygen
+ heat evolved in cell

i.e. $$IV = I\eta V_1 + I(1-\eta)V_2 + \text{heat}$$

where I and V are the current and voltage at the terminals of the cell or battery of cells, η is the current efficiency and V_1 and V_2 are the decomposition potentials of ammonium bisulphate and water.

A large battery of type-A cells has 100 cells in electrical series and consumes 2 kA at 500 volts, i.e. 1 MW. For a current efficiency of 0·8 and with V_1 and V_2 as 2·32 and 1·3 volts respectively, per cell, the heat to be dissipated by cooling is equal to 573 kW. Under these conditions the energy efficiency of the battery is only 37·1%, and to maintain even this value requires very careful attention to the factors affecting current efficiency and cell voltage.

One such factor is the temperature of the electrolyte, for which a typical case is shown graphically in Fig. 6. While increased temperatures are beneficial to the cell voltage, there is at the same time a fall in current efficiency which becomes very considerable at the higher temperatures. The energy efficiency calculated from the voltage and current efficiency curves, shows a marked reduction at temperatures above 35° C. It can be seen that maximum efficiency is reached at 30–35° C, but in practice the temperature is kept between 25 and 30° C to raise

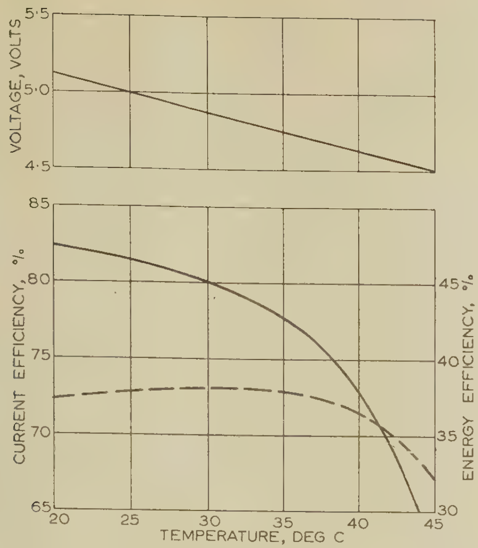


Fig. 6.—Effect of temperature on type-A cell.
—— Current efficiency. ---- Energy efficiency.

the current efficiency and increase the yield of persulphate from a given installation. The lower temperature also reduces platinum loss caused by corrosion at the anode.

It has been mentioned previously that formation of persulphate at the anode is possible only above certain critical anode potentials. An increase in anode potential is brought about by

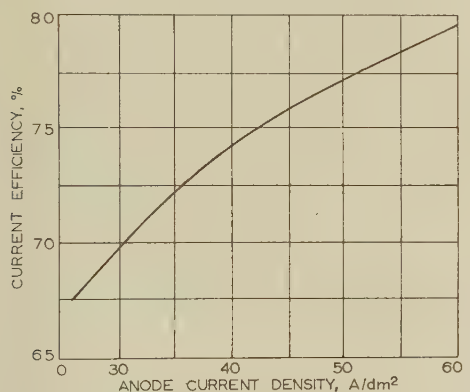


Fig 7.—Relation between current efficiency and anode current density.

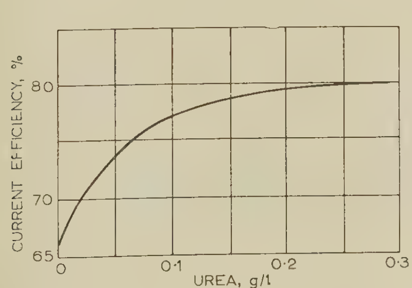


Fig. 8.—Effect of additions of urea on electrolysis efficiency of ammonium-sulphate/sulphuric acid solution.

increasing the anodic current density, and Fig. 7 shows how this changes the current efficiency. The persulphate yield is also increased by the addition of certain chemicals to the anolyte feed, which, it is thought, function by raising the anode potential. Fig. 8 illustrating the general effect.[5, 6]

(5) ELECTRICITY SUPPLY TO BATTERIES

(5.1) Rectifiers

The following Sections relate to a modern factory producing 8 000 tons/annum of hydrogen peroxide, equivalent 100% strength, and using 12 MW of electrical power. Roughly 3 MW of the power is obtained by private generation while the remainder is imported from the public supply. Over 90% of the electricity is used on the batteries, which are supplied at 500 volts d.c. from mercury-arc rectifiers and a 1 MW turbo-generator.

In this application the rectifiers carry their full load 24 hours a day for 340 days of the year, a very different proposition from intermittent traction loads or similar duties, and due regard was paid to this fact when deciding upon the rating of the equipment. Most of the rectifiers are of the pumpless steel-tank design, and each unit has two 6-anode rectifier cylinders connected to a delta/quadruple-star transformer. In general there is one rectifier unit per 500-volt battery, whose original loading was intended to be of the order of 1 100 amp. For this duty the rectifiers were specified for a continuous full load of 1 250 amp, i.e. 625 amp/cylinder, although the manufacturers supplied their nearest standard with a nominal rating of 750 amp. Improvements in the design of the electrolytic cells led to an increase in battery loads of up to 1 450 amp, which was catered for by modifications to the transformers; the original cylinders were retained, which were therefore approaching the nominal loading defined by the manufacturer. Although this has proved fairly satisfactory there has been a greater tendency for backfiring to take place, and if the development towards increased loads on the batteries had been anticipated, larger units would have been ordered in the original scheme. However, the occurrence of a backfire is not looked upon as a serious matter, since the rectifier involved can be switched on again and brought back to full load within a few minutes. Table 4 gives a record of the backfires experienced over the last few years, and it will be seen that there is a preponderance on rectifiers 5–10. These are the more heavily loaded units operating at 1 450 amp, whereas rectifiers 1–4, which have had fewer backfires, carry only 1 200 amp, since the d.c. supply at this point is supplemented by the 1 MW d.c. generator. The incidence of backfires seems to have no relation to the years of operation and does not appear to affect the life of the rectifier cylinders. The two cylinder failures that have occurred have been due to vacuum leakage at the anode seals.

(5.2) D.C. Connections

Two methods of d.c. connection have been employed for the rectifier units. The original installation of four units utilizes parallel connection via d.c. busbars, since this arrangement allows for the connection of the 1 MW d.c. turbo-generator, as shown in Fig. 9, and enables the load on the generator to be adjusted in accordance with the steam supply rather than battery current. Furthermore, it was desired to give a reliable supply, not dependent upon one source, for d.c. auxiliaries in the boiler house, which are connected to these busbars. This diagram also shows that a battery unit consists of four cascade banks of cells in series, each cascade of 25 cells forming a single battery, so far as liquor flow is concerned, while for the complete unit there are 100 cells in electrical series. To facilitate maintenance on one cascade without losing production on the other three, a system of 375/500-volt switching has been incorporated, but only one rectifier transformer out of the four has the facility and in practice this has proved sufficient.

Table 4

RECORD OF RECTIFIER BACKFIRES

Year	Rectifier number													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1950	1	1	—	3										
1951	—	—	—	1										
1952	—	—	—	—										
1953	—	1	—	—	2	1	—	1	1	—				
1954	—	—	1	3*	1	2	1	3	1	1				
1955	—	—	—	2	—	1	—	—	6*	—				
1956	—	—	—	—	—	—	7	1	1	—	—	—	3	—
1957	1	—	—	—	—	2	5	2	1	—	—	—	4	—
1958	1	—	1	—	—	—	—	2	1	—	—	—	2	—

* Includes one rectifier bulb failure. Rectifiers 11 and 12, multiple glass-bulb type. Remainder, twin pumpless steel-tank type.

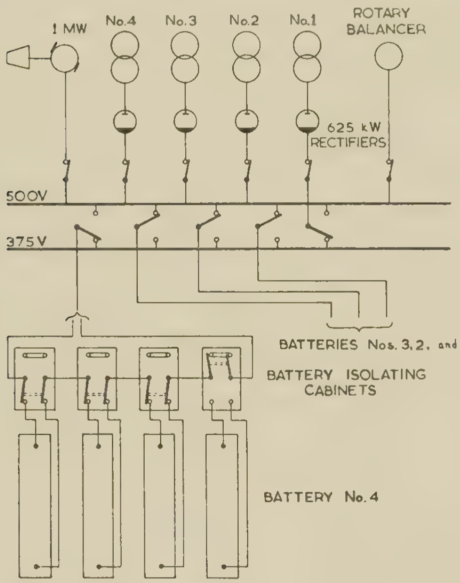


Fig. 9.—D.C. diagram of original 4-unit plant.

Extensions to the plant meant additional rectifier equipment, which is connected individually to separate battery units. As can be seen from the general layout of the buildings (Fig. 10), a parallel system would have entailed the provision of very long busbars or cables on the d.c. side and the provision of high-speed circuit-breakers to deal with the increase in fault capacity.

(5.3) A.C. Switching of Rectifier Units

For economy and simplicity of operation it is preferred that the switching of a rectifier when connected to an individual battery shall be carried out on the h.v. alternating-current side of the installation. A saving can also be made if the anode fuses are omitted. These were necessary on the first four units, since the parallel connection gave a source for reverse current on a faulty cylinder, and therefore a quick clearance of the fault was essential. With the individual connection, the only source of reverse current is either from the battery or the second cylinder of a unit, and it was decided that anode fuses could be omitted provided that there was no reverse power available from the battery.

To determine this, tests were carried out using a cathode-ray oscillograph, and measurements were taken of the current surge when switching on a new battery and of the reverse current

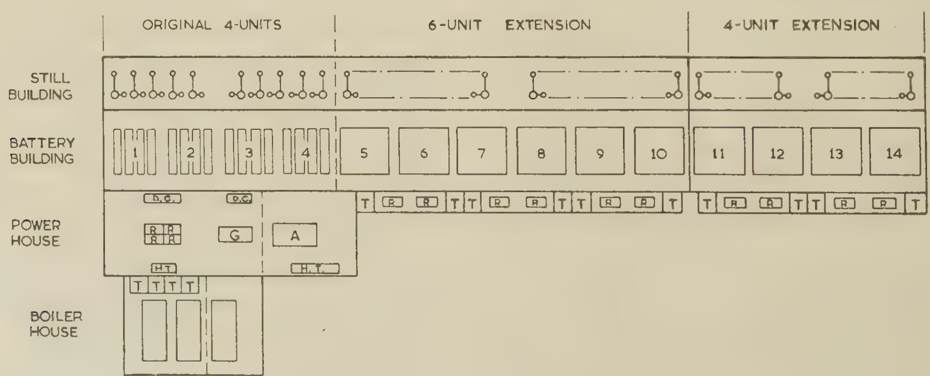
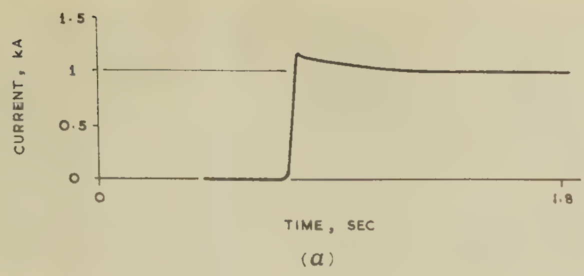


Fig. 10.—Arrangement of plant.

R. Rectifier.
T. Transformer.
D.C. D.C. switchboard.
H.T. H.T. switchboard.
G. D.C. turbo-generator (1 MW).
A. Turbo-alternator (2 500 kW).



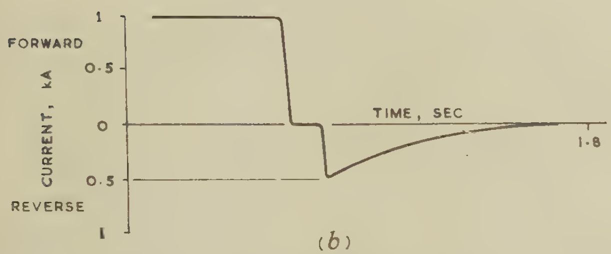


Fig. 11.—Oscillograms of battery transients.
(*a*) Current taken when switching-on a battery.
(*b*) Reverse current on a short-circuited battery.

when a battery was disconnected from its d.c. supply and a short-circuit applied across the battery terminals.

Curve (*a*) of Fig. 11 shows the current taken when switching on a battery after it has been out of use for a short period, during which time the battery was short-circuited to discharge any stored energy. It will be seen that, although the rise of current at switching on is very rapid, as would be expected from the small time-constant of the rectifier circuit, the amount of over-current at the instant of switching is practically negligible, which means that the rate of build-up of electrode potential in the battery is extremely fast. Consequently there is little danger of voltage surges being set up in the transformer windings by arc extinction in the rectifier cylinders, and h.v. switching can safely be used.

Reverse current on a short-circuited battery, curve (*b*), shows a very rapid collapse to zero, thus demonstrating that the effect of the cells as storage batteries is extremely small. Under these circumstances the current flow during a backfire is limited by transformer reactance and, owing to the robust nature of the steel rectifier cylinder, the period of fault clearance can be the same as that required for the circuit-breakers to open, and anode fuses can therefore be omitted.

(5.4) Harmonics

Comparison between type-A batteries operated on fully smoothed and unsmoothed d.c. supplies has shown that the ripple content has no detrimental action as far as energy consumption per pound of product is concerned.

Investigation of the harmonic contents of voltage and current waveforms on the a.c. side has been carried out, first to satisfy the supply authorities that excessive harmonics were not being introduced into their system and secondly to determine the effect on input metering.

The first test was carried out when the plant consisted of the 1 MW generator and the original four 625 kW rectifiers connected to the Electricity Board's 6 kV system by 3 200 yd of 0·3 in^2 cable. Oscillographs of the waveforms on the h.v. supply were analysed to determine the various harmonics, and those of importance are shown in Table 5. It was considered that these values were not excessive, and the Electricity Board agreed that identical rectifier phasing could be used on the 6-unit extension. Further extensions would, however, be carried out with the phases displaced by 15° to give the equivalent of 24-phase rectification on the supply system.

In the second test, a tuned analyser was used to determine the harmonics on one of the four feeders for the final installation

Table 5

Harmonic Voltages and Currents on Original and Final Installations

Harmonic	4-unit plant		14-unit plant	
	Current	Voltage	Current	Voltage
	amp	volts/phase	amp	volts/phase
1	232·00	3 500	240·00	3 500
3	5·08	26·2	0·16	9·0
5	4·36	24·2	0·06	4·6
7	0·92	16·7	0·22	4·6
11	12·68	35·2	2·00	27·6
13	8·00	27·3	0·90	15·4
23	3·88	5·4	0·92	19·2
25	2·40	4·3	0·78	18·0

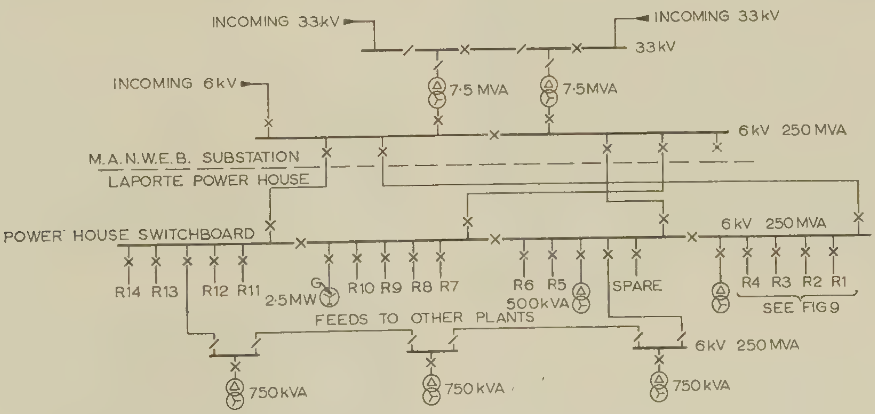


Fig. 12.—H.V. diagram of 14-unit plant.
R1–R10, 625 kW.
R11–R14, 900 kW.
Phase displaced 15°.

shown in Fig. 12. In this case the rectifier load was equivalent to 7 200 kW of 24-phase rectification and 2 650 kW of 12-phase rectification in terms of nominal rating. The significant values are also listed in Table 5 and, as would be expected, there is a considerable reduction in the eleventh and thirteenth harmonics.

It is well known that the presence of harmonics has an influence on the accuracy of induction-type kilowatt-hour meters,[8] which can read fast or slow depending upon the magnitudes and power factors of the respective voltage and current components and the excess or otherwise of braking torques. If the current wave only possesses harmonics, the meter is liable to read slow owing to the increased braking torque with no increase in driving torque, since fluxes of different frequencies cannot interact to produce a steady torque.

In the present case the voltage waveform was not seriously distorted, and it was expected that the meter reading would be on the low side. This was checked by comparing the actual power, using dynamometer wattmeters, with the readings of the induction-type kilowatt-hour meters connected in the h.v. supply to each rectifier. The average error was found to be 0·46% slow, thus confirming the previous conclusion.

(5.5) Voltage and Current Control

At an early stage in the design of the factory it was decided that a 12% control range would be sufficient to cover supply-voltage variations and adjustments to direct voltage to suit battery characteristics. With a tapping range of ±6%, steps of 1% were considered sufficient for accurate control and an economical design for the tap-change gear and transformer windings was possible. This has not proved entirely satisfactory, as seen from the following explanation.

From Fig. 4 it is seen that a 1% change in voltage brings about a 3% change in current. A good-quality commercial-grade voltage relay will have a sensitivity of about 1%, and it would not, of course, be worth while fitting a relay with greater sensitivity, since hunting between transformer taps would take place. It is therefore possible that the direct voltage can be almost 1% away from the desired value and the current supplied to the battery will then be either 3% above or below the correct load. Furthermore, since an increase in current will cause the temperature of the cell to rise, there will be a fall in the back voltage from the cells (Fig. 6), leading to a further increase in current, which in practice can be 5–6% away from the desired value.

It was appreciated that automatic current control would be more effective in obtaining steady conditions, and relays with a sensitivity of 1% are readily available. The equivalent voltage sensitivity would be 0·33% and, bearing in mind that the tapping steps are 1%, the use of such a relay in conjunction with the on-load tap change would give continuous hunting. Thus it was essential to provide a finer degree of control at the rectifier transformer and the stepless regulator was adopted. On such a current-control scheme there is always the possibility that in response to a low current signal, caused for instance by overheating of copper connections, the regulator will run away to its high-voltage position. To prevent this, a back-up voltage relay has been incorporated, which can be preset to take over control at a predetermined level.

(6) ELECTRICITY SUPPLY AND DISTRIBUTION

(6.1) Plant Layout

Private generation using back-pressure turbines is an economical proposition, and the power house becomes the focal point for both steam and electrical supplies to the process. The plant arrangement is shown in Fig. 10, dotted lines indicating the extent of the original factory and the two extensions. The boiler house is equipped with three 30 000 lb/hour boilers generating steam at 385 lb/in^2 (gauge). The power house is designed with a suspended floor, all pipework and cables being accommodated in the basement. In the battery house the cells are arranged in cascade as previously described. A concrete basin is provided beneath the batteries for collecting the cooling water after it has passed through the cells and to act as a down-draught chamber for ventilating the battery house.

(6.2) H.V. Supply

A simplified diagram of the main h.v. connections is shown in Fig. 12. The metal-clad triple-pole single-busbar air-insulated switchgear has vertical isolation and horizontal draw-out oil circuit-breakers having solenoid closing mechanisms. The 6 kV switchboard is run solid, i.e. with all busbar section switches closed, in order to give equal load sharing on the four feeders and to allow transfer of generated power to any section.

Fault protection on the 6 kV side of the supply consists of over-current relays at the sending end and reverse-current overload relays at the power house end of the four feeder cables. Rectifier switches have over-current and earth-leakage relays and instantaneous 500% over-current relays for backfire protection. Buchholz relays are fitted to the rectifier transformers to give gas alarm and automatic tripping in the event of an internal fault. The two 500 kVA power and lighting transformers have over-current and earth-fault protection in the primary and back-up earth-leakage relays connected to current transformers in the secondary neutral earthing lead, to protect against sustained earth faults in the l.v. system.

(6.3) Maximum Demand Control

Under normal circumstances the demand on the incoming supply is substantially constant apart from variation in the day and night lighting loads. Generated power will supply 3 MW of the total factory load, and again this is nearly constant because of the steady steam demand for distillation. However, anything leading to curtailment of generated power could increase the imported electricity, and if this happened during the registration periods for chargeable maximum demand, the financial penalty would be several thousand pounds. Loss of generated power can result from troubles with the boiler plant or mechanical and electrical breakdowns on the turbine sets. Many of these difficulties are of only short duration and fairly rare occurrence, but even so it has proved worth while to guard against any unforeseen rise in the imported demand and avoid the increased charges.

It is fortunate that the electrolytic batteries can be switched off for periods of up to two hours without seriously upsetting the conditions in the cells. This is done when the demand is excessively high, using a continuous-load control meter in conjunction with time-delay relays. For small excess demands the load is reduced by short-circuiting a resistance in the voltage relay circuit of the automatic tap-change gear on rectifier transformers, this action being initiated by the first in the series of time delay relays.

Automatic switching of the battery loads is also desirable for another reason. As the 6 kV busbars are run solidly with all section switches closed, a failure on one of the main 7·5 MVA supply transformers or a fault on one or more of the four feeder cables could overload the remaining supply equipment. To prevent this, intertripping circuits are installed so that when any one of these switches opens automatically, a predetermined section of battery load is switched off.

(6.4) Other Services and Electricity Usage

The l.v. supply (3-phase 4-wire 50 c/s) for general power and lighting on the factory is at 415 volts, taken from the two 500 kVA auxiliary transformers in the power house. All l.v. switchboards and distribution boards are equipped with h.r.c. fuses with a rupturing capacity of 25 MVA. Totally enclosed fan-cooled motors are used throughout and direct switching is employed whenever possible.

Apart from electricity the main services on the plant are steam at 5 lb/in^2 (gauge) for distillation, and water for cooling batteries and condensing equipment on the stills. Most of the boiler-house auxiliaries are operated on 500 volts d.c. in order to provide a simple means of speed control on the fans and chain-grate stokers. Cooling water is obtained from deep bore-holes, two of which are situated on the factory site while others have been sunk at distances up to 1¼ miles from it.

A rough estimate of steam requirements can be made by considering the amount of water which has to be evaporated in concentrating peroxide from 4 to 35% strength. The annual output of peroxide is 8 000 tons at 100% strength or 200 000 tons at 4% strength, all of which has to be evaporated in the steam-jacketed vaporizers. With a still efficiency of 95%, i.e. 5% loss of peroxide, and allowing 1% for the heat in the steam condensate, the total steam for distillation will be 213 000 tons per annum, equivalent to 60 000 lb/hour of steam at the boiler house in summer conditions.

Nearly all the water pumped from the deep bore-holes is used first for battery cooling. From Section 4 it is seen that the heat dissipation is 573 kW per megawatt of power supplied to the batteries; hence, for 11 MW of battery load the heat to cooling water will be 12×10^6 C.H.U./hour and the total quantity of water 114 000 g.p.h., since the water enters the batteries at 11·5° C and leaves the cooling circuit at 22° C. This water is then used for condenser cooling on the distillation plant, but the quantity has to be supplemented by recirculation through cooling towers. Cooling-water temperature at the condenser outlet is limited to 32° C by the vacuum requirement, and since most of the heat content in the steam for distillation finally passes to the condenser, it is simple to calculate the total quantity of cooling water. This works out at 320 000 g.p.h., of which 114 000 g.p.h. comes from the batteries and the remainder from two 100 000 g.p.h. cooling towers.

Electricity for these steam and water services and small power for chemical pumps amounts to 1 MW. The electrolytic batteries consume 11 MW, equivalent to 92% of the total usage

(7) PRIVATE GENERATION

The thermal economies which can be affected by the generation of high-pressure steam, which is used to produce electric power before serving as a process-heating medium, do not need amplification here. Consideration of the figures for steam and electricity consumption show that a hydrogen-peroxide factory of this type gives an ideal opportunity for the utilization of this principle. Batteries and distillation run simultaneously, there being only small storage capacity between the two sections of the process. Thus 12 MW and 60 000 lb/hour of steam are required at the same time.

At first sight it would appear that the higher the steam pressure the greater would be the financial return, but other matters have to be taken into consideration. The decision to use 385 lb/in^2 (gauge) was based on the need for obtaining a reasonable return without encountering the special problems associated with extra-high-pressure boilers. Because only a portion of the steam generated can be returned in the form of condensate to the boiler feed, it is necessary to use a fairly high proportion of make-up feed water, and 385 lb/in^2 (gauge) was selected as being a suitable pressure for use with the quality of the water expected from chemically treated feed.

A good deal of development work was done on the design of the stills to reduce the steam pressure required for vaporization. This was finally fixed at 5 lb/in^2 (gauge), for which purpose turbine exhaust was adopted at 7·5 lb/in^2 (gauge). At these values of inlet and exhaust pressures the steam throughput averages 20 lb of steam per kilowatt, resulting in a generated power of 3 MW. Two turbines were installed; the first machine, rated at 1 MW, drives a 500-volt d.c. generator while the second, rated at 2·5 MW, drives a 6 kV alternator. Usual loadings are 800 kW and 2 200 kW, respectively.

Both turbines are of the impulse-reaction type, No. 1 turbine driving the d.c. generator through a gearbox to reduce the speed from 6 500 to 750 r.p.m. No. 2 turbine runs at 3 000 r.p.m. and is direct-coupled to the alternator. A rather higher efficiency could have been obtained by using a high-speed turbine and reduction gear, but, within limits, this is not important since it does not represent a loss of efficiency over the complete steam cycle. The extra heat in the exhaust steam, occasioned by the lower blade efficiency, is given up to the process and the overall thermal efficiency remains unaltered.

The steam throughput and therefore the electrical output is controlled by a back-pressure relay operating through an oil-pilot system on to the main steam throttle valve, thereby adjusting the steam load to the process requirements.

A speed governor is also included, normally set higher than the running speed but available to take over control should the speed increase above the set value. With this arrangement the turbo-generator will automatically alter its speed to vary the output voltage and follow any variations of voltage on the d.c. busbars. In the same way the turbo-alternator can alter its speed to follow changes in Grid frequency and does this without change in load. The choice between a.c. and d.c. generation depends mainly on the layout of the plant and circuit arrangement rather than on comparison of the relative efficiencies of the two methods. For this particular installation the efficiencies are 93% and 94·8% for the generator and alternator, respectively. However, the alternator has to deliver its load to the batteries via the rectifier equipment with a conversion efficiency of 93–94%. Hence so far as overall efficiency is concerned, d.c. generation has a slight advantage. This fact, together with the increased reliability of supply to the boiler house, led to the selection of a 1 MW generator for the first machine. However, when the plant was extended, the d.c. busbar scheme was discontinued for reasons given earlier, and it would have been difficult to absorb all the additional generated power on the first section of plant. By using a.c. generation this power becomes available on any part of the system via the 6 kV busbars.

When considering the economic value of private generation, due allowance must be made for the additional cost of high-pressure boilers, as compared with low-pressure ones, that would have been used if steam from the boiler plant was sent direct to process. Cost figures for private generation are based on taking 15·8% of the total cost of high-pressure steam supplied to the turbine, this being the percentage heat drop in the steam passing through the turbine to a final sink temperature of 65° C, i.e. the temperature of condensate return from the still building. Cost of high-pressure steam includes depreciation on buildings and boilers, fuel, wages and repairs. The power-house costs for depreciation and running charges are then added to this steam allocation to give the total costs for generated electricity. Typical figures are given for the year ending 31st March, 1956, selected as a year of average conditions when the process plant consisted of 10 battery units and 10 rectifiers.

The estimated difference in the cost between high- and low-pressure boiler plants and buildings is £50 000, for an installed capacity of 90 000 lb/hour of steam. Depreciation is at the rate of 9% per annum which for the first ten years averages at 6%.

Generated electricity	$19 \cdot 06 \times 10^6$ kWh
Works cost excluding capital cost on boiler plant	0·332 d/kWh
Depreciation on extra cost of boiler plant: 6% of £50 000	0·038 d/kWh
Total equivalent cost of generated electricity	0·370 d/kWh
Average cost of imported electricity including capital contributions paid to the Electricity Board	0·981 d/kWh
Saving in cost by private generation ..	0·611 d/kWh
Total annual saving	£48 500

Capital investment to give the saving is of the following order:

Extra cost on boiler plant and buildings ..	£50 000
Turbine plant and building	£112 000
Total	£162 000

Thus the saving of £48 500 represents a return on capital of 30% which can be considered quite a good investment.

(8) SAFETY

(8.1) Hydrogen Risk

An electrolytic cell operating at 80% current efficiency for the formation of persulphate is producing oxygen at a current efficiency of 20%. The hydrogen evolved at the cathode is produced at 20% current efficiency for the decomposition of water and 80% efficiency for the decomposition of ammonia bisulphate, giving a total current efficiency for hydrogen of 100%. At these current efficiencies the gases evolved will be in the ratio of 10 volumes of oxygen to 100 volumes of hydrogen, i.e. an oxygen concentration of 9·1%, which is an explosive mixture.

The explosive range of hydrogen-oxygen-nitrogen mixture is between the limits 5% oxygen and 4% hydrogen. If nitrogen is used as a diluent, it will be necessary to add an amount equal to the volume of hydrogen evolved in the cell. This will reduce the oxygen concentration to 4·75% which is just outside the explosion limit. On the other hand, if air is used, it will be necessary to dilute with 25 times the volume of hydrogen evolved to reduce the hydrogen concentration to 4%. Since nitrogen is not readily available, air must be used as the diluent and a large excess is provided to give a safe margin.

On the type-A design the top of the cell is more or less open to the atmosphere and the gases are diluted and drawn away by large volumes of ventilating air which sweep downwards past the cells into the cooling-water collecting chamber, from which it is drawn away by fans exhausting to the atmosphere at the top of the building. When the cells are totally enclosed, it is preferable to keep the hydrogen and oxygen separated inside the cell as in the type-B design. The gases are led separately to a common ventilating duct through which a large volume of air is drawn to dilute the gases to a safe concentration.

(8.2) Electrical Risks

Owing to the cooling-water connections taken to the lead cathodes of the type-A batteries installed in the factory, there are electrical leakages to earth on every cell throughout the battery. These leakages tend to balance out so that the centre-point of the 500-volt supply is substantially at earth potential, and under normal conditions this coincides with the physical centre-point of a unit comprising four batteries. If an additional current leakage takes place on one side of the physical centre, the earth-potential point is displaced to one side and the outer ends of the battery rise above and below 250 volts to earth. This out-of-balance earth leakage could be caused by direct metallic contact to earth, but more usually it is set up by a leakage of electrolyte through a porous or cracked stoneware bath on to the lead-covered supporting step. Should it occur at one end of the battery unit, the cell connections at the opposite end can reach a potential of 500 volts to earth and would increase the danger of electric shock to the battery attendants. This is minimized by the use of timber flooring throughout but, in addition, warning devices are installed to give an alarm at a preset value of unbalanced earth leakage.

The simplest method is to earth the physical centre-point of a battery unit using the circuit shown in Fig. 13, employed for

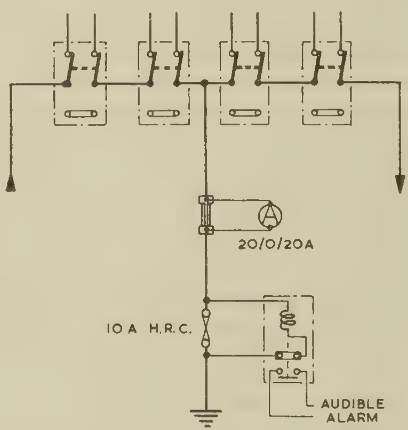


Fig. 13.—Mid-point earthing of units 5–14.

units 5–14. Visible indication of out-of-balance leakage is shown on the ammeter and an audible alarm is initiated when the connection to earth is broken by blowing of the centre-point fuse. In practice, remedial action is taken when the leakage indicator shows a reading of only 2 or 3 amp. If this is neglected, it may either become very much worse as the fault develops or be cancelled by an equal and opposite leakage on the other side of the unit, thus losing all indication that more than the normal amount of current is being lost by taking parallel paths via structural metalwork.

When batteries are connected to common busbars, as with the first four units, it is not possible to use this method of earthing since it would allow circulating currents to flow between batteries, owing to the slight variance between the potential at the physical mid-point of each battery unit. In this case, centre-point earthing is obtained by running a rotary balancer set on the 500-volt busbars with the centre-point brought out for earthing. Excessive out of balance brings about the automatic switching-in of a limiting resistance in the earth connection (Fig. 14). Visual indicators and audible alarms are provided as before. Before remedial action can be taken each battery unit has to be switched off in turn in order to locate the faulty unit.

Another reason for the installation of the balancer set is to provide a path for earth-leakage currents set up by faults on d.c. motors in the boiler house. Automatic switching of the limiting resistance is timed to allow a sufficient period for motor supply fuses to clear, thus pointing to the faulty gear. In addition, a current-balance relay is connected to measure any out-of-balance between positive and negative feeds to the boiler house. This initiates an alarm signal and provides means for

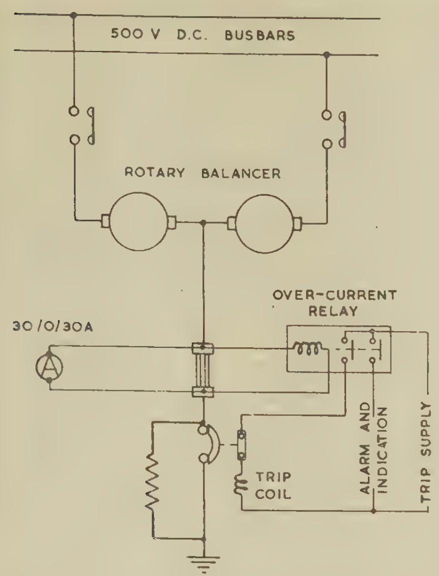


Fig. 14.—D.C. system earthing of units 1–4.

deciding whether an earth fault indicated at the balancer set is from the batteries or boiler-house auxiliaries.

(9) CONCLUSION AND ACKNOWLEDGMENTS

In view of the significance of electricity charges on the price of product, the electrochemical process is at a disadvantage in this country as compared with countries where cheaper hydro-electric power is available. Although it would appear that this industry, because of its particular type of load, warrants a special tariff which truly evaluates the actual cost of supply, negotiations in this respect have not been entirely successful. The result has been that other methods of producing hydrogen peroxide are being developed on a large scale, which may finally supersede the electrolytic process in spite of its many attractive features.

The authors wish to thank Laporte Chemicals, Ltd., for permission to prepare the paper and gratefully acknowledge the help given by their colleagues in the Chemical and Engineering Sections of the Company.

(10) REFERENCES

(1) THENARD, L. J.: *Ann. Chim. et Phys.*, 1818, **8**, p. 306.
(2) BERTHELOT, M. P.: *Comptes Rendus*, 1878, **86**, p. 71.
(3) SCHUMB, W. C., SATTERFIELD, C. N., and WENTWORTH, R. L.: 'Hydrogen Peroxide', *American Chemical Society*, 1955, Monograph Series No. 128, p. 89.
(4) FRANCIS, A. J., and CARR, T. H.: 'Electricity in the Wool-Textile Industry', *Proceedings I.E.E.*, Paper No. 1563 U, October, 1953 (**101**, Part II, p. 291).
(5) WOOD, W. S.: 'Manufacture of Hydrogen Peroxide', *Chemistry and Industry*, 1953, p. 2.
(6) WOOD, W. S.: 'Hydrogen Peroxide', *Royal Institute of Chemistry*, 1954, Monograph No. 2, p. 7.
(7) C.I.O.S. Report: 'Hydrogen Peroxide Production at Kufstein', 1945, Item 22, File No. 33.
(8) DANNATT, C.: 'The Metering of Mercury-Arc Rectifier Supplies and Outputs', *Journal I.E.E.*, August, 1937, **81**, p. 256.

DISCUSSION BEFORE THE UTILIZATION SECTION, 18TH FEBRUARY, 1960

Mr. D. B. Hogg: It is a pleasant change to have a paper about a process plant which is not only interesting electrically but of great educative value to engineers in the engineering, manufacturing and academic fields.

I am surprised that the factory inspectors allow the use of 500 volts d.c. We consider that such direct voltages are dangerous, especially in plants where there are liquids and vapours about. It is extremely difficult under such conditions to prevent damp insulating surfaces becoming 'live' from leakage currents, and for this reason we work our electrolytic plants at about 250–280 volts. Is not this high voltage on electrolytic plants taken from American practice? There are static means of rectifying more efficiently than by using mercury-arc rectifiers, especially at the lower voltages.

Why have the authors used the complicated high-voltage distribution system shown in Fig. 12? Such a system was tried and discarded in one of our works in the late 1920's and early 1930's. The trouble was caused by circulating currents between cross-connected sections when run closed. It may be that, as so often happens, the authors built a small works and then extended it, and so were committed to the system stated earlier, but could they not reconnect their sections to give direct feeders? Have they considered putting meters between their sections to find out the magnitude of the circulating currents where the main busbars are run as separate sections? They have added to their difficulties by having a second ring main in parallel which ties sections 1 and 3 (reading from left to right) together. It is hard to see why they do not have feeders tripping frequently owing to overloading by circulating currents, particularly as their protection system seems to be somewhat primitive, with over-current on one side and reverse current on the other. Did they consider using Merz–Price protection in one of its modern forms?

In Section 5.5 it is stated that from Fig. 4 it is seen that a 1% change in voltage brings about a 3% change in current. However, from Fig. 4, if the voltage changes from 4 to 5 (a change of 25%) the current changes from 38 to 100 (a change of 250%), which is a ratio of 10 : 1. Will the authors please comment?

I strongly support the authors' view that private generation in these conditions is economic and worth while. Where there is large steam usage and good electrical load there are great economies in it. But why do they not use pressures greater than 385 lb/in^2? It has been possible for quite a few years to have boilers of this size working at 900 lb/in^2 without any of the complications they mention. The authors could probably obtain 6 MW with a pressure of 900 lb/in^2, or, at 600 lb/in^2, at least much more than their 3 MW, and it would pay handsomely. One of their stated reasons for not doing so is the fear of trouble with boiler feed water when high make-up is involved. In a station which I ran a few years ago they are now running at a pressure of 1700 lb/in^2 with at least 80% make-up feed water, without any trouble at all on the feed-water system. The boilers are much larger than the authors', but I think that the authors could safely have gone to 900 lb/in^2 without any trouble.

It seems a pity that no use is made of the hydrogen and oxygen given off. The great strength of the integrated chemical industry is that the by-products and end-products of one plant are used as raw materials in neighbouring ones, leading to efficient and profitable combinations. Both hydrogen and oxygen are in great demand and the extremely pure forms from electrolytic plants have an enhanced value.

Dr. W. G. Thompson: The growth of the electrochemical industry has been due to widespread applications of mercury-arc rectifiers, but these are now being superseded by the more efficient semiconductor types.

In Section 9 the authors imply that electrochemical methods may be replaced by purely chemical means of producing hydrogen peroxide. A semiconductor-rectifier plant could be expected to show an annual saving of from £8000 to £10000 compared with the existing equipment. Would this saving make the electrochemical process competitive with the chemical one?

Analysis of Table 4 shows the backfire rate to be about 0·25 per cylinder per year for the first group, and 0·6 per cylinder per year for the second group (with the loadings increased), which compares favourably with similar information from the United States and the Continent.

Nominal ratings of rectifiers are based on B.S. 1698 class II, but in the present case they have been run successfully at something like a class V rating but without the normal cylinder derating for electrochemical service.

In these circumstances the statistical probability of backfire can be accounted for by known physical effects. Rectifiers similar to those mentioned in the paper have operated in traction plants for many years without backfires.

Grid control could have contributed to voltage and current regulation, but free-firing rectifiers were used for simplicity of design and operation. In retrospect do the authors agree with the choice?

The constant loading of the plant facilitated the harmonic studies and measurements which enabled it to be exonerated from certain harmonics produced elsewhere in the network.

Mr. D. P. Sayers: The paper is of considerable interest from the supply point of view because it describes an electrochemical process peculiarly well-suited for integration into a public electricity supply system. The cost of current is about 50% of the total cost of production so that the economics of electricity supply are extremely important. The authors point out that the plant can be shut down at any time for about two hours without suffering damage, and this should enable the public supply undertaking to quote unusually favourable terms for a supply on a restricted-hours or a disconnectable-load basis. On the latter basis the consumer might get a useful reduction in return for accepting a risk of disconnection, but the actual interference with his production would be negligible.

The authors say that the electrolytic method of producing hydrogen peroxide is being superseded by more economic chemical methods. Can they give figures of the present and potential market for this product and some idea of the relative costs involved in the two methods? If the differences are only marginal the electricity supply industry ought to consider very carefully what could be done to encourage additional load of this character where the characteristics appear to be so exceptionally favourable.

Referring to Section 7, the method of costing electricity seems a little devious. The most straightforward method is to compare the overall operating costs, including capital charges, with and without private generation. A true comparison of the merits of public and private generation must be assessed over the whole life of the plant. The private owner having built his station has to live with it for the rest of its economic life, but the public supply system has the advantage that new and more efficient plant is constantly being brought in and the older and less efficient plant relegated to reduced working so that the average works costs are declining all the time.

As an example, over three years the delivered fuel costs to a particular station increased by some 16%, but charges to the Area Board during the same period increased by only about $3\frac{1}{2}$% owing to newer and more economic stations coming into operation.

Mr. C. W. Hayes: The authors mention that in the private generation of power, importance is attached to back-pressure turbines. Such turbines can be designed which are rugged in construction and will fulfil the requirements of high efficiency, extreme reliability and relative simplicity. These factors, together with the saving made possible by using process steam, make out a strong case for the private generation of power.

A higher cycle efficiency could have been obtained by using a high-speed steam turbine and reduction gear. With the turbine steam inlet and exhaust conditions quoted, an increase of turbine efficiency of 5%, and hence power output, will result in a loss in process steam output of only approximately 1%. This is owing to the high proportion of latent heat in the exhaust steam.

Preliminary calculations show that, by installing a high-speed geared turbine and raising the steam temperature by a further 50°F, approximately 10% more power can be produced. Alternatively, if the boiler pressure is raised at the same time from 385 to 500 lb/in^2, the power increase will be of the order of 18%.

Admittedly there will be an extra charge for fuel, boiler and steam turbine plant to suit the higher conditions, but it is estimated that additional savings of £5500 for 385 lb/in^2 and £12400 for 500 lb/in^2, at a steam temperature of 750°F, can be effected. Considering these figures against the increased capital investment needed to obtain the higher steam conditions, the returns on capital will be $32\frac{1}{2}$% and 35%, respectively.

A pass-out condensing turbine would offer great advantages. It would permit flexibility in balancing process steam and electrical load requirements and also would conserve some of the feed water. Where process steam is needed at two different pressures, a pass-out back-pressure turbine would meet the requirements.

Mr. K. C. W. Pedder: The authors have referred to hydrogen peroxide as being used as a source of stored energy, and mentioned that the efficiency is rather low. Taking figures given in the paper, and comparing the amount of electricity required to make hydrogen peroxide with the amount of heat liberated on decomposition of the compound, the conversion efficiency is only of the order of 10%. Why is it so low? Possibly it is because formation and decomposition are not truly reversible reactions.

In Figs. 1–3 the flow of anolyte and catolyte is in the same direction. In most chemical processes where there is an interchange of material between one liquor and another it is usual to employ counter-current flow. Would there be any advantage in this instance, and, if so, why has it not been used? The effect of counter-current flow would be to maintain a constant difference in ion concentration between the two streams.

The figures given for battery efficiencies imply a knowledge of direct-current consumption to a degree of accuracy considerably higher than is common in, for instance, distribution systems. Knowing the difficulties of continuous and accurate measurement of fairly high direct currents, can the authors give some information about the methods used to monitor continuously the efficiency of this plant?

The large installation of rectifiers, working under similar conditions, should give an excellent opportunity for statistical investigations into the prevalence of backfires, etc.; have any such investigations been made?

Mr. J. W. Binns: In Section 1 mention is made of two alternative processes. How do they compare with the electrolytic process, and what are the chances of their being used commercially?

With regard to the four cascades which form one battery, how often do the authors find it necessary to take out one cascade for maintenance purposes?

In Fig. 9, where the rectifiers feed a d.c. busbar, are there any circuit-breakers on this d.c. switchboard? What is the fault level on the busbars?

From Fig. 12 there are two $7\frac{1}{2}$ MVA main transformers which look after the 12 MVA load. I can understand that if one of these transformers is faulty it is necessary to reduce the load, but I am concerned about the need for load reduction with the loss of one of the four 6·6 kV feeder cables. It would have been equally economical to design the system with four feeders, three suitable for carrying the full load.

Why was it decided to use 6·6 kV? 11 kV would seem better, particularly if future extensions are ever necessary. Another point is the provision of two 6·6 kV switchboards; these could have been combined to form one complete switchboard. If it is necessary to reduce the load it might be better policy to operate the second switchboard as separate sections. I was surprised at the type of protection provided on the 6·6 kV feeders. Why not use some form of unit protection? In reducing load, inter-tripping is mentioned. Does this take place on the h.v. supply to the rectifiers?

In the electrolytic plant itself the overhead lighting presumably has special consideration. Have the authors used pressurized fittings? On the question of the location of the switchgear, the fault capacity of the 6·6 kV switchboards is 250 MVA, although the actual fault level is presumably considerably lower than this. The switchgear seems to be housed along with some of the generating equipment in one room without fire segregation. The 250 MVA switchgear could provide an operational danger if locally controlled. Is there a remote-control board?

Mr. J. J. Thompson: I note with surprise that only two quantities are automatically controlled, namely cell current and maximum demand. Surely there is scope for automatic control of the process as a whole in order to maintain the quality of the end-product; was such an arrangement considered?

No mention is made of the means adopted for earthing the alternator star-point. If the star-point is in fact permanently earthed, presumably through a resistance, it would be interesting to learn whether trouble has been experienced with third-harmonic circulating current, as this provides a common operational difficulty in such cases.

THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

Messrs B. E. A. Vigers and **R. O. Fletcher** (*in reply*): Adequate spacing between cascades and the prohibition of long metal implements in the battery house have satisfied the factory inspector regarding the use of 500 volts d.c. Higher voltages are in use for modern electrolytic plants in this country and we feel that Mr. Hogg has been too conservative in his choice of voltage. A comprehensive planned maintenance scheme is in force under which each cascade is overhauled every two years. Splash barriers and other safeguards receive more frequent attention as found necessary. Fault level on the d.c. busbars (Fig. 9) is calculated at 56 kA, ignoring resistance of bars and connections. Circuit-breakers are fitted on rectifier and battery feeders for this section and faults have been cleared without excessive burning of contacts, although time discrimination has not proved very reliable.

High-efficiency semiconductor rectifiers were not available when this plant was designed, but the saving mentioned by Dr. Thompson would not alone make the electrolytic method competitive with the chemical process. We regret that policy reasons make it impossible to discuss the relative costs of the electrolytic and chemical processes. Grid-controlled rectifiers would have permitted a finer degree of current regulation, but at the expense of increased harmonics, and we believe that the simple free-firing rectifier with stepless regulator should have been adopted throughout.

With regard to the high-voltage distribution scheme, the original supply was only available at 6 kV, although, given a free choice, we should have preferred 11 kV, as suggested by Mr. Binns. The provision of two 6 kV switchboards is in line with the requirements of the supply authority, who prefer to avoid joint ownership of equipment and wish to meter near their 33 kV injection point. It is admitted that the protection system is very simple, and the trouble-free experience has proved this to be sound policy. Balanced protection was considered for the four incoming feeders, and pilots were laid but so far are not in use. Factory ring-main systems are run open at one point, and therefore all load-sharing currents flow through the section switches as planned. Reactive circulating currents between the two 7·5 MVA transformers can occur only with faulty selection of tap changing on these transformers.

Load shedding is carried out by tripping the h.v. supply to the rectifiers. Full load can be resumed on three feeders after a check on generation and load balance. This also explains the reason for cross-connecting the feeders. All 6 kV circuit-breakers are equipped for remote control either from power house or battery control boards. The alternator star-point can be earthed direct or through a resistance during synchronization. Third-harmonic currents have not been troublesome on this particular machine.

Mr. Pedder is correct in his suggestion that the low energy-storage efficiency is due to the non-reversibility of the reactions, which can be simplified to the following equations:

$$\text{Formation}\quad 2H_20 \rightarrow H_20_2 + H_2$$

$$\text{Decomposition}\quad H_20_2 \rightarrow H_20 + \tfrac{1}{2}0_2$$

No reliable method of metering the d.c. power to the batteries has been found, although pendulum meters and other methods have been tried. At present it is considered sufficient to meter the a.c. side and assume a constant conversion efficiency.

Regarding the collection of hydrogen and oxygen, the type A cell is not entirely suitable, although designs such as type B can be used for the purpose. Adequate ventilation assures the dispersion of gases without recourse to special lighting fittings. The type B design will also allow counter-current flow of electrolyte, but no definite advantage has been established.

Automatic control of battery voltage and current has been found sufficient at this stage of the process. The relation between voltage and current deduced from Fig. 4 refers to the working-point of the curve in the region of 100 amp per cell. On the physical side, conditions are such that open-loop control is perfectly satisfactory, but we can assure Mr. Thompson that throughout our organization full use is made of automatic process control.

The improved financial return by private generation at higher steam pressures has been apparent to us for a long time, and in retrospect we would now prefer 600 lb/in^2 or even 900 lb/in^2. It is to be expected that the high-speed geared turbine at these inlet pressures will show the savings quoted by Mr. Hayes. As pointed out by Mr. Sayers, the private owner has to live with his earlier decisions, but it should be remembered that as the plant gets older it also appears at a lower written-down figure for capital charges. It is interesting to hear this speaker refer to the unusually favourable terms that should be given for the main supply, e.g. off-peak and disconnectable load basis. Great efforts have been made by us to obtain such terms, but so far only the off-peak tariff has been allowed, and that only recently.

621.313.333.4

The Institution of Electrical Engineers
Paper No. 3273 U
Oct. 1960

FURTHER DEVELOPMENTS OF THE SELF-OSCILLATING INDUCTION MOTOR

By E. R. LAITHWAITE, M.Sc., Ph.D., Associate Member, and G. F. NIX, M.Sc., Student.

(*The paper was first received* 30*th November*, 1959, *and in revised form* 26*th March*, 1960.)

SUMMARY

The self-oscillating induction motor described in an earlier paper, in which two linear motors placed back-to-back are capable of producing a stable amplitude of oscillation of a moving runner without any switching device, cannot be made to operate in small sizes without careful design. Among the possible applications of such a system a traverse mechanism for textile package winders appears to be the most attractive. One of the principal requirements of traverse mechanisms is that the oscillating member should be as small as possible.

The paper describes the development of small self-oscillating motors of several types. In some of these the rotors or runners contain iron, while in others they consist of slabs of conducting material. The system is found to operate under transient conditions at all times, and it has not been possible to establish a complete theoretical analysis. Nevertheless, an experimental approach has indicated some of the rules by which a small oscillating motor may be designed. Rotors of the order of only 30 g mass have been made to oscillate successfully, and speeds of over 500 traverses/min over a 14 in length have been achieved with power input of the order of 100 watts.

LIST OF SYMBOLS

a = Acceleration.
a_m = Maximum acceleration.
e = Coefficient of restitution of end springs.
F = Force.
f = Supply frequency.
g = Acceleration due to gravity.
l_g = Air-gap length.
J_s = Stator surface current density.
K_g = Ratio of air-gap to thickness of conductor.
n = $1/\tau\omega$.
p = Pole-pitch.
s = Distance travelled.
u = Linear velocity of oscillating member.
u_s = Synchronous speed of travelling field.
w = Stator width.
μ_0 = Permeability of free space.
ρ_r = Equivalent surface resistivity of the rotor.
σ = Fractional slip.
τ = Rotor time-constant.
ω = Angular frequency of supply.

(1) INTRODUCTION

In an earlier paper[1] it was shown that, if two polyphase linear motors are placed end-to-end with their field velocities directed towards each other, as shown in Fig. 1(*a*), a mobile conducting element or 'runner' under the influence of the system can be caused to oscillate with a stable amplitude, provided that its speed/force curve is of the form shown in Fig. 1(*b*), i.e. the peak of the curve lies to the right of the origin. A theory of this type of oscillation was developed on the assumption that the steady-state equations of the conventional induction motor apply, and reasonable agreement between theory and practice was realized.

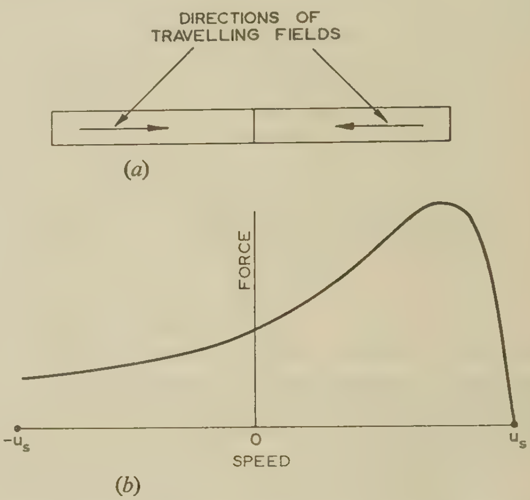


Fig. 1.—Principle of the 'back-to-back' linear motor.
(*a*) Arrangement of tracks.
(*b*) Speed/force curve of an induction motor.

The possible application of the system to weaving looms was discussed, and the paper concluded by saying that it had not been possible to assess at that time whether or not the device could be economically incorporated into a loom. After further investigations it appears that the initial cost would be prohibitive in all but possibly the very wide looms used for making carpets and paper-makers' felts. During discussions on the matter with textile engineers it was suggested that the self-oscillating motor might be useful in another branch of the textile industry, namely package winding. A package of yarn is made by winding the yarn on to a core of cardboard or similar material which may be cylindrical or conical in shape. The main feature of such packages is that the yarn is not supported by end-cheeks, as in the case of the household cotton reel or a reel of wire. For such an application the motor required would move the eyelet which guides the thread on to the package and would be much smaller than the motors which were envisaged at the time the earlier paper was written. It was also thought possible that a small reciprocating device devoid of cams or linkages might be of interest in other branches of engineering, and to these ends the present work was begun. The particular requirements of a package winder were kept in mind throughout the work, and the problem appeared always to be that of obtaining a stable running system with a smaller moving part and a lower power input. As the work proceeded, machines were developed which could be useful at somewhat higher powers than those required in package winding. The development work is described in a more or less historical sequence.

(2) THE PROBLEM OF THE TRAVERSE MECHANISM IN PACKAGE WINDERS

In winding wire, cotton or other yarns on to bobbins by mechanical means, two separate motions are employed. First,

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.
Dr. Laithwaite and Mr. Nix are in the Electrical Engineering Laboratories, University of Manchester.

the bobbin on which the material is being wound is rotated, and secondly, the thread or wire is guided so as to ensure a neat packing of the material, layer upon layer. This second motion is essentially reciprocating. Packages of yarn are often wound with no supporting end cheeks, so that the yarn may be pulled off in an axial direction at high velocity, and to assist in this process packages are often conical rather than cylindrical. Stability of such types of package is achieved by making the side-to-side traverse of the thread guide sufficiently rapid that the traverse time, end to end, is of the same order of magnitude as the time of revolution of the package. In winding a cylindrical package the thread guide must spend the same amount of time in all positions along the traverse, and therefore the ideal motion is one of uniform velocity across the surface of the package in both directions with zero reversing time at each end. In winding conical packages the thread guide may be required to increase its velocity as it moves towards one end of the traverse, so as to wind less yarn at that end.

Package winders may be divided into two classes, namely

(*a*) Those in which the traverse motion is linked or synchronized with the spindle motion so that the thread is laid in a particular desirable pattern.

(*b*) Those in which the thread is laid on the package in a random manner; in this class of winding it is necessary to ensure that no accidental synchronism between the two motions occurs, which might result in undesirable 'patterning'.

Traverse mechanisms for package winders may also be divided into two classes:

(i) The thread passes through an eyelet in a thread guide which is driven mechanically, usually by a cam-operated mechanism.

(ii) The thread passes over a roller which carries a helical groove. The thread runs in the groove and as the roller revolves the thread is moved laterally by the sides of the groove. Fig. (2) shows a traverse roller.

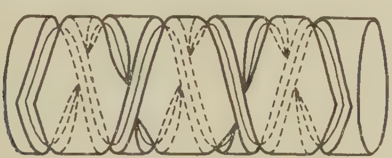


Fig. 2.—Traverse roller for a package winder.

There is no doubt that the second class of mechanism achieves one ideal of such a system in that there is no reciprocating mass requiring large reversing forces at each end. Only the inertia of the thread must be reversed. The roller type of traverse mechanism, however, has certain disadvantages. A comparatively high tension on the yarn is necessary to keep the thread in the groove in many cases, which prohibits the use of the method for fine yarns. A redistribution of twist may also occur through the rolling of the yarn on the sides of the groove, and this effect is liable to introduce periodic twist variations which, with filament yarns, could easily cause patterning in woven cloths. Furthermore, the yarn rubbing on the sides of the groove is liable to be damaged by abrasion. For these reasons, cam-operated rather than rotating-drum types of traverse mechanism are generally used with filament yarns.

The disadvantage of the cam-operated traverse is that it can be used only for comparatively low speeds. As the traversing speed is increased the cam life falls rapidly, owing to the high wear at the reversing positions caused by the large forces required to accelerate and retard the oscillating mass of the thread guide. Limits are therefore imposed by the impact properties of the material used for the cam or its follower.

For comparison, rotating-drum mechanisms are used for speeds up to 2200 traverses/min. The yarn is wound on to the package in these machines at approximately a mile a minute. Cam-operated winders rarely achieve more than 600 traverses/min. The fact that it takes hours to wind one ounce of 15-denier nylon on a commercial winding machine at 600 traverses/min serves to emphasize the need for a high-speed mechanism carrying a thread guide. The problem is accentuated for cam-driven systems in which the package is long, since the cam and its associated equipment tend to become very large.

A recent patent[2] described a package winder in which the thread guide was driven by a linear induction motor. Reversal of drive was obtained by reversing two phases of the supply to the motor stator, and a rapid reversal at each end was obtained by allowing the runner or 'rotor' to impinge on stiff springs. The use of a switching device introduces new problems, notably those of synchronizing the mechanical oscillation to the switch-operating mechanism and the maintenance of the switch itself, bearing in mind that it may be called on to switch a highly inductive circuit perhaps 20 times per second. This equipment is complex and expensive, but it contains some of the essentials of the system about to be discussed, namely a mechanical arrangement consisting of a mass moving between two end-springs and an inductive method of supplying the friction, windage and end-spring losses during the traverse. The self-oscillating induction motor appears to offer several advantages over the switched system.

(2.1) The Requirements of a Traverse Mechanism

The linear induction motor is clearly not suitable for a synchronized traverse but appears to be quite well suited to winding non-patterned cylindrical packages, although it is possible that it could also be adapted to cone winding. In the first instance, however, it will be assumed that it is to be used to wind a cylindrical package. The principal requirements are therefore as follows:

(*a*) The thread guide should travel at approximately constant speed throughout the traverse and be reversed in as short a time as possible.

(*b*) The length of the traverse should be constant, but the machine should be capable of being easily reset to wind a different length of package.

(*c*) The speed of the traverse should be readily controllable.

(*d*) The oscillating mass should be sufficiently small that excessive vibrations in the structure caused by the reversals are avoided.

(*e*) Parts which are liable to wear out should be cheap and easy to replace.

(*f*) The cost of the whole mechanism should be reasonably competitive with existing mechanisms.

(*g*) The power consumed should be small (no more than, say, 200 watts).

(*h*) If possible, the motor should be capable of operating directly from 50 c/s mains supply.

The experimental procedure adopted in this investigation was carried out with all the above requirements in mind.

(3) THE SELF-OSCILLATING INDUCTION MOTOR

It was shown in the original paper[1] that, so long as the peak of the speed/force curve occurs at a slip of less than $0\cdot5$, the oscillations will build up to such an amplitude that the maximum velocity attained by the rotor, which occurs as it crosses the centre of the track, is greater than 90% of the synchronous speed. It was also shown that the average speed is about five-eighths of the synchronous speed under these conditions. The rotor spends over half its time in the 'plugging' condition, with a slip greater than unity and is liable to become overheated. It can be shown that the energy dissipated as heat in the rotor in one traverse is approximately equal to four times its kinetic

energy as it crosses the centre. The free-running oscillating motor is clearly unsuited to a traverse motion.

The operation of the self-oscillating motor under the action of end-springs has been mentioned briefly in a recent paper.[3] Provided that the end-springs have a very high coefficient of restitution, it appears that the system could be made to operate on a cycle such as that represented by ABCD on the speed/force diagram of Fig. 3, in which the negative-speed portion of the

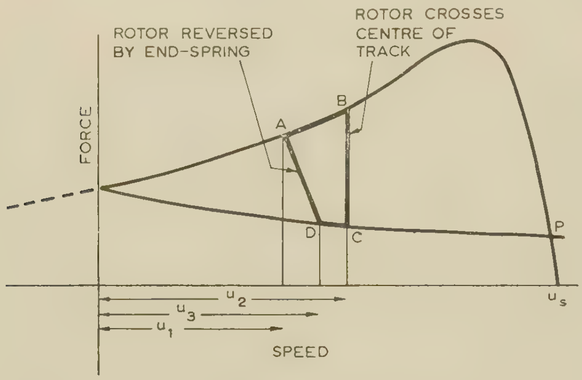


Fig. 3.—Cycle of operation of a self-oscillating motor.

curve has been plotted as a mirror image for convenience. The coefficient of restitution, e, is seen to be u_1/u_3 in this case, and it appears that if e is near to unity the speed should go on increasing until the cross-over point, P, is passed. The variation in velocity should then be small and the system should run ideally at between 90 and 100% of synchronous speed. The only conditions which must be met are that the peak of the speed/force curve should occur at a slip of less than $0 \cdot 5$ and that the end-springs should be efficient energy stores.

A conventional induction motor fed from a constant-voltage source is known to have peak torque at a value of slip given by R/X, where R and X are the rotor resistance and leakage reactance per phase (ignoring the stator impedance). This suggests that to make a more effective oscillating motor the rotor leakage reactance should be increased. To do this, however, is to make a *worse* machine electrically. If the same motor could be arranged to work at constant current (for example by connecting it to a high-voltage source through a high impedance) the speed/force curve would have a much greater ratio of peak value to standstill value and the peak force can be shown to occur at a value of slip, σ_m, given by[3] $\sigma_m = 1/\tau\omega$, where τ is the rotor time-constant. In this case, more effective oscillation is achieved by making the motor *better* electrically. The rotor time-constant can be evaluated in the first instance in terms of the parameters of a somewhat idealized machine with no stator losses or leakage reactance. When this is done, the criterion $\sigma_m < 0 \cdot 5$ becomes

$$\tau\omega = \frac{8p^2\mu_0 f}{\rho_r l_g} > 2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In a current-fed machine, the better the machine, the smaller is the slip at which maximum force occurs and the bigger the ratio of peak to standstill forces.

One of the many differences between a self-oscillating motor and the conventional rotating induction machine is that the rotor occupies but a small fraction of the stator surface. If the coils of the stator are all connected in series, the high magnetizing impedance of all the stator coils which are not opposite the rotor causes the active part of the machine to run at a virtually constant current at all speeds. A series-connected stator has been found to be far superior in producing stable oscillation to a parallel-connected one for this reason.

(3.1) Construction and Operation of the Linear Motor

Various possible forms of construction are shown in Fig. 4. Fig. 4(*a*) is a single-sided motor in which the inactive coils operate on an open magnetic circuit with very high reluctance.

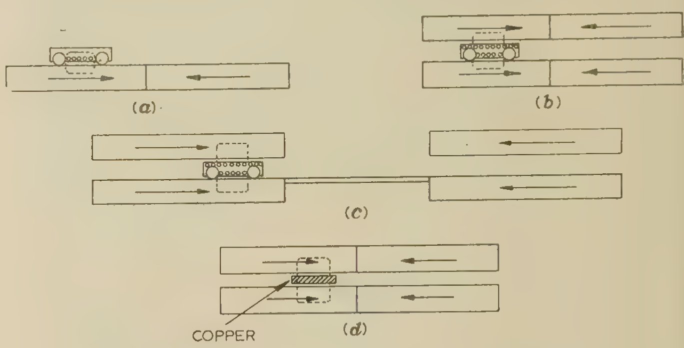


Fig. 4.—Types of linear oscillating motor.
(*a*) Open-sided.
(*b*) Double-sided.
(*c*) Double-sided with central gap.
(*d*) Double-sided with non-ferrous rotor.

Although the rotor contains iron to close the magnetic circuit around its conductors, this system is not a good one for two reasons: first, there is a very large magnetic attraction between rotor and stator, so that the rotor mechanical arrangement is under great pressure, and secondly, the inactive stator coils do not constitute a sufficiently high impedance to produce true constant-current operation. Fig. 4(*b*) shows a double-sided motor with slotted iron rotor; the first disadvantage is now removed and so to some extent is the second, provided that the air-gap between the inactive coils is reasonably small and the coils in upper and lower stators are connected in like senses to drive flux across the gap (i.e. north pole faces south pole). Fig. 4(*c*) is similar to Fig. 4(*b*), but a gap has been introduced between the oppositely moving fields, so that the rotor is not entirely under the influence of stator m.m.f.'s throughout its traverse. Fig. 4(*d*) shows a double-sided stator with a rotor which consists merely of a solid slab of conductor.

To illustrate the complex nature of the problem of studying the build-up of rotor oscillations, a system such as that shown in Fig. 4(*c*) is first considered. If the gap between the two oppositely directed fields is so large that the rotor has time to become demagnetized between leaving one block and entering the other (the operative time-constant of flux decay in this region is the rotor leakage time-constant, which will, in general, be very short), the situation as the rotor enters is similar to that of an arch motor in which unmagnetized rotor iron is being continually fed into a short stator block. The resulting behaviour of such a system under the action of the transients set up at the entry edge has been analysed in some detail,[4] the analysis resulting in somewhat complicated expressions for such relationships as speed/torque. A similar situation arises as the rotor leaves a stator block, and this, too, has been analysed;[5] the transients arising from such stator discontinuities are found to have a profound effect on performance at small values of slip. The system shown in Fig. 4(*c*) is different from the arch motor, however, in that the rotor is also discontinuous. It was believed that transient phenomena resulting from rotor conductor discontinuities are less potent than those resulting from stator discontinuities,[3] provided that the rotor is at least a pole-pitch in length. In this case, however, purely magnetic forces will also

occur as the rotor iron leaves or enters the stator blocks, since the rotor iron, as well as the rotor conductor, is discontinuous.

As an example of the kind of phenomena which can occur, the behaviour of an early experimental model of an oscillating motor, built according to Fig. 4(*a*) but having a central gap, will now be described. The flux in the end tooth nearest the centre of a stator block from which the rotor was emerging was naturally a pulsating one. If the rotor emerged at the instant the flux was a maximum, a very large braking force occurred. If, on the other hand, the flux was zero, the rotor passed out unimpeded. When the retarding impulse was large, the rotor would not proceed far into the reversed field before coming to rest. The resulting amplitude of oscillation was modulated at a frequency which was determined by the relationship between the frequency of the supply and the natural frequency of the oscillation. Shorter rotors produced a much greater depth of modulation, since the same check-force had a much greater decelerating effect on the smaller mass.

The transients occurring in machines built according to Figs. 4(*a*), (*b*) or (*d*) with no central gap are certainly different from the short-stator transients of the arch motor, but nevertheless equally important in determining behaviour. Further discontinuities occur when end-springs are introduced and the drive torque immediately after reversal by a spring is likely to be very different from that calculated by conventional induction-motor theory. Discontinuities apart, the rotors of self-oscillating induction motors are generally accelerated and decelerated very rapidly, so that conventional steady-state theory may not apply, even when the rotor is moving wholly within a stator block and discontinuity transients are ignored. In this connection the work of Treschev[6] is of considerable value.

The self-oscillating motor is thus seen to be one of the most complex of electromagnetic mechanisms, and at the time the development work on package winders was first proposed it seemed as if any form of rigorous analysis would be virtually impossible. The work therefore began with a long series of experiments to collect together a mass of information which would enable a package winder to be designed by optimizing one parameter at a time. Some of the more important of these experiments are now described.

(4) EXPERIMENTS WITH SELF-OSCILLATING MOTORS

The requirements listed in Section 2.1 give a reasonably good indication of the form of the device required. Items (*a*) and (*b*) can be met by the use of end-springs which cut down the amplitude to a very small fraction of the natural amplitude to ensure reasonably constant speed. Traverse length may be adjusted simply by the positioning of the springs. Items (*d*)–(*g*) all point to the fact that the system should be small and in particular that the moving mass should be as small as possible. Opinion as to possibilities of complying with items (*c*) and (*h*) could not be given until several workable systems had been examined.

The first aim was to produce a device capable of giving a reciprocating drive over a 6 in traverse at a speed approaching perhaps 1 000 traverses/min. The average linear speed of such a system would thus be about 8·3ft/sec, which with a 50c/s supply would require a pole-pitch of perhaps 1·5 in, allowing for some slip. The first machine built was of the type shown in Fig. 4(*b*). The total stator length was 12 in and the stator slots were ¼ in wide with a slot pitch of ½ in. A 1 slot/pole/phase winding on this structure gave the necessary 1½ in pole-pitch and the stators contained 4 pole-pitches on each side of the centre-line. In an effort to minimize transients due to short-rotor effects the double-sided rotor was made as long as possible

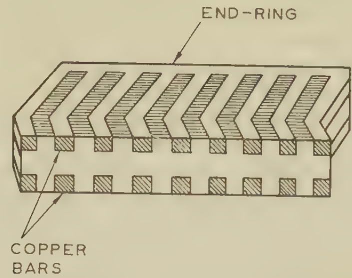


Fig. 5.—Arrangement of bars in a herring-bone rotor.

so that it contained several pole-pitches. The end-springs were located 1 in from each end and the rotor was 4 in long, so that a 6 in traverse was possible. The slots in the rotor iron were cut in a herring-bone pattern to avoid magnetic locking, and the angled copper bars were brazed to end-rings as shown in Fig. 5. It was found that this rotor produced only damped oscillations, indicating that the speed/force curve was not of the right shape and the rotor time-constant inadequate. A new rotor was made with the same overall dimensions but without the central plane of iron. This was done by starting with a solid block of copper, cutting herring-bone slots right through it and filling them with packets of laminated iron impregnated and baked to hold them in place. This rotor was found to have adequate time-constant and it oscillated. Thinner rotors were tried without success, showing that the time-constant was very little above the critical value. The discouraging things about the first machine were that the rotor had to be made large before an adequate time-constant could be obtained (the only successful rotor weighed 2 lb), and that a very large stator current density was necessary to produce oscillations which approached even one-third of the synchronous speed. In fact, at 900 traverses/min the stator rating was about ½ min, although the slots were 1 in deep. The device consumed over 2 kW. The only encouraging result was that the speed was controllable by adjustment of supply voltage. The failure of the rotor to approach synchronous speed may, it was thought, have been due to a poor mechanical arrangement. The rotor carried four wheels which ran in grooves along the sides of the air-gap, and any tendency of the rotor to twist produced excessive frictional forces.

The system was therefore redesigned with a better mechanical arrangement. The rotor was now carried on a long arm which was pivoted at the end remote from the rotor. The rotor carried two wheels which ran in a circular groove, the amount

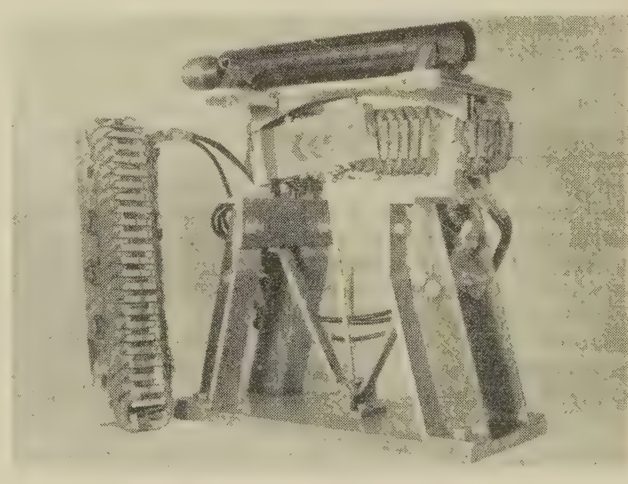


Fig. 6.—Experimental package winder.

of rotation about the pivot being limited to such a small arc that the rotor could still be contained within the linear stators, which were now arranged with the air-gap in a vertical plane. The completed machine is shown in Fig. 6, with one half of the stator removed to display the rotor and end-springs. This system was found to crawl. The crawling was thought to be due to the lining-up of half of each rotor slot with the stator slots when the rotor was in the extreme positions, although this was not completely established. Rotors were then manufactured by filling a copper slab with iron rivets, carefully spaced to give no preferred positions, and these rotors oscillated quite well, attaining speeds of 900 traverses/min over a 4 in traverse. Although the power consumed at this speed was smaller than in the previous machine, it was still far too high for continuous rating (1200 watts), and the smallest rotor which could be made to operate, after variations in length, thickness and number of rivets had been tried, weighed just over 13 oz and is shown in Fig. 7.

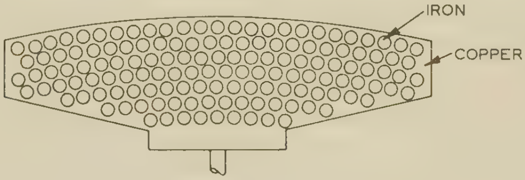


Fig. 7.—Arrangement of iron rivets in a rotor.

(4.1) Rotors Containing no Iron

At the same time as the experiments on the machine shown in Fig. 6 were being carried out, a second series of experiments was undertaken with a machine of the type shown in Fig. 4(*d*), which appeared to have some advantages. Eqn. (1) shows that, for a given pole-pitch and supply frequency, i.e. a given synchronous speed, the highest value of τ is given by the minimum product of rotor resistivity and air-gap length. The normal method of obtaining a low value of $\rho_r l_g$ in any kind of induction motor is to employ a slotted structure in which large cross-sectional areas of copper can be embedded, the only penalty on air-gap being to multiply the actual gap by Carter's coefficient. However, if all the rotor conductor is contained within the air-gap itself, $\rho_r l_g$ is independent of l_g provided that a constant fraction of the air-gap is filled with copper. If $p^2 f$ is large enough to satisfy eqn. (1) for one value of air-gap it will do so for any other value, which suggests that very thin, light rotors might be made to oscillate. For a given synchronous velocity $p^2 f$ is proportional to p, and increased time-constants may be obtained by increasing p and reducing f, even though the product pf is constant.

Eqn. (1) was obtained by considering an ideal machine with no rotor end-connections. If end-connections are taken into account, the ratio of pole-width to pole-pitch must be taken into account and the type of end-connection used will affect operation. The flow lines of end-ring current in an induction-motor rotor which consists of a copper cylinder may be plotted by an experimental technique due to Russell and Norsworthy,[7] and it appears that so long as the copper extends about one-half of a pole-pitch on either side of the stator, the effective resistivity may be calculated approximately as $[\rho_r(2p + 3w)]/3w$. ρ_r is the surface resistivity of the active conductor under the stator. If the ratio of the air-gap to the thickness of conductor is K_g, eqn. (1) is modified to become

$$\tau\omega = \frac{8p^2\mu_0 f}{K_g\rho_r\left(\dfrac{3w + 2p}{3w}\right)l_g} > 2 \quad . \quad . \quad . \quad (2)$$

To give some idea of the limits imposed on p in practice, a value of 1·25 for K_g and a stator width of 1 in together demand that, for 50 c/s supply using a copper rotor, p must be not less than $2\frac{1}{4}$ in.

An experimental machine was built in which as many parameters as possible could be changed. The two halves of the track could be separated, as in Fig. 4(*c*), over a range of distances, or be continuous, as in Fig. 4(*d*). The air-gap between upper and lower stator blocks was adjustable. The guide rails for the rotor could be moved in or out to accommodate different widths of rotor, and the end-springs could be placed at different positions inside the air-gap. Both the pole-pitch and the pole-width were fixed at 4·5 in to allow some latitude over the minimum value of $\tau\omega$. To avoid as many peculiarities as possible which are caused by transients, four poles were contained in the stator on each side of the centre, making the whole machine 3 ft long without centre gap. The method of assessing the effect of changes in parameters was to measure the value of $\tau\omega$ by the following technique.

The expression for the force, F, tangential to the rotor surface of a conventional induction motor run under conditions of constant current, ignoring leakage, is

$$F = \frac{\frac{1}{2}\rho_r J_s^2}{u_s}\left[\frac{\sigma}{\sigma^2 + \left(\dfrac{1}{\tau\omega}\right)^2}\right] \text{ per unit square of surface} \quad (3)$$

If the rotor is held at standstill and the supply frequency varied, the torque/frequency curve has a maximum value when $\tau\omega = 1$. Hence $\tau\omega$ at 50 c/s can be evaluated. Measurements of $\tau\omega$ were carried out to assess the effect of the following parameters:

(*a*) Rotor width.
(*b*) Rotor length.
(*c*) Rotor thickness.
(*d*) K_g.

Fig. 8 shows the effect of rotor width. Once the rotor is substantially wider than the stator, further increase in width gives

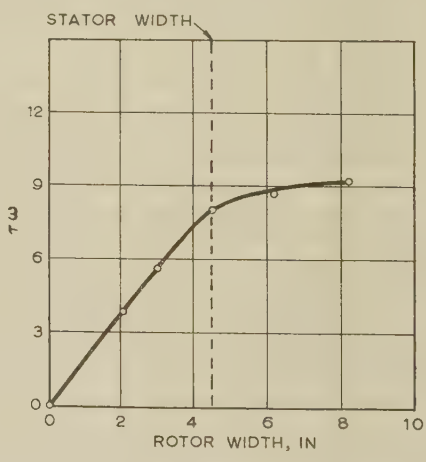


Fig. 8.—Effect of rotor width on effective time-constant.

only a marginal increase in $\tau\omega$, whereas serious reductions in this quantity occur as the rotor width is reduced below that of the stator.

Fig. 9 shows that reduction of rotor length also incurs a penalty, once the length becomes comparable with a pole-pitch.

The rotor discontinuities in a machine with a rotor containing no iron have a much greater effect on the performance than those of a machine with an iron-cored rotor. In the latter case the flux density outside the rotor is likely to be somewhat less

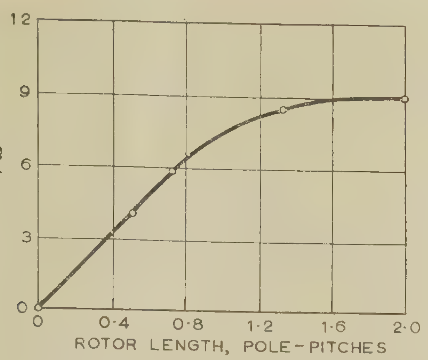


Fig. 9.—Effect of rotor length on effective time-constant.

than that inside, since the stator is series-connected. The flux density outside a rotor which contains no iron, on the other hand, is likely to be many times greater than that operating on the rotor. The leading edge of the rotor therefore cuts into a much stronger flux. The resulting transients have been theoretically investigated[8] for the case of the rotor with no iron. A similar investigation for the short iron-cored rotor is more difficult and it has not been possible to predict the performance in this latter case, except to say that it is not likely to be very different from that of a short-rotor machine with a parallel-connected stator and a rotor containing no iron. This latter case has also been investigated,[8] and it appears that rotor discontinuities are negligible for rotor lengths greater than 1·5 pole-pitches.

For rotors of shorter length the transient effects may be simulated by ascribing to the rotor conductor an increased resistivity which is a function of rotor length. For the series-connected stator the theoretical expression for the force is much more complicated and performance cannot be predicted simply by an effective increase in rotor resistivity. However, it may be stated generally that the speed/force curve is modified to an increasing degree as the rotor is made shorter and that these modifications are always such as to cause the performance as an oscillator to deteriorate. An increase in pole-pitch, although at first sight beneficial, can in fact produce effectively a lower value of $\tau\omega$, because of the combination of increased end-turn resistance and the short-rotor effect due to making a fixed size of rotor occupy less than a pole-pitch.

The effects of rotor thickness and K_g are perhaps best studied together. A very small gap between rotor conductor and stator surface increases the detrimental parasitic losses, and the best results are obtained when this gap is considerably larger than would be required from purely mechanical considerations. This being so, reducing rotor thickness only produces serious effects once the size of the gap becomes comparable with rotor thickness, so that K_g is seriously affected. In the experimental machine described the rotor thickness could be reduced without serious effect until purely mechanical considerations prevented further reduction, i.e. thinner rotors would have buckled on impact with the end-springs.

In the light of these experiments a second set of stator blocks of similar dimensions to the first, but only $2\frac{1}{2}$in wide, was built and a rotor measuring 6in × $4\frac{1}{2}$in × $\frac{1}{8}$ was made to oscillate successfully. Further reduction in length or width of the rotor caused it to perform damped oscillations only. Only one further weight reduction proved possible: it was found that slots could be cut across the active part of the conductor to make the linear equivalent of a squirrel cage. This removed 20% of the mass, and this rotor was the lightest conducting sheet which was made to oscillate, weighing just over 13 oz.

Attention was next turned to the problem of the cross-over point at the centre of the track. It was soon discovered that the introduction of a gap at the centre produced a deterioration in performance. The system was now set up without a central gap but with a series of different phase connections at the centre. For example, slot currents corresponding to the phases shown in Fig. 10(*a*) imply a zero phase change in current as the centre-

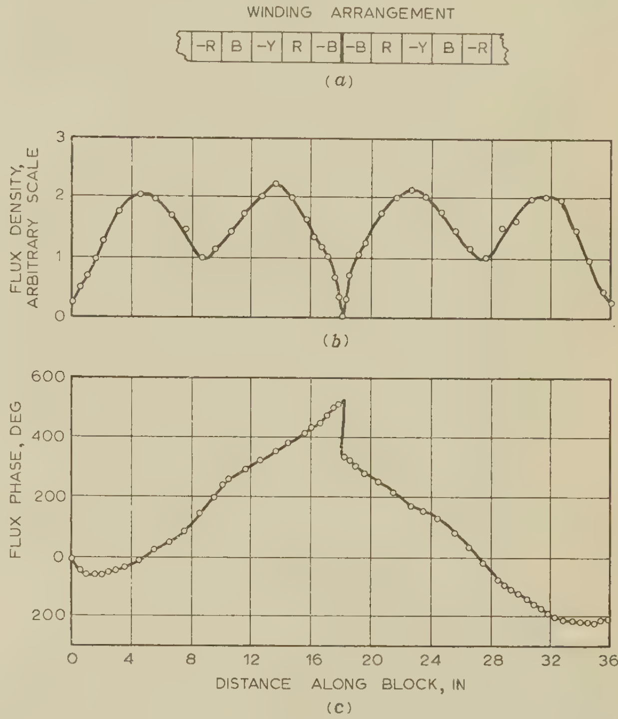


Fig. 10.—Flux distribution with a back-to-back system (no phase change of current at the centre)

(*a*) Winding arrangement.
(*b*) Flux amplitude.
(*c*) Flux phase.

line is crossed. A flux plot was made along the air-gap with the rotor removed, the magnitude and phase of the flux being shown in Figs. 10(*b*) and (*c*). Similar curves were plotted for other connections, a 180° change in current phase resulting as shown in Fig. 11. These last two Figures show the extreme cases. Other phase connections gave graphs which contained some of the features of both Fig. 10 and Fig. 11. The flux plots in both these figures are similar to those obtained by Shturman[9] and others,[10] the modulation being due to short-stator effect. As regards running performance, the 180° phase change was undoubtedly the better and a slightly higher speed of running was obtained with this connection. It was also found possible to construct a rotor which operated near the critical value of $\tau\omega$ which would not run at all on the zero-phase-change type of connection but would oscillate quite well on the 180° type.

The slotted rotor with the 180° connection was capable of producing 420 traverses/min over a length of $2\frac{1}{2}$ft, giving an average speed of 17·5 ft/sec, compared with the synchronous speed of 37·5 ft/sec. Friction and spring losses in this system were quite small. When the end-springs were moved inwards towards the centre so as to reduce the traverse length, the number of traverses per minute was almost constant over a considerable range, i.e. there was a reduction in average speed.

(4.1.1) Summary of Results of the Tests on Non-Ferrous Rotors.

While the results were disappointing in that the minimum weight of rotor achieved was still as high as $\frac{3}{4}$lb, some useful

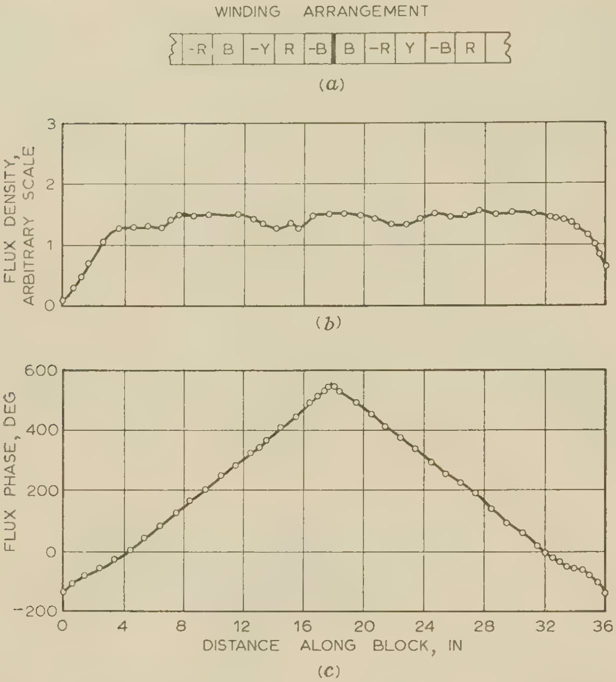


Fig. 11.—Flux distribution with a back-to-back system (current reversal at centre.)

(*a*) Winding arrangement.
(*b*) Flux amplitude.
(*c*) Flux phase.

information had been obtained, particularly about the method of connection at the centre. It was also clear that the stability criterion of eqn. (2) could not be met by increasing the pole-pitch alone. Increase in pole-pitch, if it is to be of value, should be accompanied by increase in width of both stator and rotor as well as increase in rotor length. Thus eqn. (2) appears to set a limit to the minimum weight of conductor which can be oscillated in this way. Such limitations are imposed by the resistivity of the best conducting material known and the permeability of the best magnetic material available.

Different forms of construction for the self-oscillating motor were now examined with a view to further weight reductions. It is possible that the two forms of motor which have now been described, namely the types shown in Figs. 4(*b*) and 4(*d*), may be useful in other applications and that the latter is even possible as a traverse mechanism for future developments in package winding where very large packages of coarse yarn are involved. The apparatus, however, would be fairly costly and the power consumption high. The 420 traverses/min over a $2\frac{1}{2}$ ft length was achieved only with an input of 5kW at a power factor of 0·3.

One surprising result which emerged was the low average speed attained by both the ferrous and the non-ferrous rotor types of machine, despite the use of good end-springs and a low frictional loss. It was found that speed control was easily obtained by supply-voltage adjustment. The speed increased as the voltage was increased until saturation of the iron was reached, causing the time-constant to become inadequate, when the oscillation became damped.

One other aspect of the non-ferrous rotor was disappointing: it was hoped that there might be a tendency for the rotor to 'float' between the upper and lower tracks, thereby reducing frictional forces. It was found that, in systems where the m.m.f.'s of the two stators are additive, the conductor is attracted to whichever stator it happens to be nearer. The mechanism of such attraction and, in fact, the whole problem of electromagnetic leviation is beyond the scope of the present paper. Despite this effect, the vertical forces on non-ferrous rotors are much less than those on iron-cored rotors.

(5) THE TUBULAR MOTOR

Tubular motors in which the flux is axial have not found many applications, despite their inherent advantage of eliminating all end-turns on both rotor and stator.[3] The iron cores of such machines are difficult to construct, but one form has been developed for use as a liquid-metal pump.*[11] In the case of a traverse mechanism, as with many other possible applications, the chief drawback of the tubular motor is the inaccessibility of the rotor. Some form of mechanical connection which is at least as long as the motor must be made axially, and this alone offsets the advantage of any possible reduction in the size of the rotor itself. There is no reason, however, why the principle of axial flux should not be exploited in a double-sided linear motor, in an attempt to eliminate one of the major difficulties in designing a small self-oscillating motor, namely end-turn resistance. Motors of the types shown in Fig. 12 were therefore built and tested, and such motors have been described as 'mock tubular style'.[12]

The stator windings are arranged so that north pole faces north pole and the flux is forced along the axis. Since the core of the rotor is likely to be small, e.g. $\frac{1}{2}$ in × $\frac{1}{2}$ in section, a large-pole-pitch stator will carry a very low flux density, because the limiting value of flux per pole is set by the rotor section. This implies that the power output will be low for the size of stator used, but this is not important for the package-winder application, where the sole object is to provide the reciprocating motion and the major part of the power supplied to the rotor

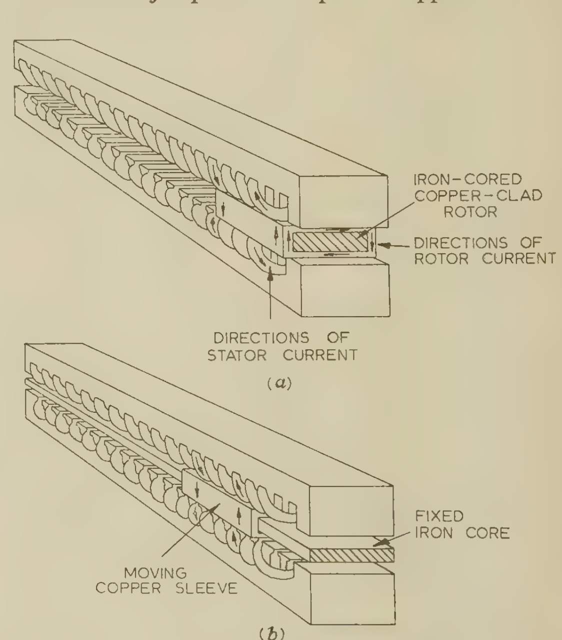


Fig. 12.—Linear motors of 'mock tubular' style.

(*a*) Moving-iron system.
(*b*) Moving-sleeve type.

is used in supplying end-spring and friction losses. The induced e.m.f.'s in the rotor cause circulating currents to flow, as shown in Fig. 12, and it will be seen that the only additional loss incurred by not adhering strictly to a tubular structure is the small end-

* The annular linear induction pump.

turn loss in the sides of the rotor conductor. Stator width in relation to pole-pitch is no longer a limitation. Instead, the stator width is related to the rotor core thickness, the end-turn resistance being equal to the active-length resistance for a square core section.

The first experimental machine built was of the type shown in Fig. 12(*a*), except that the whole motor was made as an arc of a circle of large radius, so that the rotor could be suspended like a pendulum from the centre of curvature. In this way frictional forces were reduced to a minimum, so that the behaviour could be studied without fear of the results being affected by load. The radius of the arc was so large that the natural period of the pendulum formed by the rotor was larger than the period of the induced oscillations, and the curvature of the rotor path had a negligible effect on the action. The stator width was ½ in and the total gap between the stators was also ½ in. The 180° current phase-change connection was used at the centre. A stator pole-pitch of 3¾ in was used with two pole-pitches on each side of the centre. A series of rotors each one pole-pitch long was manufactured with different ratios of conductor and core area.

An assessment of the stability criterion was made by means of a dynamic test rather than the standstill test described in Section 4.1. The machine was run at a gradually reduced frequency until the oscillations became damped, the critical frequency being a measure of the 'quality' of the rotor. This test may not give a true measure of $\tau\omega$, but it is more relevant to the problem since the greatest difficulty in developing a small machine is to obtain build-up of oscillations. Fig. 13 shows the results of this

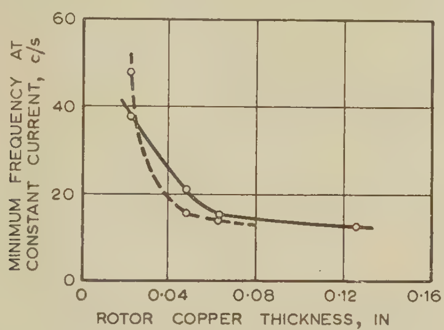


Fig. 13.—Critical operating frequency for various copper sizes.

—— Fom 0·187 in thick.
– – – From 0·283 in thick.

test, and the behaviour of the rotor is found to be quite critical as regards thickness of copper and section of iron core. One fact emerges very clearly: the tubular type of structure enabled much lighter rotors to be oscillated than had been possible previously; the rotors used in the first test weighed less than 2 oz and the power input could be reduced below 100 watts without losing stability.

Reduction of rotor length below a pole-pitch produced an apparent reduction of $\tau\omega$, as predicted in Reference 8. The smallest rotor which was made to operate after the optimum size of core had been chosen and the length reduced as much as possible weighed just over 1 oz.

The package-winder requirement is that the rotor should have the highest possible running speed with the lowest input power at 50 c/s. Further tests on tubular motors revealed the rather unexpected results shown in Fig. 14. Increase in field velocity beyond a certain point produces reduced speed in some cases; increase in current beyond a certain point can have similar effects. While the latter are almost certainly due to saturation, the former cannot be so easily explained. In no case did the average rotor velocity exceed 50% of the synchronous speed, and

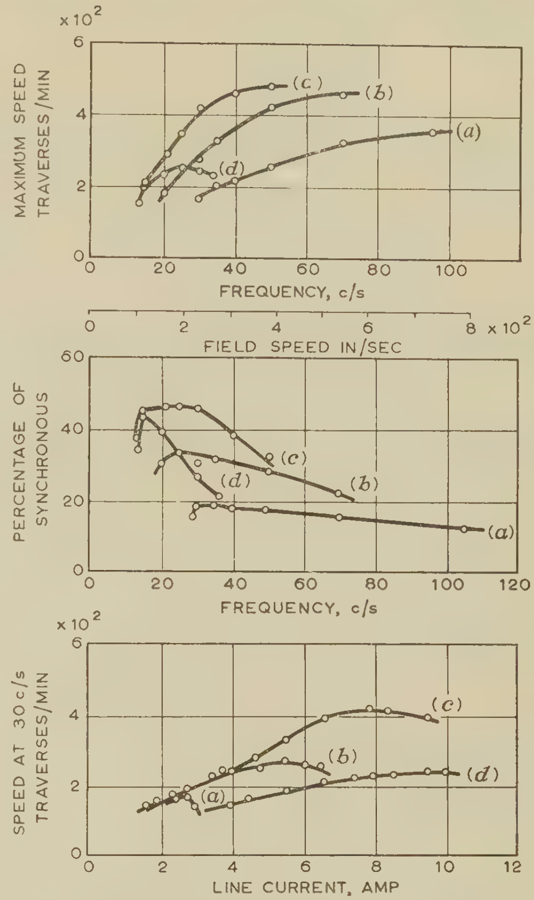


Fig. 14.—Experimental results for various rotors with iron cores 0·187 in thick and 3½ in long.

(*a*) Rotors with copper 0·023 in thick.
(*b*) Rotors with copper 0·047 in thick.
(*c*) Rotors with copper 0·063 in thick.
(*d*) Rotors with copper 0·125 in thick.

the greatest percentage of synchronous speed was attained at frequencies only slightly in excess of the critical frequency required for stability. At this stage it was decided to attempt analysis of the motion on the grounds that the transient effects at such low speeds were liable to be small. Treschev has shown[6] that at values of slip above 50% the dynamic speed/torque curve of a conventional induction machine is indistinguishable from the steady-state one. The earlier work on the short-stator effect[4, 5] has shown that transient effects are negligible for slips greater than 50% (on a 2-pole block).

(6) AN ATTEMPT TO PREDICT THE PERFORMANCE OF SELF-OSCILLATING MOTORS WITH END-SPRINGS

The starting assumption is that the speed/force characteristic is that predicted by steady-state theory for a current-fed machine, neglecting leakage reactances, namely that the force F is given by eqn. (3).

A typical cycle of operation is shown in Fig. 3. The distance, s, travelled during the accelerating period from A to B, is given by

$$s = \int_{u_1}^{u_2} \frac{u\,du}{a_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where the instantaneous acceleration, a_1, may be written

$$a_1 = \frac{2a_m n}{n^2 + \sigma^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $n = 1/\tau\omega$ and a_m is the maximum acceleration. Likewise the rotor travels the same distance s in decelerating from C to D, so that

$$s = \int_{u_3}^{u_2} \frac{udu}{a_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where a_2 is given by

$$a_2 = \frac{2a_m n(2 - \sigma)}{n^2 + (2 - \sigma)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting for a_1 from eqn. (5) and a_2 from eqn. (7) and writing $u = u_s(1 - \sigma)$, eqns. (4) and (6) become, after integrating,

$$s = \frac{u_s^2}{2a_m n}\left[\frac{\sigma_2^3 - \sigma_1^3}{3} - \frac{\sigma_2^2 - \sigma_1^2}{2} + n^2(\sigma_2 - \sigma_1) - n^2 \log\frac{\sigma_2}{\sigma_1}\right] \quad (8)$$

$$s = -\frac{u_s^2}{2a_m n}\left[\frac{\sigma_2^3 - \sigma_3^3}{3} - \frac{3}{2}(\sigma_2^2 - \sigma_3^2) + (n^2 + 2)(\sigma_2 - \sigma_3) + n^2 \log\frac{2 - \sigma_2}{2 - \sigma_3}\right] \quad (9)$$

If the rotor has reached the stable cycle of operation, eqns. (8) and (9) will be related by the spring equation $u_1 = eu_3$. σ_3 can therefore be replaced in eqn. (9) by

$$\sigma_3 = 1 - \frac{1}{e} + \frac{\sigma_1}{e}$$

yielding, together with eqn. (8), two simultaneous equations for the maximum and minimum values of slip, σ_1 and σ_2, attained in the stable state, namely

$$K = \left[\frac{1}{3}(\sigma_2^3 - \sigma_1^3) - \frac{1}{2}(\sigma_2^2 - \sigma_1^2) + n^2(\sigma_2 - \sigma_1) - n^2 \log\frac{\sigma_2}{\sigma_1}\right] \quad (10)$$

$$K = \left\{\frac{\sigma_2^3 - \left(1 - \frac{1}{e} + \frac{\sigma_1}{e}\right)^3}{3} - \frac{3}{2}\left[\sigma_2^2 - \left(1 - \frac{1}{e} + \frac{\sigma_1}{e}\right)^2\right] + (n^2 + 2)\left(\sigma_2 - 1 + \frac{1}{e} - \frac{\sigma_1}{e}\right) + n^2 \log\frac{2 - \sigma_2}{1 + \frac{1}{e} - \frac{\sigma_1}{e}}\right\} \quad (11)$$

where $K = 2a_m ns/u_s^2$.

Eqns. (10) and (11) were solved for σ_1 and σ_2 by a digital computer using several sets of values for n, e and K, and the results are shown in Figs. 15 and 16. Fig. 15 shows the maximum speed attained in a stable cycle, expressed as a fraction of synchronous speed, for various values of a_m and e. The values of u_s and s used in these computations were appropriate to the experimental tubular motor (950 cm/sec and 19 cm) and were constant; a_m is expressed in terms of the acceleration, g, due to gravity. These results go a long way towards explaining the failure of all the early experimental machines to run at speeds comparable with the synchronous speed, even with end-springs for which $e > 0 \cdot 9$. The fact is that to run at low values of slip the value of K must be high. For a 50 c/s supply a limit is set on the minimum value of u_s and the maximum value of n by

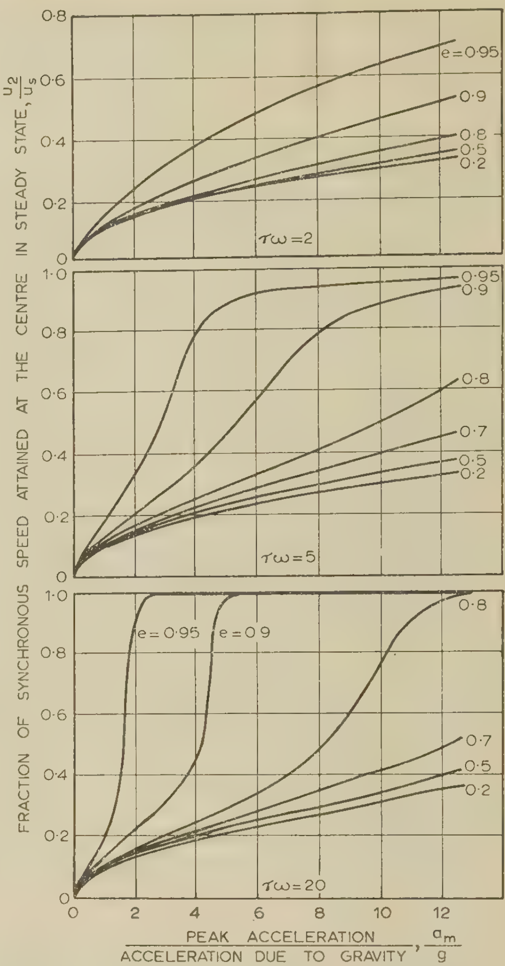


Fig. 15.—Theoretical curves of fractional speed against peak acceleration.

eqn. (2). For a given traverse length, therefore, K can be increased only by increasing the peak acceleration. Peak force is proportional to the maximum flux which can be contained within the rotor. For rotors longer than a pole-pitch, doubling the rotor length doubles both total flux and mass, and therefore the acceleration is unaffected. Increase in rotor cross-sectional area has a similar effect. For rotors shorter than a pole-pitch, increasing the length must be accompanied by an increase in the sectional area, with resulting reduction in acceleration.

A change in pole-pitch has a very marked effect on peak acceleration. Reduction of pole-pitch enables a greater air-gap flux density to be maintained for the same cross-sectional area of core without saturation. Halving the pole-pitch therefore results in approximately four times the value of a_m. At the same time, n is increased by 4 and u_s reduced by 2, so that K increases four times. It can be shown from eqns. (10) and (11) that an increase in pole-pitch may produce a reduction in speed in some circumstances and that there is in general an optimum value of pole-pitch for each design of system, depending on the rotor core area, rotor conductor size, track length, air-gap length and supply frequency.

Fig. 16 shows the predicted degree of uniformity of velocity across a traverse by plotting the ratio of maximum to minimum speeds calculated from eqns. (10) and (11). Comparison of Figs. 15 and 16 shows that conditions which enable the rotor to attain a high proportion of synchronous speed also ensure small variations in velocity from centre to end of a traverse.

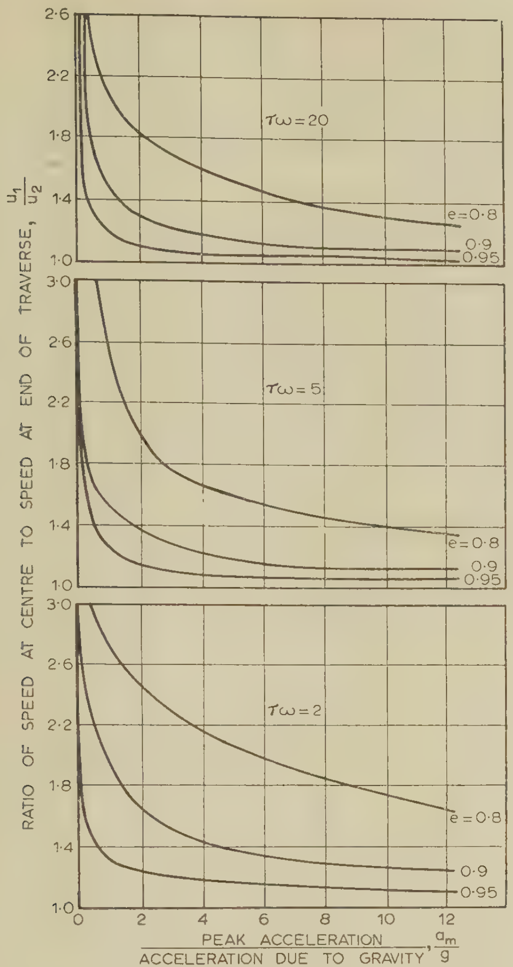


Fig. 16.—Theoretical curves of velocity variation across a traverse.

(7) THE 'COPPER SLEEVE' AND 'WOUND ROTOR' TYPES OF TUBULAR MOTOR

Figs. 15 and 16 show that peak acceleration is a potent factor in achieving high speeds of traverse and reasonably uniform velocity. The problem is to increase the peak acceleration without increasing *n*. The scheme illustrated in Fig. 12(*b*) appears to have considerable advantages in this respect. The same amount of conductor as that in the motor of Fig. 12(*a*) can be used on the same section of core, with the moving mass reduced by perhaps three times. Models of this type were built and tested. It was found that the effective value of $\tau\omega$ for sleeve rotors was several times less than that of the moving-iron type, which was undoubtedly due to the short-rotor effect as predicted in Reference 8. Further evidence of this was provided by constructing a moving-iron rotor in which the core projected some distance beyond each end of the copper. This rotor also appeared to have a low value of $\tau\omega$. In order to ensure stable running, the dimensions of the sleeve rotor had to be increased beyond a point where its weight exceeded that of the moving-iron type.

One further type of rotor was investigated, and its constructional details are shown in Fig. 17. Both winding and core move together, as in the machine illustrated in Fig. 12(*a*). The conductor consists of insulated wire wound continuously from end to end and finally short-circuited on itself. Such a series winding, when used with a series-connected stator, produces a uniform

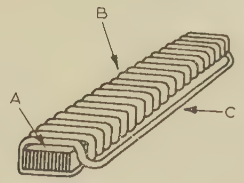


Fig. 17.—Wound-rotor construction.
A. Laminated core.
B. Rotor winding.
C. Short-circuiting connection.

rotor current-density at any instant. An experimental rotor of this type was found to have an effective value of $\tau\omega$ at least twice that of a motor of the type shown in Fig. 12(*a*) containing the same amount of copper and iron. Moreover, it was found to operate 30% faster at a speed corresponding to 60% of synchronous speed, with a power intake of only 80% of that of the earlier type.

Although the mechanism of this type of rotor is not fully understood, it is clearly superior to all other types. One such rotor, weighing $2\frac{3}{4}$ oz, has been operated at 540 traverses/min over a length of 14 in with a power input of 120 watts. Rotors of the type shown in Fig. 17 will normally be of the order of one pole-pitch in length. Longer rotors should have the short-circuiting bar connected to other points in the winding approximately one pole-pitch apart.

(8) CONCLUSIONS

The problem of making a small self-oscillating induction motor is comparable with that of making any small conventional induction motor with a high efficiency, and is perhaps more directly comparable with the problem of making a small single-phase induction motor run without a phase-splitting mechanism. The maintenance of a high rotor time-constant with reduced overall size is always the difficulty. If applications in the medium-power range are ever forthcoming, both the iron-impregnated rotor and the conducting-sheet type could be useful, but for package winding clearly only the mock-tubular style is worth considering. It has been established that rotors of the order of an ounce in weight can be oscillated and that high speeds are possible with low power input with careful design, particularly with the wound-rotor type. The electrical performance of these machines is certainly good enough to enable faster package winders to be constructed. Continuous speed control may be achieved at the cost of a voltage controller which, in view of the low power consumption, might well be a simple variable series resistance. The low flux density in the stator suggests that thick, low-grade laminations might be used. Development work is continuing with a view to reducing the initial cost of construction. Further experiments will be of an essentially textile nature rather than an electrical one.

(9) ACKNOWLEDGMENTS

The authors are indebted to Messrs. Leesona Holt and Co., Ltd., Rochdale, for the loan of package-winding equipment and for their interest generally, and in particular to Mr. A. Norris for advice on the textile aspects of the problem.

The experimental work on the ferrous-rotor machine was carried out by Mr. J. Bina of the Textile Department, Manchester College of Science and Technology, under the direction of Mr. D. Brunnschweiler, to both of whom we are indebted. We should also like to thank Dr. Payne of Manchester University for his assistance with the computation.

(10) REFERENCES

(1) LAITHWAITE, E. R., and LAWRENSON, P. J.: 'A Self-Oscillating Induction Motor for Shuttle Propulsion', *Proceedings I.E.E.*, Paper No. 1988 U, February, 1956 (**104** A, p. 93).

(2) Celanese Corporation of America: British Patent No. 808850, February, 1959.

(3) LAITHWAITE, E. R.: 'Linear Induction Motors', *Proceedings I.E.E.*, Paper No. 2433 U, December, 1957 (**104** A, p. 461).

(4) WILLIAMS, F. C., LAITHWAITE, E. R., and PIGGOTT, L. S.: 'Brushless Variable-Speed Induction Motors', *ibid.*, Paper No. 2097 U, June, 1956 (**104** A, p. 102).

(5) WILLIAMS, F. C., LAITHWAITE, E. R., and EASTHAM, J. F.: 'Development and Design of Spherical Induction Motors', *ibid.*, Paper No. 3036 U, December, 1959 (**106** A, p. 471).

(6) TRESCHEV, I. I.: 'Investigation of Variable Speed A.C. Machines', *Elektrichestvo*, February, 1957, p. 49.

(7) RUSSELL, R. L., and NORSWORTHY, K. H.: 'Eddy Currents and Wall Losses in Screened-Rotor Induction Motors', *Proceedings I.E.E.*, Paper No. 2525 U, April, 19 (**105** A, p. 163).

(8) LAITHWAITE, E. R., TIPPING, D., and HESMONDHALGH, D. E.: 'The Application of Linear Induction Motors to Conveyors', *ibid.*, Paper No. 3225 U, June, 1960 (**107** A, p. 284).

(9) SHTURMAN, G. I.: 'Induction Motors with Open Magnetic Circuits', *Elektrichestvo*, 1946, p. 43.

(10) WILLIAMS, F. C., LAITHWAITE, E. R., EASTHAM, J. F., and FARRER, W.: 'Brushless Variable Speed Induction Motors using Phase-Shift Control', *Proceedings I.E.E.*, Paper No. 3262 U, May, 1960 (**108** A).

(11) BLAKE, L. R.: 'Conduction and Induction Pumps for Liquid Metals', *ibid.*, Paper No. 2111 U, July, 19 (**104** A, p. 49).

(12) LAITHWAITE, E. R.: British Provisional Patent No. 1597 1959.

DISCUSSION ON 'SUBMERSIBLE PUMPING PLANT'*

Mr. H. K. Whitehorn (*communicated*): A little of the early history of the subject may be of interest. I was in at the birth. Mr. W. R. Macdonald obtained British Patent No. 19384/1908 for an a.c. motor whose stator winding was rubber-covered cable. He had made a model which worked under water coupled to a small pump, and showed it to Mr. B. T. Rumble. Then he had a larger motor made, about 3 h.p. 3-phase, but it took excessive current. About 1912 Mr. Rumble contacted Mr. Reed Cooper with whom I was employed. I asked for details of core and windings, and predicted that the motor would run satisfactorily at half the rated voltage—and, of course, half power. It was tested at Gwynnes, Hammersmith, and ran as predicted. Reed Cooper took on the manufacture; we made a 15 h.p. 3-phase set with a 4 in pump. All went well. Rumble and Reed Cooper met Mr. C. H. Wordingham, Chief Electrical Engineer at the Admiralty, who became very interested. Some sets were ordered. There was then no alternating current on battleships, so we also designed and made motor-alternators to give a 220-volt 3-phase 50 c/s supply from 220 volts d.c. Of several sets installed in battleships the luckiest was that in H.M.S. *Lion*. The *Lion* was holed at the battle of Jutland, but with its two submersible sets running day and night just managed to get to Dover with bows awash.

There were 'teething troubles' with the rubber cable. Standard cable was unusable because the conductors would bulge at places. Callender's produced a special submersible winding cable with all the strands twisting the same way, and cab-tyre sheathed. Then by turning over the main hank for each turn wound on the stator, all bulging ceased. In most motors the cable diameter was about $\frac{1}{2}$ in overall and the slot width slightly greater, so the slots were sausage-shaped, with no break in the tooth at the air-gap. The $\frac{1}{16}$ in bridge there produced rather excessive leakage. The rotor was squirrel-cage—no rubber.

Messrs. H. H. Anderson and **W. G. Crawford** (*in reply*): Mr. Whitehorn's notes are most interesting. We were quite unaware that submersible pump sets had such a long and distinguished history. Our own knowledge of this type of plant goes back no further than the years immediately preceding the Second World War. Even then, submersible motors were still being wound with rubber-covered wire, and it was not until a few years after the start of the war that p.v.c.-covered wire came into general use. The change to p.v.c. resulted in a considerable improvement in the reliability of submersible motors, and since then there has been a steadily increasing demand for them.

* ANDERSON, H. H., and CRAWFORD, W. G.: Paper No. 3147 U, November, 1959 (see 107 A, p. 127).

.313.1.013.24

The Institution of Electrical Engineers
Paper No. 3327 U
Oct. 1960
©

THE APPLICATION OF THE METHOD OF IMAGES TO MACHINE END-WINDING FIELDS

By C. J. CARPENTER, M.Sc.(Eng.), Associate Member.

(*The paper was first received* 15*th March, and in revised form* 4*th July*, 1960.)

SUMMARY

Although a variety of methods have been used to calculate the fields ;sociated with the end-windings in rotating electrical machines, there pears to have been little attempt to exploit the method of images. iis provides a means of solving the field problem by integration with atively high accuracy. By making use of the vector potential, a mparatively simple calculation is possible of the flux linkages due to mplex end-winding arrays. The image principle is extended in the per to circuits which are partly embedded in the reflecting surface. ie effects of various complexities in the boundary conditions, cluding the presence of the air-gap, are considered. The method is ell suited to both turbo-type alternators, with magnetic and non- agnetic end-rings, and to induction motors. It is illustrated by lculating the end-winding inductance of a 2-layer induction-motor inding.

LIST OF SYMBOLS

a = Coil overhang, coil separation.
b = Coil pitch.
g = Air-gap length.
h = Length of overhang conductor.
l = Length.
r = Distance of field point, conductor radius.
x = Distance measured parallel to current filament (Fig. 14).
y = Distance measured perpendicular to current filament (Fig. 14).
A = Magnetic vector potential.
H = Magnetic field intensity.
I = Current.
K = Constant (Section 5.2).
L = Inductance.
X = Function defined by eqn. (6*b*).
η_0 = Primary magnetic constant.
θ = Angle.
μ = Permeability.
Φ = Magnetic flux.

(1) INTRODUCTION

Developments in the design and application of electrical machines have drawn attention in recent years to various problems of machine analysis which require more exact solutions than have hitherto been necessary. Among the more important are those arising from the magnetic effect of the end-winding, such as the calculation of end-winding inductance, of conductor forces under short-circuit conditions and of eddy losses in the core end-plates. An essential prerequisite of a study of these phenomena is a means of calculating the magnetic field in the end-region, i.e. the field produced by a winding which is partly embedded in iron and whose ends protrude through the iron surface. The field due to the winding alone can be obtained by integration, but the magnetic effect of the iron and of any other magnetic surfaces which may be close to the end-winding cannot be dealt with so directly. One method recently adopted[1, 2] is to solve Laplace's equation so as to obtain an expression for the magnetic scalar potential in a finite end-region, all of whose boundaries must be defined by surfaces at which the flux is either normal or tangential. An alternative method is to make use of the virtual-image principle as a means of replacing the iron or other boundary surfaces by equivalent circuits in air, so that the entire field problem is reduced to one of integration. The principle is well known and has been extensively applied by Hague,[3] for example, to the calculation of 2-dimensional magnetic fields in machines, but its application to end-winding fields—which are not only 3-dimensional in form, but are due to circuits which intersect one of the reflecting surfaces—has been less adequately explored. Image circuits have been suggested[4–6] but these are valid for only one type of boundary condition; in general they are incomplete, since one of the essential conditions, namely that of continuity of current flow, is not satisfied.*

The object of the paper is to extend the image principle to circuits which are partly embedded in one of the reflecting surfaces, and in particular to end-winding problems in which this surface is broken by an air-gap. The method of approach is to consider first a semi-infinite circuit protruding from an infinite, flat iron block without an air-gap, and then to take into account the more important of the complexities of the end-region boundaries which are met with in practice. The image circuits can be derived for all permeabilities, provided only that these are constant for each of the reflecting surfaces, but generally the effect of saturation is less important than that of induced currents. The two limiting permeability values of infinity and zero are therefore specifically examined, corresponding on the one hand to the assumption that the induced currents have a negligible magnetic effect and on the other to the complete suppression of the normal component of the field at the surface.† At present there appears to be little direct experimental evidence to suggest which of these two is the more nearly correct, in particular at the core end-surface, and in the literature both values have been used, although the permeability of end shield and other stationary surfaces has been generally assumed to be infinite. The rotor surface, which moves with the flux wave in a rotating-field alternator or stator-fed induction motor, has no appreciable eddy currents induced in it under steady-state operating conditions, and its effective permeability will usually be high, although saturation effects may be significant in a turbo-alternator end-ring. However, if a solution of the end-field problem is required under transient and, more specifically, sub-transient conditions, it is necessary to consider also rotor surfaces which are effectively impermeable.

The discussion relates specifically to machine end-fields, but the method is generally applicable to the calculation of fields due to circuits and iron surfaces, more particularly when they

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.
Mr. Carpenter is in the Department of Electrical Engineering, Imperial College of Science and Technology, University of London.

* The correct form of the image, when there is no air-gap present, has, however, been given in a recent paper by Hammond.[13]
† In practice the surface will have anisotropic properties when laminated, so that the permeability measured in the tangential direction may be large. However, this does not affect the field in the air if the normal field component is zero.

intersect. The M.K.S. rationalized system of units is used, with the notation suggested by Carter,[7] in that symbols μ and η_0 denote permeability and primary magnetic constant, respectively.

(2) VIRTUAL-IMAGE PRINCIPLE

(2.1) Reflection of Closed Circuits

The image principle, as applied to a closed current-carrying circuit in the vicinity of a semi-infinite block of iron, states that the magnetic field in the air is the same as that which would result if the iron were removed and replaced by the mirror image of the circuit. The current in the image circuit is given by

$$I' = -\frac{\mu - 1}{\mu - 1}I \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

where the negative sign indicates that I' has an angular sense opposite to that of I when both circuits are viewed in a direction tangential to the reflecting surface, and also when they are separately viewed along the normal towards the surface (or away from it). This result, which has been given by Searle,[8] Smythe[9] and others for 3-dimensional circuits, may be derived very simply by first ascertaining the virtual image of an isolated pole using the method given, for example, by Jeans[10] and then relating the circuit to a distribution of dipoles by means of the magnetic-shell principle. Thus the rectangular circuit ABCD shown in Fig. 1(*a*)

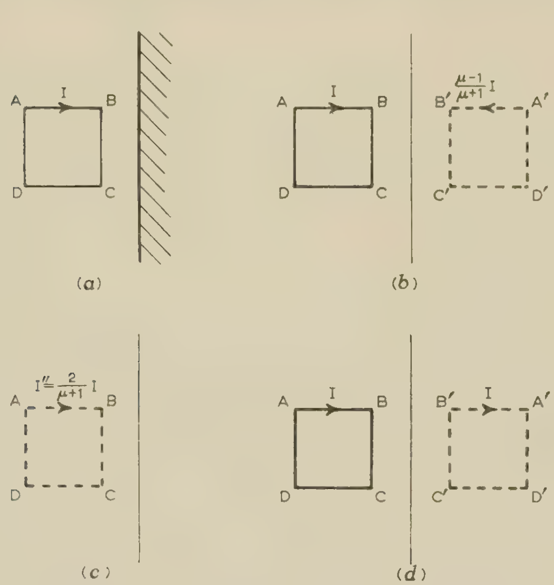


Fig. 1.—Images of circuits closed outside iron.
(*a*) Circuit adjacent to iron block.
(*b*) Image giving field in air.
(*c*) Image giving field in iron.
(*d*) Zero-permeability image.

sets up a field which may be calculated by replacing the iron by the image A′B′C′D′ [Fig. 1(*b*)] carrying a current I' whose positive sense is in the direction shown. A line circuit is shown, for simplicity, in the diagram, but the principle applies equally to one of any cross-section.

The field within the iron is the same as that which would result if a second semi-infinite block were placed in contact with the first, so as to fill the whole space, and the current in the circuit reduced to a value

$$I'' = \frac{2}{\mu + 1}I \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

For the purposes of field-strength (H) calculations the iron m be ignored, since it now extends to infinity in every directic The problem is thus again reduced essentially to that of the fie of a circuit in air, but the permeability of the iron must be tak into account in computing flux and flux density. The seco image current associated with the circuit ABCD in Fig. 1(*a*) shown in Fig. 1(*c*).

The images simplify for three particular values of μ. In t limit, when the permeability is infinite, I' is equal in magnitu to I but has the reverse sense. All flux lines then link bo circuits and the iron surface becomes a surface of consta scalar potential. I'' is zero, which means that there is no H-fie associated with the flux inside the iron. On the other har when the surface is completely impermeable in both the norm and the tangential directions, because of eddy currents, I' aga has the same magnitude as I, but this time circulates in the sa sense, as shown in Fig. 1(*d*). None of the flux which is no produced links both circuits or crosses the iron surface. Th condition $\mu = 0$ gives a current I'' which is twice I, but th result has no physical significance since the effect of eddy curren is accounted for by assigning zero permeability to the iron or in so far as the external field is concerned; inside the iron t field strength, H, as well as the flux density, B, is suppressed. finally, the material has unit permeability, i.e. there is no iro present, the value of I' in eqn. (1*a*) is zero and I'' is the actu current I.

(2.2) Application to Current Elements

The magnetic effect of a circuit may be analysed into t (supposed) individual contributions of its elements, and it evident that each of these elements may be regarded, separa e as being reflected in the iron surface, since the correspondi part of the image is determined entirely by the element and independent of the remainder of the circuit. A current eleme such as AD (Fig. 1) which is parallel to the surface is, for t purpose of external field calculations, associated with an ima A′D′ whose current is in the same direction and of the sa magnitude when the permeability is infinite. An element su as AB, which is perpendicular to the surface, has an ima carrying the same current, but now in the opposite directic When the current-density vector is inclined at any arbitrary an to the surface, this principle applies to its components.

This result appears to be a remarkable one when examined detail, and it requires further investigation before the ima principle can be applied with confidence to part-circuits, such end-windings, which are terminated by the iron surface. T effect of adding A′D′ to the parallel element AD is to produc field which everywhere intersects the plane of symmetry in normal direction, as is consistent with the boundary conditi imposed by the iron surface. But the field due to the perpe dicular element AB is everywhere tangential to the surface a can induce no surface polarity in the iron. The iron therefc contributes no additional field component, which suggests th there should be no image. The apparent image, A′B′, produ a magnetic field which is inconsistent in form with any fi component which could result from polarization in the irc being everywhere parallel to the iron surface. On the oth hand, the resultant field can have no tangential component the iron surface, and the element A′B′ is clearly necessary neutralize the tangential contribution from AB.

Evidently the principle which is valid for the closed circuit not strictly valid for its parts. It is not difficult to see why this so. The field associated with a current element $I\boldsymbol{\delta l}$, at a po distance $\boldsymbol{r}$ from it is

$$\delta H = \frac{I}{4\pi r^3}\boldsymbol{\delta l} \times \boldsymbol{r} \quad \text{or} \quad \delta H = \frac{I\delta l}{4\pi r^2}\sin\theta \quad . \quad .$$

d this field has the property that its curl is non-zero for all ite values of r. As was shown by Heaviside, the field is the me as that which would be produced by a small, linear current urce if immersed in a large bath of conducting fluid within ich the current flow lines are closed, and it is only by this vice (or by introducing displacement current) that eqn. (2) comes consistent with Maxwell's equations. In other words, e current element field has sources, or effective sources, at ery point, and, in consequence, the image principle cannot be plied to it. It cannot be expressed in terms of a scalar tential, and the vector potential has non-zero divergence;* in ese respects the field due to the element differs from that due a closed circuit (and from that due to magnetized iron). e image method can be justified only for closed circuits, and though the image of such a circuit can be analysed into parts, ch of which appears as the reflection of the corresponding rt of the circuit, these must add together in such a way that e current flow is everywhere continuous.

) IMAGE OF CIRCUIT PARTIALLY EMBEDDED IN IRON

(3.1) Boundary Conditions at Iron Surface

Before examining further the possibility of applying the image ethod to the external part of a partially embedded circuit, it is portant to formulate the boundary conditions set up at the on surface. These differ in a most important respect from ose imposed by the iron when the circuit is wholly external to . The iron boundary is then, supposing infinite permeability, a rface of constant scalar potential, whereas in the more general ase the field must satisfy the condition

$$\oint H \, . \, dl = I \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

ound all contours in the surface, including those which enclose onductors passing through the surface. Thus the tangential omponent of the field is not zero and the flux lines do not enter he iron at right angles, however large the permeability. In a otating machine this condition is modified by the presence of n air-gap, but it is convenient to ignore this gap in the first nstance. The assumptions imply a circuit inductance which ends to infinity in the limit of infinite permeability; nevertheless he idealization is not an impractical one, provided that the ircuit current is sufficiently small.

Eqn. (3) is a more general form of boundary condition, eplacing the condition of zero tangential field which is imposed y the iron when the circuit is wholly external to it. It suggests t once what the form of the image circuit may be (assuming that one exists) which will satisfy the required field and boundary conditions in the region outside the iron. This can be illustrated most simply in terms of the semi-infinite rectangular circuit shown in Fig. 2(*a*), penetrating a semi-infinite block of high permeability. The element BC will be associated with an image element B′C′ [Fig. 2(*b*)], provided that both form part of a closed circuit. These two elements together contribute (by symmetry) nothing to the tangential field component in the iron surface, and this must therefore be accounted for by currents normal to the surface. But integration of eqn. (2) shows that the circuit elements BF and CE do not, alone, produce a tangential field sufficient to satisfy eqn. (3); additional elements B′G′ and C′H′ are required to extend BF and CE to infinity in both directions. So far as the boundary conditions are concerned, these additional elements may be placed on either side of the surface representing the iron boundary, but if the field conditions within the end-region are to be satisfied, the elements must obviously be on the

* At least, if the customary expression for vector potential,[9,10] namely $\delta A = \eta_0 I \delta l / 4\pi r$ is used; this is not the only possible one.

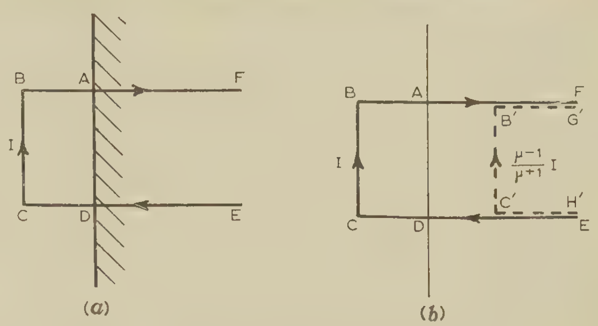


Fig. 2.—Image of partially embedded circuit.
(*a*) Circuit arrangement.
(*b*) Image giving field in air.

opposite side of the surface. In this position they join with B′C′ to form a closed circuit, and this configuration therefore forms the desired image. The image is a valid one, since the two (closed) circuits together produce a field which satisfies the same conditions everywhere in the end-region as the field due to the actual circuit and the iron; in addition, the field satisfies eqn. (3) round every contour in the iron surface, so that the tangential field at the boundary is given correctly.*

(3.2) Derivation of Image Circuit by Superposition

The above method gives the form of the image for a circuit which is partly embedded in iron of infinite permeability, but a more general method of attack is required before the air-gap, in particular, can be taken into account (Sections 4 and 7). The approach which has been found to be the most generally useful is to resolve the circuit, e.g. that shown in Fig. 2(*a*), into two parts by the superposition principle. The parts consist of a pair of infinitely long straight conductors GF and HE, as shown in Fig. 3(*a*), and a semi-infinite circuit GBCH [Fig. 3(*b*)] which is

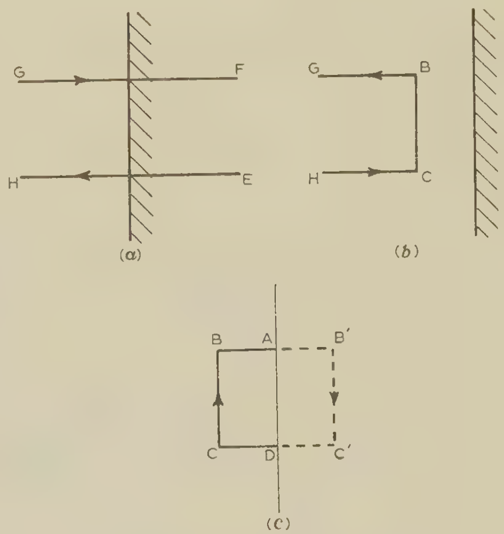


Fig. 3.—Superposition method.
(*a*) Infinite circuit.
(*b*) Superposed circuit.
(*c*) Reflection in zero-permeability surface.

wholly external to the iron. Both circuits are closed (at infinity). The first sets up a 2-dimensional field which satisfies eqn. (3) at the iron surface and which is entirely unaffected by the presence of the iron, so that there is no image circuit. The effect of the

* This boundary condition is a sufficient one, as has been shown by Hammond.[13]

iron on the field of the second circuit, GBCH, is that of an image in which the current is given by eqn. (1*a*); this image is the circuit G′B′C′H′ shown in Fig. 2(*b*). Thus the combined field, in the end-region, of the original circuit and the iron is the same as that of the three circuits GFEH, GBCH and G′B′C′H′. When these are superimposed the result is in accordance with Fig. 2(*b*).

This result is valid for all permeabilities, provided that the current in G′B′C′H′ is chosen in accordance with eqn. (1*a*). If the effective permeability is zero, the current has the same magnitude as the actual circuit current but is reversed in sign, so that B′G′ and C′H′ neutralize the corresponding parts of AF and DE. The end-winding ABCD then has a simple mirror image in the iron surface, but with the current reversed [Fig. 3(*c*)], and this reversal makes the current flow continuous at the junction between the part-circuit and its image. The 'slot conductors' inside the iron have, under these conditions, no magnetic effect in the end-region.

The method is applicable, without restriction, to any shape of end-winding. For example, the field in the vicinity of the 2-layer coil shown in Fig. 4(*a*) may be calculated from the equivalent

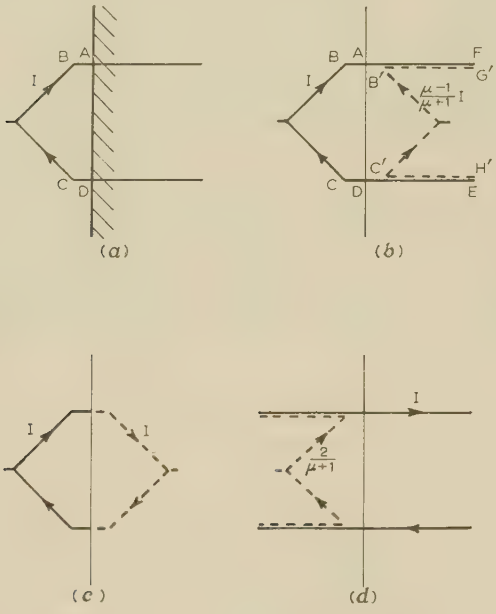


Fig. 4.—Images of 2-layer end-winding.

(*a*) Non-planar circuit.
(*b*) Image giving field in air.
(*c*) Zero permeability image.
(*d*) Image giving field in iron.

circuits shown in Fig. 4(*b*),* and this reduces to the arrangement shown in Fig. 4(*c*) when the effective permeability is zero. Furthermore, the same principle gives the field inside the iron, but using eqn. (1*b*) instead of eqn. (1*a*). The equivalent circuits shown in Fig. 4(*d*) are derived in this way for the circuit in Fig. 4(*a*).

(4) EFFECT OF AIR-GAP

(4.1) Method of Analysis

In a rotating machine the windings do not project from a continuous block of iron but from two blocks separated by a small air-gap, and this air-gap has a drastic effect on the field near the iron when the permeability is large. The tangential field at the iron surface disappears and the m.m.f. which it represents is concentrated in the form of a potential differen between the two sides of the gap.

The effect of a short air-gap, which is, for the present, assum to divide the large, infinitely permeable, block into two halv may be allowed for by an extension of the superposition princip used in Section 3.2. This shows that the end-winding field m be analysed into the two components represented by the circu shown in Figs. 3(*a*) and 3(*b*). It is only the first of these co ponents which, in the absence of an air-gap, produces a tangenti field at the iron surface, and it is therefore this part of the tot field which is most affected by the air-gap. The second fie component—that due to the circuit shown in Fig. 3(*b*) or equivalent for other shapes of end-winding—is normal to t iron surface, both with and without the air-gap, so that t boundary conditions are altered only along the line at which t gap intersects the iron surface. Provided that the air-gap small compared with its distance from the circuit, it introdu only a small additional reluctance into those flux paths whi intersect the iron, and its effect can be ignored.

Thus the magnetic effect of the air-gap in an infinitely pe meable block can be determined, to a close approximation, examining its effect on the field set up by the two infinitely lo conductors shown in Fig. 3(*a*). This field component is entir unaffected by the shape of the end-winding.

(4.2) Effect on 2-Dimensional Field Component

The problem may be further reduced by separating the ma netic fields due to the two straight conductors [Fig. 3(*a*)]. T field due to either, in regions relatively close to the conduct surface, decays inversely as the distance from the conductor. regions relatively close to the iron, on the other hand, the fl lines are drawn into the iron along paths of low reluctance abo and below the conductor, in the manner indicated in Fig. 5(

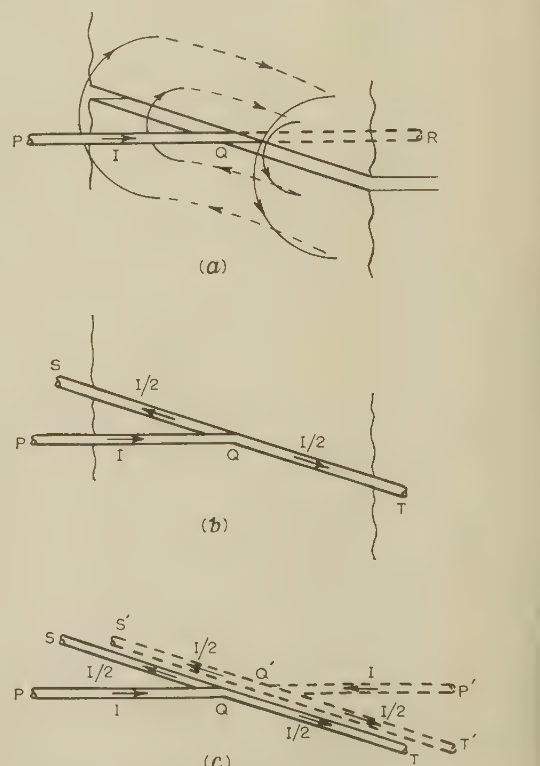


Fig. 5.—Line conductor penetrating infinitely permeable block with air-gap.

(*a*) Field due to straight conductor.
(*b*) Circuit equivalent of air-gap.
(*c*) Circuit equivalent of block with air-gap.

* This result, expressed in a slightly different form, has also been derived by Hammond.[13]

iewed from the outside, the boundary of the iron forms two ırfaces of constant, but different, scalar potential. The otential difference between these surfaces is $\frac{1}{2}I$, where I is the rcuit current, and it changes sign at the conductor.

Provided that the air-gap is small (i.e. at points sufficiently mote from it) these boundary conditions are the same as those btained by replacing the two iron blocks by a single, continuous ock of infinite permeability, on the surface of which is a line urrent of $\frac{1}{2}I$, as shown in Fig. 5(*b*). The two currents on oppote sides of the point of entry of the circuit are in opposite irections and the equivalent circuit, PQST, does not penetrate ıe iron. This circuit, placed against an infinite block, is quivalent to the arrangement shown in Fig. 5(*a*) since (except at oints close to the air-gap) the field set up satisfies the same onditions everywhere outside the iron and on the iron surface, nd the potential discontinuity along the line SQT is the same.

The effect of the iron may now be allowed for in terms of the nage circuit P′Q′S′T′ [Fig. 5(*c*)], which follows at once by omparison with Fig. 1(*b*). If this is added to PQST and the esult combined with that for the second conductor, the complete quivalent circuit corresponding to the go-and-return circuit in 'ig. 3(*a*) is that shown in Fig. 6(*a*). This conductor arrange-

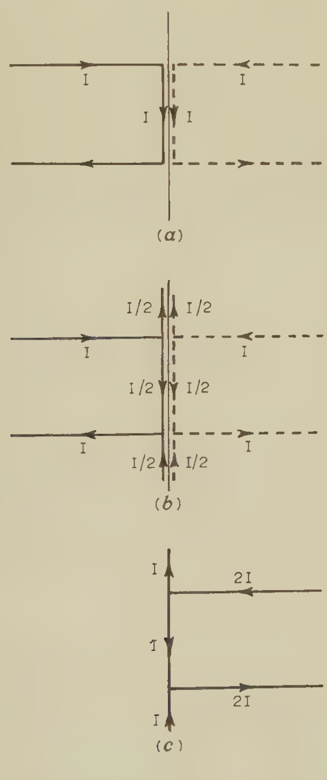


Fig. 6.—Field due to two line conductors.

(*a*) Line conductors penetrating block with air-gap.
(*b*) Effect of air-gap of finite length.
(*c*) Circuit equivalent of air-gap.

ment, in air, produces the same field in the end-region as that in Fig. 3(*a*) in combination with the iron.

The circuit does not, however, represent the end-field produced in an actual machine by two line conductors, since the air-gap does not, in fact, extend to infinity, but forms a closed circle. In a 2-pole machine the line conductors are diametrically opposite and the air-gap flux density is everywhere the same, whereas it would be zero outside the circuit if the gap extended to infinity. Where there are more than two poles the single coil shown in Fig. 3(*a*) must be supplemented by additional conductors under each pole, so as to maintain symmetry, and the equivalent circuit in Fig. 6(*a*) is then replaced by that in Fig. 6(*b*).

When there is no air-gap the iron does not affect the *H*-field due to the two long conductors. It therefore follows by subtraction that the magnetic effect of the air-gap in an infinitely permeable block on the end-region field is that of the semi-infinite double circuit, carrying a total of twice the actual circuit current, which is shown in Fig. 6(*c*).

(4.3) Reflection of End-Winding in Block with Air-Gap

The field due to the end-winding is obtained by adding to the component just considered that due to the external circuit represented, for example, by GBCH in Fig. 3(*b*). As was pointed out in Section 4.1, the form and magnitude of the image [Fig. 2(*b*)] which is associated with this circuit is not significantly affected by the air-gap. The magnetic field in the end-region, due to the circuit in Fig. 2(*a*), is therefore the field of the closed circuit ABCD and its image A′B′C′D′, as shown in Fig. 7(*a*).

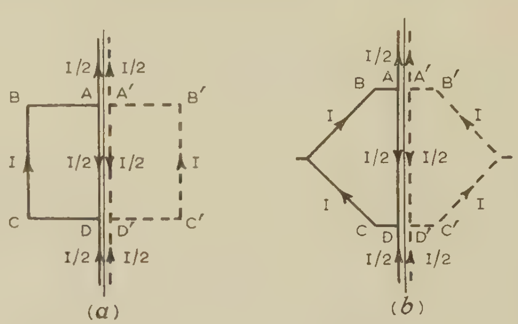


Fig. 7.—Reflection of end-winding in infinitely permeable block with air-gap.

(*a*) Rectangular end-winding.
(*b*) Two-layer end-winding.

This result is to be compared with that shown in Fig. 2(*b*), which is valid when there is no air-gap, and may be derived from it by adding the circuit in Fig. 6(*c*).

The method applies to other end-winding shapes and to air-gaps which are not restricted to a plane. The winding shown in Fig. 4(*a*), for example, sets up a field in the end-region which is given by the circuits in Fig. 7(*b*) in place of those in Fig. 4(*b*). The cylindrical form of the air-gap modifies the shape of the equivalent conductor SQT in Fig. 5(*b*), so that the conductors AD and A′D′ in Fig. 7 become circular arcs. In general, AD and A′D′ must follow the curve of intersection of the air-gap and the iron surface, and this may be of any arbitrary shape provided that it satisfies the necessary conditions of symmetry* and passes through the points A and D at which the circuit intersects the surface. This last condition is, of course, only approximately true of a practical machine, in which the conductors are placed in slots and thereby offset from the air-gap, but if the effect of the displacement is appreciable it may be readily taken into account. The slots are essentially extensions of the air-gap (closed slots being regarded as magnetically equivalent to open slots because of local saturation) and are therefore represented by a local distortion of the equivalent circuit in the manner indicated in Fig. 8. The resultant modifications in the equivalent circuits for the rectangular end-winding are shown in Fig. 8(*c*).

The equivalent-circuit representation of the air-gap, which has been proposed by Douglas[4] in the non-symmetrical form repre-

* If the air-gap were not symmetrical the conductor current would not divide equally into two parts at the iron surface.

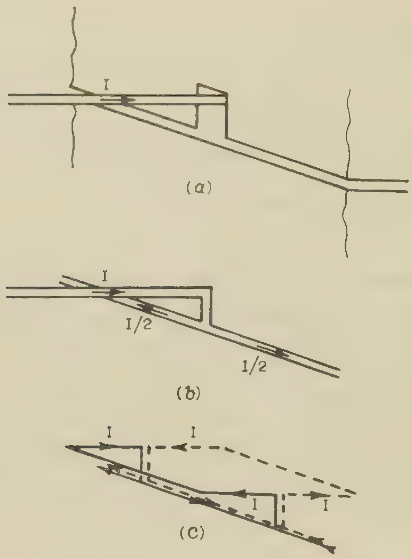


Fig. 8.—Conductor offset from air-gap.
(*a*) Arrangement of conductor in slot.
(*b*) Circuit replacing slot and air-gap.
(*c*) End-winding image.

sented by the circuit in Fig. 6(*a*), is not an exact one, since it assumes a gap of negligible length, and it becomes invalid near the gap entrance. In this region other approximations also become inadequate, in particular the assumption that the core end-plates form a flat surface, and the complexity of the local boundary conditions are such that no simple analytical solution is possible. For the purposes of inductance calculation, the contribution to the flux linkage from this part of the field is best estimated by the method discussed in Section 7.1, instead of in terms of the equivalent circuit, and the image circuits in Figs. 2(*b*) and 4(*b*) are then used in place of those shown in Fig. 7. But for other purposes, and in other parts of the end-region, the equivalent circuits in Fig. 7 can be expected to give a good approximation to the field when the permeability on both sides of the air-gap is large.

(4.4) Derivation of Image in Terms of Magnetic Shell

The magnetic-shell equivalent[10] of the circuit provides a more direct, but perhaps less convincing, method of deriving the circuits shown in Fig. 7, although it is much less simple if there is no air-gap present. The surface layer of dipoles which constitutes the shell may be of any arbitrary shape provided only that it is bounded by the circuit. It may therefore be chosen to pass through the air-gap (assuming open slots), so as to avoid any intersection with the iron surface. Since the air-gap is not infinite but is symmetrical, the shell may be separated into two components corresponding to the two parts of the air-gap, and each component then represents half the total circuit m.m.f. Those parts which lie within the air-gap can have no effect on the field in the end-region, provided that the permeability of the iron is large, since the iron forms a magnetic screen. The remainder of each of the magnetic shells is a surface (of arbitrary shape) bounded by the closed contour ABCD (Fig. 7), and this is reflected in the iron surface in a manner which is not significantly affected by the air-gap. Both the two part-shells, ABCD, and their images, A′B′C′D′, are equivalent to closed circuits, and these are the circuits shown in Fig. 7.

(4.5) Iron Surfaces of Zero and Finite Permeability

When the effective permeability of the iron is zero, the presence of the air-gap is clearly unimportant, so that the circuits [Figs. 3(*c*) and 4(*c*)] which were derived by ignoring the air-ga remain valid. It is noteworthy that this result is obtained fro the circuit in Fig. 6(*b*) if the image current is reversed in acco dance with eqn. (1*a*), but this result is fortuitous and it does n appear to be possible to extend the equivalent circuit method any simple manner to include permeability values between th two extremes. Nevertheless an analysis of the effect of th component circuit such as is shown in Fig. 3(*b*) is possible terms of images for finite permeability values, and if the field d to the infinite line conductors can be established by experime or by any other method, a complete solution is possible for a end-winding configurations.

(5) EXTENSION OF METHOD TO OTHER BOUNDARY CONFIGURATIONS AND CONDITIONS

(5.1) Multiple Boundary Surfaces

In a practical machine surfaces other than those of the co may influence the end-winding field. Provided that these a flat, and either parallel or at right angles to the core end surfac they may be allowed for by the multiple-image method discusse for example, by Hague[3]; generally, no new principle is intro duced, since the conductors do not intersect the additional surface A surface which is parallel to the core end introduces an infini series of image circuits, but these can probably be represente by the first terms only without introducing errors larger tha those already inherent in the representation of the surface as on of uniform permeability.

The increase in the number of image circuits required represent additional reflecting surfaces provides a practic limitation to the degree of boundary complexity which can b taken into account. On the other hand, it is doubtful wheth stationary surfaces in which eddy-currents are induced a satisfactorily represented by the usual assumption that the permeability is infinite, and more accurate results may b obtained by ignoring these surfaces altogether. Whereas th analysis in terms of images is simplified by approximations this kind, other methods require the assumption of simp boundary conditions over a closed surface so as to make th end-region finite. Since the neglect of end shield and simil surfaces is necessarily accompanied by the replacement of th core end by a surface of infinite extent, and since the effecti permeability of the two surfaces is likely to be the same, th second of these two approximations provides a measure compensation for the first.

(5.2) Magnetic Effect of Rotor End-Ring

In high-speed turbo-type alternators the rotor end-windin retaining ring provides a radial limit to the end-region and has relatively large magnetic effect. It provides an important illu tration of a type of boundary consisting of two surfaces inte secting at right angles and, moreover, two surfaces which hav different effective permeabilities under steady-state operatin conditions. Here both surfaces are not flat, but one is a cylinde whose radius is not necessarily large compared with the windin pitch, and this does not permit any exact image solution. Th cylindrical surface can, however, be taken into account approx mately by treating each element of the end-winding separate and calculating its image as if the rotor surface in its neighbou hood were part of an infinite plane. The components of th image must then be distorted slightly in the circumferenti direction so that they join to form a closed circuit. If the imag is to reduce to the exact solution for an infinite line parallel t the axis, the elements must be placed at the inverse points rathe than at points obtained as for a plane surface.

The image principle for two plane surfaces which meet at

ght angle and which have, in general, different permeabilities and μ_2 is illustrated in Fig. 9 in terms of the 2-dimensional

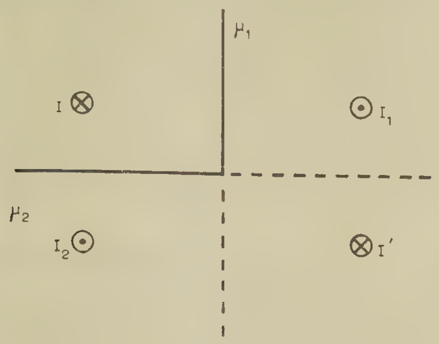


Fig. 9.—Surfaces of different permeability intersecting at right angles.

eld due to a line current I. The images I_1 and I_2 which are ormed, separately, by the μ_1 and μ_2 surfaces have the positive irections shown (so as to maintain the same circuital sense as e current I) and are both given by eqn. (1*a*). Thus

$$I_1 = -\frac{\mu_1 - 1}{\mu_1 + 1} I \equiv -K_1 I$$

nd

$$I_2 = -\frac{\mu_2 - 1}{\mu_2 + 1} I \equiv -K_2 I$$

vhere K_1 and K_2 are positive when μ_1 and μ_2 are greater than nity, so that both currents then flow in the negative direction i.e. into the paper). When the two surfaces are combined, I_1 equires an image I' in the μ_2 surface and this image is given by

$$I' = -K_2 I_1 = K_2 K_1 I$$

I_2 similarly requires an image in the μ_1 surface, and this coincides n position with that just considered. It is given by

$$I' = -K_1 I_2 = K_1 K_2 I$$

so that it coincides also in magnitude and sign. The set of three mages shown is therefore self-consistent, and it satisfies the appropriate boundary conditions at both surfaces.*

The result may be extended to 3-dimensional fields by modifying the diagram but not the equations, the circuit-elements being reflected in both surfaces in accordance with the rule given in Section 2.2. A point of interest which arises when the two surfaces have zero and infinite permeability respectively is that a small air-gap at their junction should have no effect, so that it is immaterial whether the circuit in Fig. 2(*b*) or that in Fig. 7(*a*) is used. This is not, of course, true if both permeabilities are infinite, but when one is zero all parallel current elements are neutralized by their images as they merge together, i.e. in the limit when the end-winding enters both surfaces along their line of intersection.

The possibility of choosing a finite, rather than an infinite, permeability is here a useful one, since the rotor retaining ring is relatively close to the end-windings. It therefore not only has a comparatively important magnetic effect, but it is also liable to saturation.

(5.3) Core End Surfaces of Different Effective Permeability

A difference in the effective permeability of the stator and rotor iron is common to heterapolar machines operating under steady-state conditions, and a limitation of the image method is that it can take account of this only when the two surfaces are at right angles and not when they are in the same plane, as in an induction motor. This type of boundary configuration in which the core end surface is flat, with a change in effective permeability at the air-gap, cannot therefore be treated exactly, but an obvious approximation is to assign to the whole core end the permeability of its most important part. Thus, in calculating the field due to the stator end-winding, the stator end-plate is assumed to have a sufficient relative importance for the different rotor permeability to be ignored. The magnitude of the error which this entails depends to a large extent on the amount by which the end-winding is folded back towards the core, but some idea of its maximum value can be obtained by examining the effect of a change in the permeability of the core end surface on, for example, the inductance of the rectangular end-winding shown in Fig. 2(*a*).

The total flux linkage with the end-winding* is greatly affected by the change in boundary condition, as is evident from a comparison of the equivalent circuits in Figs. 3(*c*) and 7(*a*), but this difference is of little practical importance, for several reasons. In terms of the superposition method previously employed, the circuit in Fig. 7(*a*) may be separated into two components, one of which is shown in Fig. 2(*b*) and the other in Fig. 6(*c*). The second of these sets up a field which is relatively intense close to the air-gap, and this would contribute considerably to the flux linkage; but since the whole of this flux must pass through the iron surfaces on both sides of the air-gap, it is largely suppressed when the effective permeability of one of these surfaces is small. Furthermore, the flux distribution is such that most of it links both stator and rotor windings, and it is therefore relatively unimportant, since it is primarily the leakage inductances of the two windings which are required. Finally, the magnitude of the field associated with the circuit in Fig. 6(*c*) is greatly reduced when both stator and rotor windings are taken into account. In an induction motor the air-gap is short and the resultant m.m.f. associated with it—i.e. the resultant current in the circuit in Fig. 6(*c*)—is considerably less than the full-load m.m.f. of the winding. The two equivalent circuits which represent the air-gap, one associated with the stator and one with the rotor winding, largely neutralize each other.

It therefore appears that, in an induction motor, the effect of the air-gap on the total end-winding flux linkage is small, and on the leakage reactance is negligible, so that the two equivalent circuits to be compared are those in Figs. 3(*c*) and 2(*b*), rather than 7(*a*). The flux-linkage calculations for a single coil are given in Section 11.1 and the percentage difference, due to the two extreme boundary conditions, is plotted in Fig. 10, as a function of the shape of the projecting part of the circuit, assuming an effective conductor radius of one-tenth of the coil pitch. The difference is seen to be little more than 10% when the coil projects from the core by an amount equal to its pitch. This suggests that a difference in permeability of one part of the core end surface affects the leakage inductance by an amount which is sufficiently small to be ignored.

(5.4) Approximate Representation of Winding Supports

In a large machine the stator end-windings are usually lashed to magnetic supporting rings and these may have a significant effect on the end-field, and in particular, on the end-winding inductance. Although no exact treatment of such a surface, of finite width, is possible by the image method, a simple approximation can be made and this may be useful where some measure of the effect of these rings is required.

Consider a long straight conductor parallel to the surface of a

* Since the paper was written it has become evident that the boundary conditions referred to are not sufficient, and that the image representation is, in general, only approximate. However, it is exact when the permeability of one surface has any value and that of the other is zero or infinite (or unity).

* This concept is further discussed in Section 7.

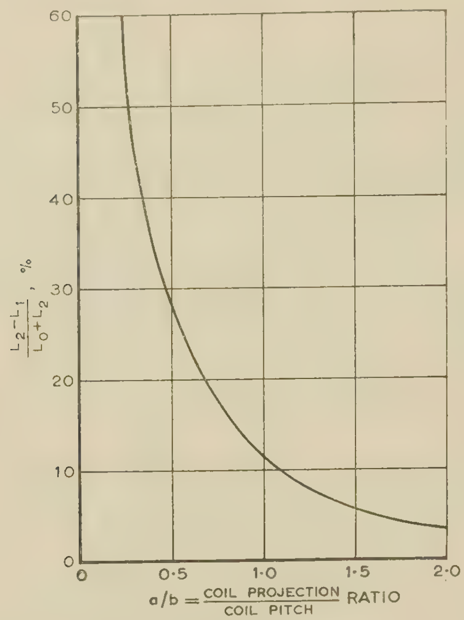


Fig. 10.—Effect of permeability on inductance of rectangular end-winding.

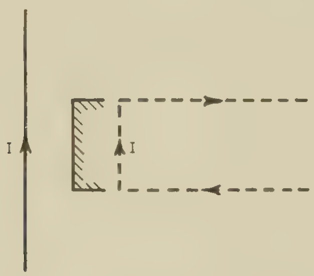


Fig. 11.—Image representation of narrow strips.

rectangular strip of iron which is infinitely long and which crosses the conductor at right angles, as shown in Fig. 11. The magnetic effect of the strip is represented approximately by the semi-infinite image circuit ABCD, since a sufficient number of the strips placed side-by-side constitute an infinite sheet in which the image is a long conductor parallel to the first, and this is also the result of the addition of a series of the image circuits placed side-by-side. The magnitude of the image current is given, in general, by eqn. (1*a*). If the conductor passes obliquely across the surface of the strip, the image circuit is similarly oriented.

The image circuit has the useful practical feature that the sides AB and DC are at right angles to the plane of the end-winding, so that they contribute nothing to the inductance calculation, while the effect of the side BC is very easily calculated by the vector-potential method (Section 7). Clearly the image representation is not a close one, since the boundary conditions imposed by the strip are obtained only approximately, but it provides a simple means of allowing for the small additional flux linkage contribution due to a narrow strip of infinite or of finite permeability.

(5.5) Effect of Finite Core Length

In machines in which the length of the core is comparable with the coil pitch the shape of the end-winding at one end of the core would have a significant effect on the field at the other end if the permeability were sufficiently low. However, if the effective permeability is zero, because of eddy currents, th magnetic field at the iron surface is determined entirely by th field sources outside that surface, and this would also be true the permeability were infinite (provided that there is an air-gap since the iron surface is then an equipotential, with a discon tinuity whose magnitude is not affected by the other end-regio Thus, in practice, the interaction between the two end-fields likely to be entirely negligible and the two sets of image circui can be treated separately, even though they may overlap wi each other and possibly project beyond the far end of the cor

(6) MAGNETIC EFFECT IN END-REGION OF EMBEDDE PART OF CIRCUIT

It has so far been implicitly assumed that the slots and th conductors in them are at right angles to the core end surface so that the embedded part of the circuit coincides with t straight conductors in Fig. 3(*a*). In machines in which the slo are skewed, the amount of skew would, if there were no air-ga effect the field outside the iron, since there are both tangenti and normal field components at the iron surface. The tangent field is, however, considerably modified by the air-gap, whi makes the end-field independent of the shape of the embedde part of the circuit when the permeability is infinite, since the tw parts of the iron then act as a magnetic screen. At the oth extreme, a core of zero transverse permeability likewise scree the end-region from the slot conductors. The effect of sl skew on the end-field is therefore likely to be entirely neglig in a practical machine and, consequently, none of the equivale circuits needs to be modified to take it into account. If, on t other hand, there were no air-gap and no induced currents, th shape of the embedded part of the circuit would be importa however large the permeability, but it could still be calculat by the image method.

Consider a circuit of any arbitrary shape penetrating t surface of a continuous iron block of uniform permeability. the whole of the circuit were embedded, the field, both insi and outside the iron, could be given by image currents who magnitude and direction can be obtained from eqns. (1*a*) an (1*b*) by replacing μ by its reciprocal. Thus the internal field that due to the actual current I together with an image

$$I' = \frac{\mu - 1}{\mu + 1} I \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

and the external field is that due to a current

$$I'' = \frac{2\mu}{\mu + 1} I \quad . \quad . \quad . \quad . \quad . \quad (4$$

in place of I. When there is a part of the circuit on both sid of the iron surface, an image representation must still be possib since the superposition method used in Section 3.2 is st applicable, giving here three component circuits including o which is confined to the interior of the iron. However, it unnecessary to find the images associated with these compone and to add them up, since the currents in the part-circuits a their images always satisfy the continuity condition at the ir surface. Thus the sum of the two currents I' and I'' on the o side of the iron surface, given by eqns. (1*a*) and (4*b*) respectivel or alternatively by eqns. (4*a*) and (1*b*), is equal and opposite the current I on the other side of the surface. The field outsi the iron is therefore given by the part of the circuit outside t iron, carrying the current I, its reflection in the iron surface, a a third part-circuit corresponding in position to the embedd conductor but carrying a current I'' given by eqn. (4*b*). T three components meet at the surface and no other currents a required to satisfy the condition of continuity of current flow.

When the permeability is infinite the current I'' given by qn. (4*b*) is twice the actual circuit current. Thus the embedded part of the circuit sets up a field in the end-region which is exactly twice the field which it would produce if there were no iron. This doubling of the effective slot current is consistent with the result previously obtained.

(7) END-WINDING INDUCTANCE

(7.1) Effect of Air-Gap

The equivalent circuits which have been derived provide a means of computing the field outside the core, and an important practical application is to the calculation of the component of circuit inductance which is associated with this field. In general, there is no precise way in which such an inductance component can be defined, and any part of the flux which passes from the air into the iron may be attributed with equal validity to either the end-region or to the core, or divided between them. The air-gap, in particular, causes a strong field component normal to the core end surface when the effective permeability is high, and there is some advantage in separating this component, by superposition, in the manner adopted in Section 5.3. Although the field is given accurately over most of the end-region by the appropriate equivalent circuit (i.e. that shown in Fig. 7), the accuracy diminishes close to the air-gap, where the field becomes progressively stronger and provides an important part of the flux linkage. In this region the equivalent circuit represents the field only approximately, partly because the boundary condition at the entrance to the air-gap is incorrect, but more particularly because the field is here very greatly affected by local irregularities in the core end surface, especially the slot openings.

There are thus two flux-linkage components which are here distinguished. The first is that produced by the circuit shown in Fig. 2(*b*), or 4(*b*), and the second is that which is due specifically to the air-gap and which is represented by the circuit in Fig. 6(*c*). The former is not greatly affected by the detailed geometry of the core end surface, but is determined primarily by the shape and diameter of the end-winding conductors. The latter, on the other hand, is intimately associated with the core, and the field from which it is derived is essentially the core 'fringe' field. It is entirely dependent on the shape of two end surfaces and the potential difference between them, and it is unaffected by the end-winding configuration. Moreover, as pointed out in Section 5.3, the greater part of this component of the flux links both the rotor and stator windings of a round-rotor machine, and in this respect also it is more closely related to the core flux than to the end-field.

When viewed as a mutual flux linkage, additional to the main core flux, and indeed, not clearly separable from it at the entrance to the air-gap, this component is evidently of no great importance, as compared with that part of the end-field which contributes to the leakage flux. The effective increase in the core flux can be allowed for by adding a small correction factor to the core length. The calculation given in Section 11.2 shows that the upper limit to this correction factor is not much more than the length of the air-gap (at both ends of the core). Once the air-gap has been taken into account in this way, the equivalent circuits in Fig. 7 must be replaced by those in Figs. 2(*b*) and 4(*b*) for the purposes of end-winding flux-linkage calculations.

(7.2) Method of Calculating Flux Linkage

Once the iron parts have been replaced by equivalent circuits in air, the flux-linkage calculation can often be made more simply in terms of vector potential (i.e. by Neumann's method) than from the flux density. The vector potential due to a line current I flowing round a closed contour l_1 is given by[9, 10]

$$A = \frac{\eta_0}{4\pi} I \oint_{l_1} \frac{1}{r} dl_1 \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

and the flux linkage with a second closed contour l_2 is

$$\Phi = \oint_{l_2} A \, . \, dl_2 \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

so that the surface integral of flux density is replaced by a line integral. When the circuit can be represented—as is usually the case—by a series of straight segments, the vector potential due to each segment is everywhere in the same direction, so that its integral along any part of the circuit is a scalar integral of one variable and the result is zero if the two parts are at right angles. The external component of the flux due to the current in the conductor is obtained by integrating along a line in the conductor surface. The method is illustrated by the calculations given in the Appendices. The general form of the integral for two non-planar filaments inclined to each other is given by Campbell.[11]

The vector-potential method is not only convenient for computation, but also makes it simpler to separate the end-region from the core flux. When fringing has been allowed for by increasing the effective core length, the field inside the iron is assumed to be 2-dimensional, whether or not the component normal to the end surfaces is suppressed by eddy currents. The corresponding vector-potential field inside the core is everywhere parallel to the slots, and the component of flux linkage which is attributed to the core flux is therefore obtained by integrating the vector potential along the slot conductors. It follows that the remaining flux linkage, which is to be associated with the end-winding, is given by the vector-potential integral along the end-winding up to the points at which it enters the iron. Unless the effective permeability is zero, this result is not the same as that obtained by integrating the flux density over a surface which is bounded on one side by the iron surface and on the others by the end-winding contour. These two calculations are equivalent only if the vector-potential integral is zero along the curve of intersection of the flux integration surface and the core end surface.

The point is illustrated in Fig. 12, which shows the field due to a rectangular end-winding as viewed in a plane drawn at right angles to both the core-end surface and the plane of the winding. A high permeability results in a field configuration like that shown in Fig. 12(*a*), and the image current at C′ is in

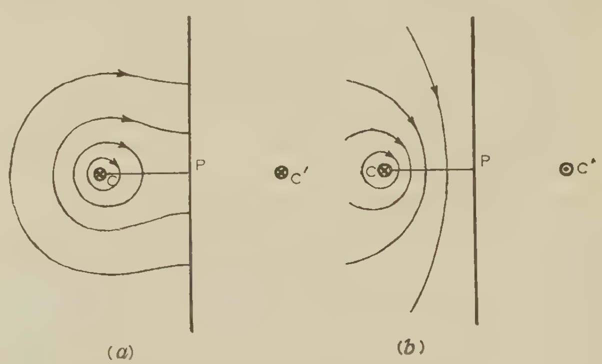


Fig. 12.—Flux-linkage calculation.
(*a*) Surface of infinite permeability.
(*b*) Surface of zero permeability.

the same direction as the current in the conductor C. When the eddy currents are such as to suppress completely the normal component of flux, the sign of the image is reversed and the flux pattern is changed to that shown in Fig. 12(*b*). Here the whole of the flux in the end-region crosses the plane represented by the line CP, whereas in Fig. 12(*a*) only a part of the flux is obtained by integrating over this plane; moreover, this flux component is reduced and not increased as the core permeability is increased.

A different result is obtained when the vector potential is integrated round the end-winding. Fig. 12(*a*) represents the field set up by the equivalent circuit shown in Fig. 2(*b*), and it is evident from the form of this circuit that the integral of the vector potential along DCBA [Fig. 2(*b*)] is greater than that obtained when the rectangle is closed along AD. It is also greater than the result obtained by integrating along DCBA in Fig. 3(*c*), which is the equivalent circuit corresponding to Fig. 12(*b*). On the other hand, the integral along AD in Fig. 3(*c*) is zero, so that, when the effective permeability is zero, it is immaterial whether the contour is closed or not; but for all other permeabilities the integral must extend round the end-winding only.

It is instructive to examine the form of the surface which will, in general, intersect all the flux lines. If the circuit is confined to a plane, there can be no vector-potential component in the direction normal to it, so that the integration can be extended in this direction without modifying the result. It follows that the same flux linkage is obtained by integrating the flux over any semi-infinite surface which is bounded by the end-winding and by two straight lines drawn in the iron surface, in a direction normal to the plane of the circuit, from the points of intersection of the circuit outwards to infinity. If the circuit is not confined to a plane, the two lines must be replaced by curves drawn so that they everywhere intersect the potential vector in a direction at right angles to it. In Fig. 3(*c*) the line AD is an example of such a curve, but the same line in Fig. 2(*b*) does not satisfy the condition.

(7.3) Flux Linkage due to Distributed Windings

The flux linkage with a coil end in a machine winding is considerably affected by adjacent coil ends, both on the same side of the air-gap and on the opposite side, and various methods have been suggested in the literature for taking the mutual interactions into account. When the coil end may be represented by a small number of straight-line elements, the image representation provides a means of estimating the mutual inductances directly by relatively simple calculations in terms of vector potential. The method used for self-inductance may be extended in an obvious manner to the calculation of mutual inductance. The integration contour l_2 of eqn. (5*b*), which is now entirely different from l_1, is again defined by the coil end and should not be closed, or flux drawn locally into the core will be excluded.

The method is illustrated by the calculation given in Section 11.3 of the end-winding inductance for a 3-phase 2-layer full-pitch winding with 60° coil groups. It is assumed that each of the winding layers is confined to a plane and is perpendicular to a flat core end of zero effective permeability, so that the result is directly applicable to induction motors. The coil-end shape is replaced by a form which is triangular in plan (Fig. 16), the sides of the triangles being joined at the apex by a short length of conductor perpendicular to the plane of the winding, and the angle subtended being taken to be 90°. This simplifies the calculation, and since the intrinsic effect of conductor inclination angle is small, it gives a close approximation to the correct result for other angles provided that the length of the end conductor is correct; this is ensured by expressing the result as a specific inductance per unit conductor length. The rotor winding is ignored, but the calculation may be readily extended to find the mutual linkage with a rotor end-winding of specified shape.

The instantaneous flux linkage with a coil end is given by the vector potential integral along the two sides of the triangle and along the short connector between the two layers. The component of vector potential required for each part is that due to the part itself plus the contributions from the parallel parts of the other coils, and this may be expressed as a simple logarithmic function which is easily evaluated. The contribution due to the connecting piece could be calculated in the same way, but it is sufficiently short and its effect is sufficiently similar to that of a corresponding length of the coil side, for a separate calculation to be unnecessary.

Once the mutual-inductance coefficient is known for each of the relative positions of the coils in the end-winding, the flux linkage with one coil, and thus the end-winding inductance, is obtained by summing contributions from adjacent coil ends. One limitation to the accuracy of the result is the relatively large coil section, but this can be allowed for, if necessary, by reducing the coil to a series of filaments, which may correspond to the actual conductors, and evaluating the vector potential along each of these.

(8) CONCLUSIONS

The image principle is applicable to coils which are partly embedded in iron, and it provides a relatively simple means of estimating the end-winding fields in non-salient-pole machines. By combining it with the vector-potential method the flux linkages in complex end-winding arrays can be obtained in a surprisingly simple manner when the end-winding shapes reduce to straight lines. Since the field and inductance calculations are expressed as summations, they are in general in a form which is particularly suitable for evaluation by a digital computer.

When considering circuits which intersect the reflecting surface it is important to obtain continuity of current flow at the points of intersection. The method becomes invalid if this condition is not complied with, as in some forms of image circuit which have been suggested. The image principle can be applied, in general, to a coil of any shape penetrating a semi-infinite plane iron block in any arbitrary manner, and when there is no air-gap intersecting the block, the method is valid for all constant permeabilities. Another type of reflecting surface which can be represented is one consisting of two planes intersecting at right angles and, since the two parts of the surface need not have the same permeability, this is of particular value in the calculation of end-fields in turbo-type alternators operating under both steady-state and transient conditions.

When the reflecting surface has a high permeability, the presence of an air-gap in the iron modifies the end-field considerably, but in a way which can be represented by a simple equivalent circuit. For the purpose of inductance calculation, however, it is preferable to separate the flux outside the iron into two components, one of which is associated with the core and takes the air-gap into account, while the other is not significantly affected by it and can therefore be represented by the image circuit derived for a core without a gap. Viewed in this way, the component of the field in the end-region which is due to the air-gap becomes a part of the core field, and it contributes to the mutual rather than to the leakage flux of the windings. The flux which it represents can be allowed for by increasing the length of the core by a factor which is largely independent of the shape or arrangement of the coil ends.

The field due to a coil intersecting a reflecting surface which is divided by the air-gap into two parts of different effective permeability cannot be treated in any exact manner by the image

method when the two parts of the surface are not at right angles. Thus the different boundary conditions due to the different flux-wave velocities on opposite sides of the air-gap cannot, in general, be represented. However, the effect on the inductance of a single coil of changing the permeability of the entire end surface from zero to infinity has been shown to be of the order of only 10% for a rectangular coil end when only those flux components which contribute to the leakage inductance are taken into account.

(9) ACKNOWLEDGMENTS

The author is indebted to Mr. S. Neville, of A.E.I. (Manchester) Ltd., for the stimulus without which the paper would not have been written.

(10) REFERENCES

(1) Smith, R. T.: 'End Component of Armature Leakage Reactance of Round-Rotor Generators', *Transactions of the American I.E.E.*, 1958, **77**, Part III, p. 636.
(2) Honsinger, V. B.: 'Theory of End-Winding Leakage Reactance', *ibid.*, 1959, **78**, Part III, p. 417.
(3) Hague, B.: 'Electromagnetic Problems in Electrical Engineering' (Oxford University Press, 1929).
(4) Douglas, J. F. H.: 'Reactance of End Connections', *Transactions of the American I.E.E.*, 1937, **56**, p. 257.
(5) Caldwell, B. H.: 'End-Winding Inductance of a Synchronous Machine', *ibid.*, p. 455.
(6) Harrington, D.: 'Forces in Machine End Windings', *ibid.*, 1952, **71**, Part III, p. 849.
(7) Carter, G. W.: 'The Electromagnetic Field in its Engineering Aspects' (Longmans, Green, 1957).
(8) Searle, G. F. C.: 'On the Magnetic Field Due to a Current in a Wire Placed Parallel to the Axis of a Cylinder of Iron', *Electrician*, 1898, **40**, pp. 453 and 510.
(9) Smythe, W. R.: 'Static and Dynamic Electricity' (McGraw-Hill, 1950).
(10) Jeans, J. H.: 'Mathematical Theory of Electricity and Magnetism' (Cambridge University Press, 1915), p. 200.
(11) Campbell, G. A.: 'Mutual Inductances of Circuits Composed of Straight Wires', *Physical Review*, 1915, **5**, p. 452.
(12) Hammond, P.: 'The Calculation of the Magnetic Field of Rotating Machines, Part I', *Proceedings I.E.E.*, Monograph No. 333, May, 1959 (**106 C**, p. 158).
(13) Hammond, P.: 'Electric and Magnetic Images', *ibid.*, Monograph No. 379, May, 1960 (**107 C**, p. 306.)

(11) APPENDICES

(11.1) Effect of Core Permeability on Inductance of Rectangular End-Winding

The influence of the effective permeability of the core on the component of inductance which is associated with the end-winding is here analysed for an end-connection consisting of a circular rod of radius r bent into the form of a semi-infinite rectangle of side b and projecting distance a from the iron surface [Figs. 2(*a*) and 13]. It is assumed that the air-gap produces a flux linkage which, since it does not contribute significantly to the leakage reactance, is allowed for by increasing the core length by an appropriate amount. The image circuits which are then required for the limiting conditions of zero and infinite permeability are those shown in Figs. 13(*a*) and 13(*b*). The end inductance associated with both circuits is given by the vector-potential integral round ABCD, excluding the line DA.

The vector potential at a point P distance y from the axis of a straight current filament carrying a current I is, from eqn. (5*a*),

$$A = I\eta_0\frac{1}{4\pi}\int_2^1 \frac{dx}{\sqrt{(x^2+y^2)}} = I\eta_0\frac{1}{4\pi}\left(\operatorname{arc\,sinh}\frac{x_1}{y} - \operatorname{arc\,sinh}\frac{x_2}{y}\right)$$

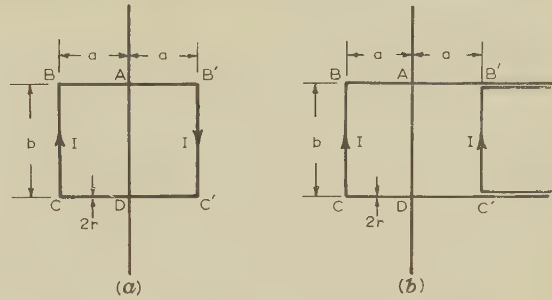


Fig. 13.—Inductance of rectangular end-winding.
(*a*) Zero permeability.
(*b*) Infinite permeability.

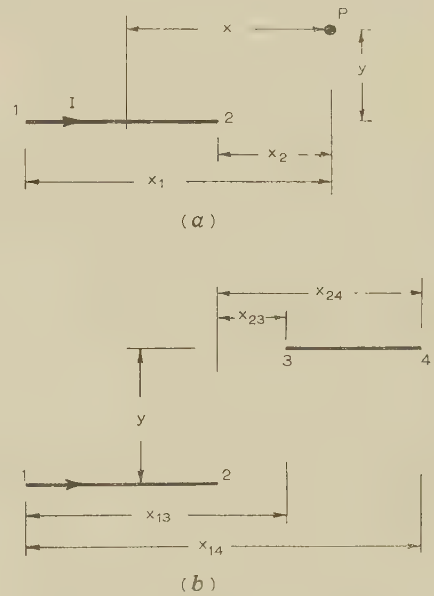


Fig. 14.—Parallel current filaments.
(*a*) Co-ordinates of field point.
(*b*) Co-ordinates of parallel filament.

where x is the axial distance from the element contributing to the field [Fig. 14(*a*)]. Integrating this expression between the two ends, denoted 3 and 4, of a parallel line [Fig. 14(*b*)], gives

$$\int_3^4 A\,.\,dl = I\eta_0\frac{1}{4\pi}y(X_{14} + X_{23} - X_{13} - X_{24}) \quad . \quad (6a)$$

where

$$X_{nm} = X\left(\frac{x_{nm}}{y}\right) = \frac{x_{nm}}{y}\operatorname{arc\,sinh}\frac{x_{nm}}{y} - \sqrt{\left[\left(\frac{x_{nm}}{y}\right)^2 + 1\right]} \quad (6b)$$

x_{14}, x_{23}, x_{13} and x_{24} being the distance between the ends, measured parallel to the filaments, as shown in Fig. 14(*b*). The function X in this equation is dependent only on the magnitude and not the sign of the distances. The function is plotted in Fig. 15.

Eqns. (6) provide a simple means of estimating the contributions from each of the elements of the circuits shown in Figs. 13(*a*) and 13(*b*) to the vector-potential integral along ABCD. Provided that the effective radius of the conductor is small compared with the other dimensions, the circuit-elements may be replaced by line currents at their axes. The integral is evaluated along the conductor surface at an effective radius r, but since this distance is small, it can be neglected except when calculating the

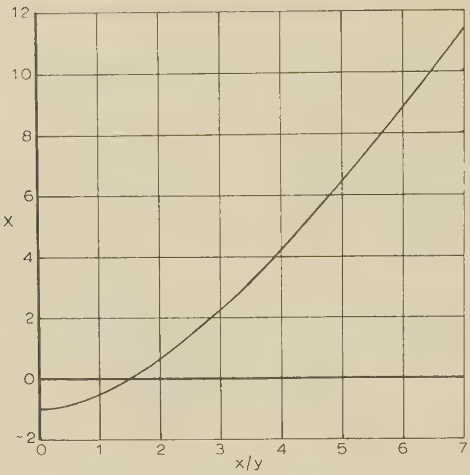


Fig. 15.—Graph of vector-potential function.

$$X = \frac{x}{y} \text{ arc sinh} \frac{x}{y} - \sqrt{\left[\left(\frac{x}{y}\right)^2 + 1\right]}$$

contribution in a given part of the circuit from the current in that part or in any other part coaxial with it. Hence, in both circuits, the contribution along BC due to the current in BC is

$$I\frac{\eta_0}{4\pi}r\left[2X\left(\frac{b}{r}\right) - 2X\left(\frac{0}{r}\right)\right]$$

which may be written

$$I\frac{\eta_0}{2\pi}b\left(\log_\varepsilon \frac{2b}{r} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

since the ratio b/r is large. The contribution along AB and CD due to the current in BB′ and CC′ is, making a similar approximation,

$$I\frac{\eta_0}{2\pi}b\left[\frac{2a}{b}\left(\log_\varepsilon \frac{4a}{r} - 1\right) - 1 - X\left(\frac{2a}{b}\right)\right]$$

so that the component of inductance which is common to both circuits is

$$L_0 = \frac{\eta_0}{2\pi}b\left[\frac{2a}{b}\log_\varepsilon \frac{4a}{b} + \left(\frac{2a}{b} + 1\right)\log_\varepsilon \frac{b}{r} - \frac{2a}{b} - X\left(\frac{2a}{b}\right) - 2 + \log_\varepsilon 2\right] \quad (8)$$

The zero-permeability image circuit [Fig. 13(*a*)] contributes an additional term due to B′C′ which is

$$L_1 = -\frac{\eta_0}{2\pi}2a\left[X\left(\frac{b}{2a}\right) + 1\right] \quad . \quad . \quad . \quad . \quad (9)$$

and the further contribution from the image circuit in Fig. 13(*b*) is

$$L_2 = \frac{\eta_0}{\pi}b\left[\frac{a}{b}\log_\varepsilon \frac{b}{a} + \frac{a}{b}X\left(\frac{b}{2a}\right) + X\left(\frac{2a}{b}\right) - X\left(\frac{a}{b}\right) + \frac{a}{b}(2 - \log_\varepsilon 8)\right]. \quad (10)$$

The difference between these two inductances, i.e. $L_2 - L_1$, expressed as a percentage of the total inductance $L_0 + L_2$, is plotted in Fig. 10 as a function of a/b. To obtain the result in numeric form the conductor radius has been arbitrarily assumed to be one-tenth of the winding pitch, i.e. $b = 10r$. This ratio does not affect the difference between the two inductances, but it does affect the percentage difference, which is reduced by a reduction in the radius r.

(11.2) Calculation of Core Fringing Factor

The equivalent circuit shown in Fig. 6(*c*), which represents the effect of the air-gap in an infinitely permeable core, produces a field having a maximum intensity close to the core surface. The greater part of the flux produced will therefore link both stator and rotor windings, so that it represents an addition to the core flux. The magnitude of this 'fringe' flux is greatly dependent on the geometry of the core end surface, but its maximum possible value can be estimated by assuming a flat end-surface without slot openings and calculating the linkage with a rectangular end-winding which projects from the core by a sufficiently large amount to embrace the whole of the flux represented by one pole-pitch. The equivalent circuits producing this linkage [Fig. 6(*c*)] consist of a series of semi-infinite rectangular circuits, each spanning one pole-pitch, but only one of these contributes significantly to the linkage. The next pair will reduce it, but by an amount which is small compared with the effect of the slot openings. The problem is therefore that of calculating the mutual flux linkage between two semi-infinite rectangular circuits placed end-to-end and having a coil span b.

The vector-potential contribution due to the adjacent sides is given by eqn. (7). The effect of the longer sides is obtained by considering filaments of finite length, which gives a result similar to that obtained by adding eqns. (9) and (10), and then approaching the limit with respect to the overhang length. The contribution is

$$-I\frac{\eta_0}{2\pi}b$$

so that the total flux linkage is

$$I\eta_0\frac{1}{2\pi}b\left(\log_\varepsilon \frac{2b}{r} - 2\right) \quad . \quad . \quad . \quad . \quad (11)$$

for a conductor of radius r. If the diameter of the conductor is the same as the length of the air-gap, eqn. (11) gives the whole of the flux passing from the end-plate into the end-region, as integrated up to a semi-circular flux line which is terminated by the two corners of the air-gap. This line may be reasonably taken as representing the limit of the air-gap—i.e. the core-flux, so that eqn. (11) represents the fringe flux when $r = \frac{1}{2}g$, where g is the gap length.

It is convenient to express the result in terms of the flux per unit length in the core, which is

$$I\eta_0\frac{1}{2g}b$$

so that the end-region flux is that which would be obtained by adding a length l_e (at both ends) to the core, where l_e is given by

$$l_e = \frac{g}{\pi}\left(\log_\varepsilon \frac{4b}{g} - 2\right) \quad . \quad . \quad . \quad . \quad (12)$$

For a ratio of b/g, i.e. of coil pitch to gap length, of 100, eqn. (12) reduces approximately to

$$l_e = 1 \cdot 3g$$

(11.3) Induction-Motor End-Winding Inductance

Fig. 16 represents the coil groups of a full-pitch 3-phase 2-layer end-winding with 60° phase groups. It is assumed that each of the winding layers may be regarded as being confined to a plane perpendicular to the core end surface and that the coil ends, seen in plan, form the two sides of a triangle each of

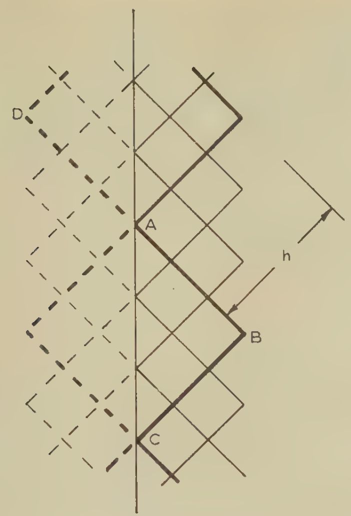


Fig. 16.—Induction-motor end-winding.

length h and subtending an angle of 90°. The other parts of the coil end, i.e. the projecting part of the slot conductor and the part joining the two layers, may be subsequently taken into account with sufficient accuracy by adjusting the result according to the actual length of the end conductor. The core end surface is assumed to have zero permeability, so that the image circuits are as shown in Fig. 16.

The total end-field linkage with a coil is given by the vector-potential integral round the two ends of the coil, i.e. round the square circuit formed by one of the ends and its image. The contribution to this linkage from a neighbouring turn, such as that shown in Fig. 17, is obtained by summing the vector-

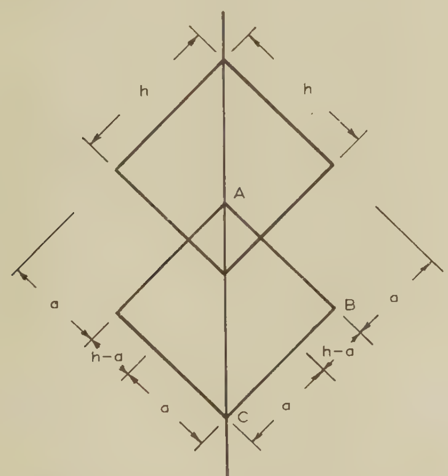


Fig. 17.—Relative positions of coil ends.

potential contributions from parallel sides, as given by eqn. (6). The summation may be written

$$\Phi = I\frac{\eta_0}{\pi}\left\{a\left[X\left(\frac{h+a}{a}\right) + X\left(\frac{h-a}{a}\right) - 2X(1)\right]\right.$$
$$+ (h-a)\left[X\left(\frac{a}{h-a}\right) - \frac{1}{2}X\left(\frac{h+a}{h-a}\right) - \frac{1}{2}X(1)\right]$$
$$\left. + (h+a)\left[X\left(\frac{a}{h+a}\right) - \frac{1}{2}X\left(\frac{h-a}{h+a}\right) - \frac{1}{2}X(1)\right]\right\} \quad (13)$$

where $X(x/y)$ is the function defined by eqn. (6b) and plotted in Fig. 15, I is the current in the coil and a is the distance between coils, as shown in Fig. 17. All distances must be interpreted with a positive sign for all positions of the two circuits.

The flux linkage given by eqn. (13), which must be summed over the adjacent coil ends of the winding, may also be expressed in terms of a vector-potential integral along a single conductor, such as DB, due to the parallel conductors, and a considerably simpler expression is obtained for each contribution. However, the resultant series converges more slowly than that given by eqn. (13), whose components diminish less rapidly than their sum, so that a considerable advantage is gained by relating the typical calculation to a closed circuit rather than to a straight current filament.

Eqn. (13) gives the self-inductance of a single end-turn when a is sufficiently small; it then becomes the effective radius, r, of the conductor and the equation reduces to

$$\Phi = I\frac{\eta_0}{\pi}2h\left(\log_\varepsilon\frac{2h}{r} - 1\cdot467\right)$$

Substituting increasing values of a, the mutual-inductance coefficients for successive coil positions in the end-winding (Fig. 16) are as follows:

a/h	1/3	2/3	1	4/3	5/3
$\pi\Phi/2\eta_0 Ih$	0·508	0·016	−0·099	−0·023	−0·01

To obtain the flux linkage with a coil at any given instant it is necessary merely to multiply the appropriate inductance coefficient by the instantaneous current in each of the coils and to sum the results. By repeating the calculation the flux linkage may be obtained as a function of time. However, when there is no relative motion between the coils, i.e. for coils on the same side of the air-gap, the flux-linkage function can contain no harmonics if the currents are sinusoidal, and it is necessary to calculate only the maximum value. This occurs when the current in the phase concerned is a maximum and that in the other two phases is one-half this value. Hence, with due regard to sign, the end-field flux linkage with one coil is

$$\Phi = I\frac{\eta_0}{2\pi}\left(\log_\varepsilon\frac{2h}{r} - 1\cdot467 + 0\cdot70\right)$$
$$\simeq 2\times 10^{-7}I\log_\varepsilon\frac{h}{r}$$

in webers per metre length of the end conductor. The corresponding inductance per phase of a winding with p poles and having T turns in series is

$$L = 8\times 10^{-7}h\frac{T^2}{p}\log_\varepsilon\frac{h}{r}\ \text{henry} \quad . \quad . \quad . \quad (14)$$

where h is, in general, the half-length of the end-winding conductor (in metres) and r is the effective radius of the conductor group forming the phase belt. For simplicity, the phase group has been made the basis of the calculation, but the method is equally applicable to individual coils.

An expression for leakage inductance is obtained by estimating and subtracting the flux linkage with the rotor winding, which is given by the vector-potential integral round a rotor coil-end. The result will depend on the machine dimensions and winding arrangement. For the present purpose it is suffi-

cient to observe that eqn. (14) gives, approximately, the stator leakage of a squirrel-cage machine in which the rotor end-rings are close to the core. The principal assumption made in the analysis, namely that each layer of the end-winding lies in a plane, may make the result relatively inaccurate when applied to a 2-pole machine. Here the field distribution is likely to be considerably different from that in a machine wound for more poles, as is indicated by the detailed calculations made by Hammond.[12]

Of the many published expressions for end-winding leakage, some give coefficients which are smaller than that predicted here and some are higher. A detailed comparison does not appear to be warranted until further experimental evidence is available.

DISCUSSION ON 'ELECTRICAL SUPPLIES TO POWER STATION AUXILIARIES'*

Before the North-Eastern Centre *at* Newcastle upon Tyne *9th November*, 1959, *and the* South Midland Supply and Utilization Group *at* Birmingham *11th January*, 1960.

Mr. R. R. Pattinson (*at Newcastle upon Tyne*): Dr. C. H. Merz was the first to advocate the unit system and it is interesting to learn that some four decades later this system is still the most favoured. The variant shown in Fig. 2(*a*) is probably of most interest oversea where load requirements can be met from relatively small machines and where stations are isolated: it could be made more attractive if the simpler and cheaper load-making isolator replaced the generator circuit-breaker.

Power station design is primarily one of reconciling the need to keep to a minimum the rating of switchgear whilst maintaining variation of voltage within defined limits. Lack of accurate loading figures, and extraneous influences such as the requirements of architects that dictate plant positioning, make power station design an exercise, not in precision, but of intelligent compromise.

On the matter of standard voltages, the latitude given under (*a*) and (*b*) of Section 9 may lead to a selection of main voltage that would not give the best results. It would be wrong to allow a few large motors to sway the decision to a voltage that favoured them if, as a consequence, many motors of smaller output had to be designed for an unnatural voltage. The correct approach would be to group the motors in the voltages that would produce the most reliable design and then decide the main voltages. Thus, a single step-up in voltage from say 3·3 to 6 kV may not be the solution and two main voltages rather than one would be justifiable.

The right voltage for one power station layout can be uneconomic for another, even although the turbo-generators are of similar size. Such questions as whether the feed pumps are to be two 100% or three 50%, and whether the boiler auxiliaries are large in size and small in number or vice versa, are decisive. Thus, the cost comparison in Section 15.3 can be accepted only with this reservation.

Finally, there are many problems which call for further thought. These include:

(*a*) The necessity for taking into account the infeed into faults of large induction motors and the effect of this on the making capacity of auxiliary switchgear.

(*b*) The drop-out voltage has over the years been raised from $\frac{2}{3}$ normal voltage to 70% and is now 75%. With the modern high-efficiency motor the danger of stalling under transient conditions of low voltage is increased, leading to serious consequential damage, particularly if such urgent auxiliaries as the boiler-feed-pump motors were involved.

(*c*) Machine and boiler instrumentation is requiring a most complex system of controls and alarms. As an indication, one modern power station required a multicore cable schedule with more than 7000 different items, some of which comprise cables with as many as 27 cores.

Mr. T. H. Morrison (*at Newcastle upon Tyne*): I agree that the distribution system is built around the starting and running of the boiler feed pumps. These are increasing in size with larger boiler units and generally two 100% pumps are employed. Although they may be more economical and efficient than a larger number of smaller ones, their effect on the electrical system must be taken into account. Larger pumps mean larger transformers and higher-voltage switchgear with a higher rupturing capacity and higher costs. Therefore the overall economics of pumps, piping, foundations, control gear, transformers, etc., must be examined, and it may be found that smaller pumps are more economical.

The effect of large voltage drops due to starting boiler feed pumps must be taken into account when deciding the type of lighting installation. Blended lighting fittings, comprising tungsten and mercury discharge lamps, are in common use. The mercury lamps will go out if the voltage drops below 80% of normal and require about 10 minutes to cool down before re-striking. Lamps going out in this fashion have a very disconcerting effect on the operating staff.

Fig. 3 is valuable, but I suggest that hot starts are more important than cold starts as they take place in a shorter time, with considerable overlapping. Most stations being constructed to-day will eventually become 2-shift stations, with the inevitable starting procedure of several sets in the early morning. Can the author give some information on the loadings imposed by multiple starts?

Mr. G. W. B. Mitchell (*at Newcastle upon Tyne*): Tripping of urgent auxiliaries or of the main circuit-breaker in the alternator circuit causes loss of output. The bigger the set, the more important it is to avoid unnecessary loss of output, and I think the author should have referred to the important question of neutral earthing on the station, unit and other auxiliary transformers.

Present C.E.G.B. practice is to earth the neutrals of alternators and station and unit transformers via resistances and 415-volt transformers solidly. This involves the provision of additional relays for earth-fault protection and any earth fault causes tripping.

A single earth fault (which is by far the most common type) in the alternator circuit, in the unit transformer or on the unit board causes complete shut-down of a machine, and this can be avoided by earthing the neutrals via the primary windings of voltage transformers with alarm relays connected to their secondaries. This scheme was successfully employed in this country for over 30 years and is still employed oversea. When proper attention has been paid to one or two details I know of no troubles.

* Dewison, D. A.: Paper No. 2759 S, December, 1958 (see 106 A, p. 451).

In recent years some engineers have suggested that the introduction of voltage transformers might cause transient overvoltages despite the fact that this point was investigated, both theoretically and practically, by certain Tyneside firms over 20 years ago.

Recently, the Tyneside findings have been confirmed by a joint C.E.G.B./E.R.A. investigation. I suggest strongly that we should revert to voltage transformer earthing, particularly having regard to the undesirability of suddenly unloading modern high-pressure and high-temperature turbines because of the possibility of valves sticking.

Mr. D. Riach (*at Newcastle upon Tyne*): Will the author state the function of the circuit-breakers in the output connections from the turbine house and boiler house unit auxiliary transformers shown in Fig. 1 and the 1·25 MVA unit auxiliary transformers shown in Fig. 6(*b*)? These circuit-breakers are unnecessary for the protection of the transformers and it is difficult to imagine how their inclusion has been justified on other grounds.

Are the selector switches for the supplies to the boiler services boards shown in Fig. 1 off-load operated? If so, are they interlocked with the respective circuit-breakers on the station boards? Such interlocking would involve tripping both these circuit-breakers to permit operation of a selector switch. Consequently the inclusion of circuit-breakers between each selector switch and its respective boiler services board might be justified. What is the author's view?

All the auxiliaries shown in Fig. 1 are controlled by circuit-breakers. From Table 1, the ratings of the coal feeders, air-heater oil pumps and the generator-transformer oil pumps are only 3·3, 1 and 6 h.p. respectively. Besides the considerable cost of circuit-breakers for such small motors a problem arises in designing current transformers of the required short-circuit rating with adequate output to operate the protective gear. Does the author agree that in such cases it would be more suitable to substitute contactor gear backed up by h.r.c. fuses?

Several of the transformers shown on the diagrams are not included in the list of preferred kVA ratings in B.S. 171. It would be interesting to know the justification.

Mr. A. T. Crawford (*at Newcastle upon Tyne*): The paper gives a useful record of the costs of equipment at May, 1958. The significance of this should be appreciated as prices will fluctuate and each individual station should be reviewed separately with the up-to-date estimates prepared to assess the overall economic position. The application of two panels in parallel for service on the heavy-current circuits has the advantage, particularly when relatively few heavy-current panels are required, that considerable duplication of electrical and mechanical type testing can be obviated, thus achieving economy in its broadest sense.

It would be useful to have the author's views on the practice which has been adopted in one or two instances of incorporating protective equipment interspersed with the auxiliary switchgear panels; when work is to be done in the relay cubicle, which also houses primary connections running between adjacent panels, covers may be left off by accident and an unnecessary hazard is introduced. It would be preferable to dissociate secondary and primary equipment more completely. Has the author any comments on the need to depart from the single busbar layouts that he shows?

In Section 15.2 costs are included for fire-fighting equipment where oil-break switchgear has been included, but with the provision of such facilities more elaborate housing of the switchgear would be necessary and it would seem justifiable to show some increased costs for civil works.

Mr. E. J. Hand (*at Birmingham*): The reliability of the auxiliary supply system is essential for continuity of generation and to safeguard the plant against damage should a fault develop. I suggest that we should not too slavishly follow the unit principle to its limits in the abolition of electrical interconnection. The unit system was introduced largely as an economic measure, but interconnection at station transformer voltage is essential for starting purposes and the extra cost of providing electrical interconnection at intermediate voltages has to be considered. It is probable that this cost would be no more than the cost of running out-of-merit plant for 2–3 days.

Another factor to which the subsidiary interconnection might well contribute is that the tendency nowadays is, at the time of annual survey, to get high-rated machines back into service in a matter of days rather than weeks, and interconnection at the subsidiary voltage can well assist this. It would also enable a better use of maintenance man-power to be made by spreading the work load.

Very careful consideration should be given to the relative importance of the unit and station transformer supplies. Several aspects enter into this consideration, which is mainly the safeguarding of the boiler and turbine in the event of operation of the main protection when the use of the unit transformer is lost.

With the larger units now becoming available, the water capacity of a modern boiler may well be less than 30 seconds on loss of feed water; boiler circulating pumps are being introduced and 'once through' boilers are under consideration. All the associated auxiliary drives, together with circulating water pumps, are essential for running as a unit load, but are equally if not more essential to safeguard the plant from damage in the event of the loss of the unit transformer. Does the author envisage any change in the principles he has laid down, having regard to these more onerous conditions, and does he envisage the use of high-speed automatic switching for selected auxiliaries?

In Section 5 the author refers to outside supplies for feeding the station transformers and mentions the attendant economies. I agree with him entirely, but query his reference to reduced reliability. Surely there is much to be gained by using a Grid supply for the station transformer feed which is entirely independent of the generator busbars?

There is little doubt that with large units, multi-voltage schemes will be necessary at both 11 kV and 3·3 kV. 110 volts is now becoming very widely used, both during construction and as a permanent supply, largely for safety reasons. Has the author made an assessment of the merits of introducing 550 and 110 volts, using standard equipment which is available in this country?

Fig. 1 shows a transformer with a very expensive 415-volt output switch. Will the author give the justification for the inclusion of this switch?

Referring to the analysis of loads, I suggest that a very severe restriction will be imposed on future 2-shift operation of power stations if the station transformer is designed on the basis of staggered starting.

Mr. W. J. Stevens (*at Birmingham*): Section 3 details essential features of unit transformer schemes. Does a unit have to continue running consequent on the loss of Grid supplies? Some interconnection is surely justified at intermediate levels with a 3-voltage auxiliary supply system. Regarding the practice of carrying spare transformers and switches in the stores, does not the cost of additional cabling justify making these items permanently available?

The author states that in general both extraction pumps should be connected to the unit switchboard, but I would suggest they be connected as are the feed pumps.

Operating staffs tend to ignore the auxiliary supply source connections in order to get equal life from plant ancillaries, i.e. to disregard standby and main plant items. Has the author

considered connection to an alternative alternator unit switchboard as a source of standby supplies or, alternatively, dispensing with unit transformers and utilizing station transformers fed from independent sources?

In Section 11 the running and starting load requirements for five 200 MW sets are quoted. Were these proved to be on the high side when the station was commissioned? The figure of 90% for the kVA rating for motors for load assessment purposes, particularly for fans, is high, 60–70% being a more correct figure.

Fig. 5 suggests an arrangement for two 550 MW sets with station transformers rated at 50 MVA. Why was it necessary to restrict the short-circuit levels of the 11 and 3·3 kV systems to 500 and 150 MVA respectively when it is stated that the maximum permissible voltage drop should be of the order of 20%? It may be necessary to consider increasing the supply voltage to enable a higher short-circuit level to be used economically in order to keep voltage drops on starting within permissible limits.

I was associated with some tests on boiler-feed-pump motors when we got approximately a 33% voltage drop on starting a motor group under the most severe possible conditions. Tests on these auxiliaries under design conditions gave an 18% voltage drop. Can voltage-drop problems be assisted by using slip-ring motors for boiler feed pumps, since this arrangement may readily assist the pump manufacturer's design problems?

What steps can be taken in the design stages to eliminate a large number of outages which are primarily due to human error failures?

Mr. G. B. Tully (*at Birmingham*): Is the use of oil-break switchgear only an economic consideration? I should have thought air-break switchgear had advantages in simplifying the maintenance and also it has less tendency to give rise to over-voltages due to current chopping. Have the last two advantages been outweighed by the extra cost of air-break switchgear in the initial stages?

Mr. G. S. Buckingham (*at Birmingham*): I am interested in the safe short-circuit currents which can be carried by mains cables. Fig. 1 shows a 6·6 kV switchboard rated at 350 MVA short-circuit rupturing capacity. Fed from this board are a number of cables supplying very small loads, such as 30 or 50 amp. The size of cable supplying these loads must be prescribed by the short-circuit rating and not by the load-carrying capacity. For example, a 3-core 0·2 in^2 cable capable of carrying 250 amp continuously would be overheated in 0·3 sec under short-circuit conditions. If the fault is allowed to run for 1 sec a 3-core 0·5 in^2 cable will be required. In all these cases a neutral earthing resistance is used, so that, if all faults could be limited to earth faults, cable sizes could be reduced to the size required to carry the load current only.

Has the author considered the possibility of segregating phase conductors, for example by the use of screened cables, so as to avoid the possibility of very onerous phase-to-phase faults?

Mr. A. W. Bowyer (*at Birmingham*): In Fig. 1 the 6·6 kV switchgear is rated at 350 MVA. With a unit and station transformer in parallel I estimate that the maximum fault contribution through the unit and station transformers is about 325 MVA. Further fault current contribution will come from induction motors. Motor fault current will die away in one or two cycles and will not therefore materially affect the required 6·6 kV switchgear breaking capacity. It could, however, affect the switchgear making or carrying capacity requirements quite appreciably. What allowances would the author make for induction-motor fault contribution when assessing breaker ratings?

Is there ever any question of a calculated risk being taken in view of the short time during which the unit and station transformers are connected in parallel?

Mr. D. A. Dewison (*in reply*): A number of speakers have raised similar questions, and these will be dealt with before replying to individual points.

Motor Infeed and Switchgear Rating.—With the electrical system outlined in the paper it is not necessary to take into account motor infeed for the making or breaking capacity of the switchgear. This is because under normal running conditions there is a considerable margin between the calculated system fault MVA and the MVA rating of the switchgear. These MVA values are, however, more nearly equal under starting conditions, when the station and unit transformers are paralleled. In these circumstances, however, the operation procedure is such that no switches are required to be 'made'; thus, motor infeed can be ignored for switchgear 'making'. There is a possibility of a fault occurring during the short paralleling period, and the breaking capacity of the switchgear is designed to deal with the system infeed, but not motor infeed. This is because for 3-phase faults the fault current from the motors will die away in two or three cycles, as mentioned by Mr. Bowyer, and the circuit-breaker will not have had time to open in this short period. It is evident that the switchgear must be capable of carrying the through current for these two or three cycles, and the switchgear rating would have to take this into account.

Interconnection at Lower Voltages.—The factors mentioned by Mr. Hand are important when deciding whether to build standby capacity into the system, and the cost of outage of a very large unit would seem to indicate that interconnections should be provided, as shown in Fig. 5.

Station Transformer Loading under Starting Conditions.—Hot starts take considerably less time than cold starts, and advantage could therefore be taken of the overload rating of the transformers if necessary. In the system described in the paper there should be no difficulty in starting four out of five sets under normal conditions.

Oil- versus Air-break Switchgear.—The question of air- or oil-break switchgear is a wide subject and the factors mentioned by Mr. Tully are significant. Generally speaking, air-break switch-gear is usually preferred where the cost differential is small.

The following comments are given in reply to individual questions.

To Mr. Pattinson.—The relationship of motor h.p. to voltage indicated in Section 9 is not strictly adhered to in all cases; thus, the factors mentioned by Mr. Pattinson can be taken into account. The stalling of induction motors under low-voltage conditions is a complex problem, and a paper on this subject is being considered.

To Mr. Morrison.—The factors mentioned with regard to boiler feed pumps certainly need taking into consideration when deciding on the number and ratings of pumps to be installed. The behaviour of lighting under low-voltage conditions is a further justification for designing the system so that the voltage does not fall below 80% under the worst conditions.

To Mr. Mitchell.—Voltage transformer earthing has the disadvantage that it is difficult to locate a fault on an individual circuit. Furthermore, there have been one or two instances where this system has given trouble due to maloperation. Solid- and resistance-earthing systems have given reasonably satisfactory service for a number of years and it would be difficult to justify a reversion to voltage-transformer earthing.

To Mr. Riach.—The question of circuit-breakers or isolators on various circuits is often related to the site location of the boards with respect to each other, and is concerned with safety for all conditions of operation and maintenance. Circuit-breakers are

usually preferred on circuits controlling urgent auxiliaries, but contactors have been used where technical or economic considerations dictate, and for the sizes quoted contactors would appear to be the most suitable. The ratings of transformers are dictated by technical and economic factors.

To Mr. Crawford.—The operation of two circuit-breakers in parallel on heavy-current circuits is not viewed with much favour, mainly because of the possibility of unequal distribution of current for various conditions of operation, and the complication of the control apparatus. It is agreed that separation of relays from the switchgear is desirable where possible. Single busbar layouts have proved to be adequate for the requirements of the auxiliary system.

To Mr. Hand.—The unit transformer system is well suited to the protection of the modern boiler, because failure of the unit supply ensures that the boiler fires are extinguished. The figure of 30 sec for water capacity seems rather low; a figure in the region of 60 sec has been more often used in recent designs.

550 volts is not a British Standard, and much discussion would be required between many branches of the electrical industry to effect a change; 110 volts does not seem attractive because larger copper section would be required for cables, etc., for a given power. The heavy-duty switch on the 415-volt system was chosen because the alternative of segregating the system into a number of circuits was more expensive and made the system more complicated.

To Mr. Stevens.—It is desirable to keep a unit running on loss of Grid supplies because the unit can be more rapidly restored to service on clearance of the fault. However, there are other factors, such as the protection of the plant, which need to be taken into account on large units. There does not appear to be much point in keeping an extraction pump running on the station supply when the unit has been lost owing to a fault on the unit supply.

The suggested alternative sources of supply have been considered, but were found to be either expensive or technically undesirable. The technical advantages of the unit transformer are given in Section 3(*c*).

The station for which the system shown in Fig. 1 was designed is not yet completed, although one set has been commissioned. The readings taken on this set in service indicate that the total unit load is approximately the same as the estimated load given in the paper, although there are slight differences for the individual auxiliaries. The system shown in Fig. 5 was found to be technically and economically the most suitable for this size of station. The ratings used keep the voltage drop to 20%. Slip-ring motors are being used for the boiler feed pumps on the larger units at present being designed.

The human error can be minimized by designing the system to be as simple as possible.

To Mr. Buckingham.—The rating of cables under fault conditions requires careful consideration, and a minimum size is often specified for circuits supplied from a board with a given MVA. On the unit system, the protection is usually designed to clear faults in approximately 0·2 sec.

The use of screened cables in the voltage ranges used on power station auxiliary systems has not been considered, but has to be reviewed now that 11 kV is being used.

DISCUSSION ON

'THE DESIGN OF ELECTRO-MECHANICAL AUXILIARIES DIRECTLY ASSOCIATED WITH POWER-PRODUCING REACTORS'*

WESTERN SUPPLY GROUP AT CARDIFF, 18TH JANUARY, 1960

Mr. P. E. Dickinson: The authors refer to the considerable efforts made to ensure the prevention of both defects in equipment which has been designed and mistakes by the operators concerned with this equipment. For example, they refer to an investigation into the probability of failure of a component which was estimated as being once in 10^5 years, and that although this degree of probability of failure was very remote, it has to be designed against. However, what efforts have been made to ensure that the design team are correct in their design outlook?

Messrs. A. E. Harwood, P. Scott and **B. H. Stonehouse** (*in reply*): The design work in a nuclear power station is carried out by two design groups on a co-operative basis. One group is responsible for determining the basic parameters in the performance of the station and the other is associated with the engineering design work required to make the station a practical reality. An interchange and cross-checking of each other's ideas naturally takes place between these two groups.

During the station construction the design is subject to checking by the customer, his consultant, and the Industrial Power Branch of the U.K.A.E.A. When the final form has been settled, the station safety report is prepared and submitted to the U.K.A.E.A. Health and Safety Branch. This document is subsequently examined by the Inspector of Nuclear Installations, and final approval to the running of the station is given by the Minister of Power.

The design work is checked by the production and proving of prototype units, by rigorous factory testing, and by extensive tests on site.

Thus, at every stage the design is subject to checking and scrutiny by a considerable number of engineers and every endeavour is made, by this means, to eliminate individual human failings.

* HARWOOD, A. E., SCOTT, P., and STONEHOUSE, B. H.: Paper No. 2812 S, December, 1958 (see **106** A, p. 262).

PAPERS AND MONOGRAPHS PUBLISHED INDIVIDUALLY

Summaries are given below of papers and monographs which have been published individually. The papers are free of charge; the price of the monographs is 2s. each (post free). Applications, quoting the serial numbers as well as the authors' names, and accompanied by a remittance where appropriate, should be addressed to the Secretary. For convenience, books of five vouchers, price 10s., can be supplied.

The Determination of the Electrical Characteristics of an Arc Furnace. Paper No. 3328 U.

J. Ravenscroft, B.Sc.

The paper describes three different methods for the determination of the electrical characteristics of arc furnaces. A comparison is made between the three methods by applying each in turn for the determination of the electrical characteristics of a 10 cwt Héroult-type 3-phase arc furnace.

In the first method the characteristics are obtained directly from observations on the furnaces during normal operation and under short-circuit conditions; in the second method the furnace curves are calculated by using an equivalent circuit for the 3-phase arc furnace; the third and most simple method treats the furnace as a balanced 3-phase load and utilizes a current-locus diagram to calculate operating points on the characteristics.

For the 10 cwt furnace the simple locus-diagram treatment gave results of sufficient accuracy for all practical purposes, and the characteristics show that the furnace is most efficient, electrically, when operated on the highest voltage tapping.

The Design and Performance of High-Precision Audio-Frequency Current Transformers. Paper No. 3296 M.

J. J. Hill, B.Sc., and A. P. Miller.

Current transformers of existing design are found to have large high-frequency errors. The causes of these errors are examined and it is shown that they are due to the high values of leakage inductance, self-capacitance and concentrated inter-winding capacitance that result from the conventional multi-layer form of construction. Greatly improved performance can be obtained if the windings are arranged in uniformly-distributed single layers with a suitable thickness of insulation between them. Approximate formulae for calculating the various constants and errors of a transformer having single-layer windings are given, together with a design procedure for any specified limit of error over a wide band of frequencies. The practical aspects of the design are also discussed.

Multi-ratio transformers in which all the ratios have virtually identical errors over a band of frequencies can be obtained by sub-dividing single-layer primary windings into sections, all of which are used by series-parallel connections for each ratio.

Details are given of the design and performance of precision single-layer multi-ratio transformers rated from 5/5 to 400/5 amp. Their errors do not exceed 5 parts in 10^5 in ratio and $0{\cdot}3'$ in phase over the frequency range 400 c/s–10 kc/s and are less than 2 parts in 10^4 and $1'$ over the range 50 c/s–30 kc/s.

Techniques for the Calibration of Standard Current Transformers up to 20 kc/s. Paper No. 3297 M.

J. J. Hill, B.Sc.

When standard current transformers are tested at high audio frequencies the errors and uncertainties in the measurements are liable to be great owing to the effects of stray magnetic fields on the large unshielded resistors which of necessity have to be used. The sources of stray fields are examined and means are suggested for substantially reducing them. It is shown that the 4-terminal resistance standards used in current-transformer testing bridges are very frequency-dependent and present techniques limit the accuracy of measurement at 20 kc/s to 2 parts in 10^4 in ratio and $0{\cdot}8'$ in phase.

New techniques are described by which the errors and uncertainties can be reduced to give an accuracy of 5 parts in 10^5 and $0{\cdot}1'$ at 20 kc/s.

The Design of an Audio-Frequency Amplifier for High-Precision Voltage Measurement. Paper No. 3335 M.

S. Harkness and F. J. Wilkins, B.A., B.Sc.

The specification and design of an amplifier required for precision audio-frequency measurements is discussed. Formulae are presented for the 'ring of three' feedback circuit and a detailed analysis is made of appropriate feedback theorems. A multi-gain amplifier which extends the voltage range of the electrostatic voltmeters, used at the N.P.L. as basic a.c./d.c. transfer instruments, is described. An increase in this range of up to 1 000 times is provided and enables voltages between 60 mV and 60 volts in the frequency range 30 c/s–30 kc/s to be measured to a few parts in 10^4.

An Analytical Review of Power-System Frequency, Time and Tie-Line Control. Monograph No. 395 S.

D. Broadbent, B.Sc., M.Eng.Sc., Ph.D., and K. N. Stanton, B.E.

Over the past decade and a half there have been proposed three schemes of control for electrical supply systems interconnected through tie lines. In chronological order they are the speed-governed system with frequency biasing, the time-governed system and the load-phase energy control.

This paper describes computer studies which compare the three systems using a performance index of the integral square error. Certain conclusions are drawn by applying this criterion both to the frequency error of the isolated system and to the tie power-flow error of the interconnected system.

Pole-Face Losses in Alternators. Monograph No. 404 S.

Prof. J. Greig, M.Sc.(Eng.), Ph.D., and K. Sathirakul, B.Sc.(Eng.), Ph.D.

A recent re-examination of the phenomenon of tooth-ripple flux pulsations at laminated pole-faces in alternators resulted in a new theoretical analysis by means of which the pole-face eddy-current loss can be calculated. The primary purpose of the present investigation has been to make measurements of tooth-ripple losses which could be compared with values calculated from the new formulae. In order to make these measurements an experimental homopolar machine was built and values of eddy-current losses were determined over a fairly wide range of flux density and frequency for four different thicknesses of lamination. It is found that there is moderately good agreement between calculation and experiment for the thicker laminations and in general the agreement is markedly better with the new formulae than with those previously developed.

The Physical Realization of Induction-Motor Equivalent Circuits. Monograph No. 406 U.

N. N. Hancock, B.Sc.(Eng.), M.Sc.Tech., and B. H. Karakaraddi, B.Sc., M.Sc.Tech.

The paper reviews the practicability of setting up physically, in the form of specialized networks, several of the equivalent circuits representing an unbalanced 2-phase induction motor. This motor is sufficiently general to include, as special cases, most induction machines with balanced secondary windings.

PROCEEDINGS OF THE INSTITUTION OF ELECTRICAL ENGINEERS

Part A. POWER ENGINEERING, OCTOBER 1960

CONTENTS

Published by The Institution, Savoy Place, London, W.C.2. *Telephone* COVent Garden 1871. *Telegrams:* 'Voltampere, Phone, London.'
Printed by Unwin Brothers Limited, Woking and London.